COMPREHENSIVE LESSON NOTEBOOK

AQA GCSE Mathematics

HIGHER TIER • 8300

UNIT 1 | NUMBER

HIGHER PURPOSE

Build exact number fluency for grades 4–9: justify classifications, manipulate fractional indices and surds, use bounds correctly, and solve multi-step numerical problems without premature rounding.

Specification map and Higher progression

Lesson	AQA	Higher focus
1	N1-N3	Number systems, ordering and calculation
2	N4	Prime factors, HCF and LCM
3	N5	Systematic listing and the product rule
4	N6-N7	Powers, roots and fractional indices
5	N8	Exact calculation with surds
6	N9	Standard form and calculator interpretation
7	N10	Fractions and recurring decimals
8	N11-N12	Fractions, percentages and reverse change
9	N13	Units, compound measures and dimensional thinking
10	N14-N15	Estimation, rounding and error intervals
11	N16	Upper and lower bounds in calculations

PROGRESSION PRINCIPLE

Higher Tier assesses all relevant Foundation knowledge as well as Higher-only material. Routine skills are therefore taught as prerequisites, then extended through exact calculation, proof, modelling and multi-step reasoning.

Formula and exact-method bank

Topic	Essential relationship
Indices	$a^m a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^{-n} = 1/a^n$
Fractional index	$a^{(m/n)} = \sqrt[n]{(a^m)}$
Standard form	$A \times 10^n$, $1 \le A < 10$
Percentage multiplier	$1 \pm r/100$
Error interval	rounded value ± half unit; upper endpoint excluded

Bounds quotient	UB = numerator UB ÷ denominator LB for positive data

LESSON 01 • N1-N3

Number systems, ordering and calculation

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N1-N3. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Number systems, ordering and calculation

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ **irrational numbers lie in $\mathbb{R}$ but not $\mathbb{Q}$**

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Natural, integer, rational and irrational numbers form nested sets. A rational number can be written as a/b where a and b are integers and b is not zero; its decimal terminates or recurs. Irrational decimals neither terminate nor recur.

To compare mixed forms, convert to a common exact form or use sufficiently accurate decimals. Preserve inequality direction unless multiplying or dividing both sides of an inequality by a negative number.

Use inverse operations and BIDMAS. A calculator line should match the intended mathematical structure: brackets protect a numerator, denominator or power.

Build the method

How the number sets fit together

Natural numbers count objects. Integers extend them to include zero and negatives. Rational numbers include every integer and every fraction of integers. Real numbers contain both rational and irrational numbers, so a value such as $\sqrt{7}$ belongs to the real set but not the rational set. A terminating decimal is rational because it can be written over a power of ten; a recurring decimal is rational because an algebraic subtraction turns it into a fraction.

Ordering without losing accuracy

Fractions, roots and recurring decimals should not be rounded too early. For 2/3, 0.67 and 0.6 recurring, write enough decimal places to see that $0.666... < 0.67$, while 0.6 recurring equals 0.666.... Negative values reverse the visual intuition: the number farther left on the number line is smaller, so $-0.75 < -0.70$.

Calculation structure

BIDMAS does not mean multiplication always precedes division; multiplication and division have equal priority and are processed left to right. The same is true of addition and subtraction. A fraction bar acts as a bracket around the whole numerator and denominator. On a calculator, enter explicit brackets rather than relying on the display to guess the intended structure.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Classify √49 and √7.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain √49 is rational (7); √7 is irrational..
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

√49 is rational (7); √7 is irrational.

Worked example 2

QUESTION

Order 2/3, 0.67 and 0.6 recurring.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 0.6 recurring, 2/3, 0.67.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

0.6 recurring, 2/3, 0.67

Guided Higher practice

GUIDED QUESTION 1

Evaluate 3 - 2(5 - 8).

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Explain why 0.1010010001... is irrational.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Classify √49 and √7.	4. Explain why 0.1010010001... is irrational.
2. Order 2/3, 0.67 and 0.6 recurring.	5. Insert <, > or = between -3/4 and -0.7.
3. Evaluate 3 - 2(5 - 8).	6. Find the reciprocal of -2 1/2.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 02 • N4

Prime factors, HCF and LCM

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N4. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Prime factors, HCF and LCM

$360 = 2^3 \times 3^2 \times 5$ $756 = 2^2 \times 3^3 \times 7$

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Every integer greater than 1 has a unique prime factorisation apart from order. Write repeated factors with indices, for example $360 = 2^3 \times 3^2 \times 5$.

For the HCF, choose each common prime with the smaller index. For the LCM, choose every prime appearing with the larger index. This explains rather than merely memorises the method.

In context, HCF usually makes the largest equal groups; LCM usually finds the first time repeating cycles coincide. Check which interpretation the wording requires.

Build the method

Why prime factorisation is powerful

The unique factorisation theorem says that every integer greater than 1 has one prime decomposition, apart from order. This creates a reliable “fingerprint” for the number. Divisibility, perfect powers, HCF and LCM can all be read from the indices in this fingerprint.

HCF and LCM from indices

Suppose one number contains 2^3 and another contains 2^2. A common factor can contain at most 2^2, so the HCF takes the smaller index. A common multiple must contain enough factors of 2 for both numbers, so the LCM takes the larger index. Apply this independently to every prime.

Choosing the model in context

“Largest equal pieces”, “greatest group size” and “maximum identical packs” signal an HCF model. “First together again”, “smallest common cycle” and “when will both repeat” signal an LCM model. State the numerical answer with the contextual unit.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Write 756 as a product of primes.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain $2^2 \times 3^3 \times 7$.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

$2^2 \times 3^3 \times 7$

Worked example 2

QUESTION

Find HCF(360, 756).

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 36.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

36

Guided Higher practice

GUIDED QUESTION 1

Find LCM(360, 756).

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Two lights flash every 18 s and 24 s. When together again?

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Write 756 as a product of primes.	4. Two lights flash every 18 s and 24 s. When together again?
2. Find HCF(360, 756).	5. Why is 1 not prime?
3. Find LCM(360, 756).	6. Prove √2 is not an integer using square numbers.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 03 • N5

Systematic listing and the product rule

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N5. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Systematic listing and the product rule

choices multiply across independent stages: m × n × p

Represent → calculate exactly → check magnitude → interpret

Understand the idea

A systematic list has a stated order so missing or duplicated outcomes can be detected. Tables and tree diagrams are useful when stages have different choices.

If one stage has m choices and a second independent stage has n choices, the product rule gives m × n possible ordered outcomes. Extend by multiplying the number of choices at every stage.

Restrictions change branch counts. Deal with restrictions explicitly—for example, after choosing one digit without repetition, one fewer digit remains.

Build the method

From lists to multiplication

For two shirts and three trousers, a complete tree has two first branches and three branches from each, giving 2×3 leaves. The product rule is therefore a compressed count of the leaves of a complete tree—not a separate trick.

Handling restrictions

Without repetition, the number of available choices changes after each selection. A three-letter code from five letters has 5 choices, then 4, then 3. If zero cannot be the first digit, separate the first position from later positions rather than using the same count everywhere.

Checking a counting argument

Ask whether order matters, whether repetition is permitted and whether any choices are forbidden. Two selections AB and BA are different for a code but may represent the same pair if order is irrelevant. A correct solution must match the definition of an outcome.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

4 starters and 7 mains: number of meals.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain 28.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

28

Worked example 2

QUESTION

Five-digit PIN using 0-9 with repetition.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 100000.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

100000

Guided Higher practice

GUIDED QUESTION 1

Three-letter code from A-E without repetition.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

How many even two-digit numbers from 1,2,3,4 without repetition?

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. 4 starters and 7 mains: number of meals.	4. How many even two-digit numbers from 1,2,3,4 without repetition?
2. Five-digit PIN using 0-9 with repetition.	5. Why is an ordered list safer than guessing?
3. Three-letter code from A-E without repetition.	6. A route has 3 buses then 2 trains, return independently. Number of round trips?

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 04 • N6-N7

Powers, roots and fractional indices

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N6-N7. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Powers, roots and fractional indices

$a^m a^n = a^{m+n}$ $a^{-n} = 1/a^n$ $a^{(1/n)} = \sqrt[n]{a}$

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Index laws follow from repeated multiplication: $a^m a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$. A negative index means reciprocal; $a^0=1$ for $a\neq0$.

A fractional index represents a root: $a^{(1/n)} = \sqrt[n]{a}$ and $a^{(m/n)} = \sqrt[n]{(a^m)}$. Choose an order that keeps arithmetic manageable.

For equations involving powers, compare common bases where possible. When estimating roots, locate the value between neighbouring powers and refine logically.

Build the method

Where the index laws come from

$a^3 \times a^4$ contains three factors of a followed by four more, hence a^7. Division cancels common factors, which explains subtraction of indices. Extending this pattern to $a^3 \div a^5$ gives $a^{-2}=1/a^2$; extending down to equal powers gives $a^0=1$.

Interpreting fractional powers

The denominator of a fractional index names the root and the numerator names the power. Thus $16^{(3/4)}$ can be read as (fourth root of $16)^3$. Root first when it creates an integer; power first is equivalent but can create unnecessarily large numbers.

Solving power equations

Rewrite both sides using the same base before equating exponents. For $5^{(x+1)}=125$, write $125=5^3$, so $x+1=3$. If common bases are not available at GCSE level, use systematic numerical reasoning or a graph rather than inventing an index rule.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Simplify $x^7 \div x^3$.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain x^4.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

x^4

Worked example 2

QUESTION

Evaluate 16^(-3/4).

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 1/8.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

1/8

Guided Higher practice

GUIDED QUESTION 1

Evaluate 27^(2/3).

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Solve 5^(x+1)=125.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Simplify $x^7 \div x^3$.	4. Solve 5^(x+1)=125.
2. Evaluate 16^(-3/4).	5. Between which integers is $\sqrt[3]{70}$?
3. Evaluate 27^(2/3).	6. Simplify $(a^3b^{-2})^2$.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 05 • N8

Exact calculation with surds

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N8. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Exact calculation with surds

$\sqrt{72} = 6\sqrt{2}$ $(a+\sqrt{b})(a-\sqrt{b})=a^2-b$

Represent → calculate exactly → check magnitude → interpret

Understand the idea

A surd is an irrational root kept in exact form. Simplify by extracting the largest square factor: $\sqrt{72}=\sqrt{(36\times2)}=6\sqrt{2}$.

Only like surds combine. Multiplication uses $\sqrt{a}\sqrt{b}=\sqrt{(ab)}$, but $\sqrt{a}+\sqrt{b}$ cannot generally be simplified. Expand brackets before collecting like terms.

Rationalising a denominator removes a surd from the denominator. For a single surd multiply by that surd; for a binomial use its conjugate so the difference of two squares becomes rational.

Build the method

What exact form means

The decimal for $\sqrt{2}$ never terminates or repeats, so a rounded decimal loses information. Keeping $\sqrt{2}$ preserves the exact value. Exact answers are essential in proofs, multi-stage geometry and questions that explicitly request an answer “in surd form”.

Simplifying and collecting

Search for the largest square factor: $\sqrt{180}=\sqrt{(36\times5)}=6\sqrt{5}$. This makes like surds visible. $3\sqrt{5}-\sqrt{20}$ becomes $3\sqrt{5}-2\sqrt{5}=\sqrt{5}$. By contrast $\sqrt{5}+\sqrt{3}$ has unlike irrational parts and cannot be combined.

Rationalising denominators

For $5/\sqrt{3}$, multiply top and bottom by $\sqrt{3}$, giving $5\sqrt{3}/3$. For $4/(3+\sqrt{5})$, multiply by the conjugate $3-\sqrt{5}$. The denominator becomes $9-5=4$, so the expression simplifies exactly to $3-\sqrt{5}$.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Simplify $\sqrt{180}$.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.

2 **Transform:** Apply the exact rule step by step to obtain $6\sqrt{5}$.

3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

$6\sqrt{5}$

Worked example 2

QUESTION

Simplify $3\sqrt{5}+7\sqrt{5}-\sqrt{20}$.

1 **Plan:** Identify every operation and preserve the correct order.

2 **Calculate:** The exact result is $8\sqrt{5}$.

3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

$8\sqrt{5}$

Guided Higher practice

GUIDED QUESTION 1

Expand $(\sqrt{3}+2)(\sqrt{3}-1)$.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Rationalise $5/\sqrt{3}$.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Simplify $\sqrt{180}$.	4. Rationalise $5/\sqrt{3}$.
2. Simplify $3\sqrt{5}+7\sqrt{5}-\sqrt{20}$.	5. Rationalise $4/(3+\sqrt{5})$.
3. Expand $(\sqrt{3}+2)(\sqrt{3}-1)$.	6. Show $(\sqrt{2}+1)^2$ is irrational.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 06 • N9

Standard form and calculator interpretation

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N9. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Standard form and calculator interpretation

$(3\times10^7)(4\times10^{-3})=12\times10^4=1.2\times10^5$

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Standard form is $A\times10^n$ where $1\le A<10$ and n is an integer. Moving the decimal left gives a positive power; moving it right gives a negative power.

For multiplication, multiply A-values and add indices. For division, divide A-values and subtract indices. Renormalise the coefficient at the end.

For addition and subtraction, first rewrite both numbers with the same power of ten. Interpret calculator E notation, and retain enough digits until the final rounding step.

Build the method

Place value behind standard form

The power of ten records scale. A positive exponent gives a value at least 10; a negative exponent gives a value between 0 and 1. The coefficient restriction $1\le A<10$ makes the representation unique.

Operations with standard form

When multiplying, treat coefficients and powers separately. If the coefficient becomes 12, rewrite it as 1.2×10 and increase the exponent by 1. For addition, exponents cannot simply be added: rewrite 8×10^5 as 0.8×10^6 before combining with 6.2×10^6.

Calculator and context checks

Calculator displays such as 1.43E-7 mean 1.43×10^{-7}. Estimate the exponent before accepting the display. In a contextual answer, include units and consider whether the result represents a count, in which case a whole-number decision may be necessary.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Write 0.0000725 in standard form.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain 7.25×10^{-5}.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

7.25×10^{-5}

Worked example 2

QUESTION

Calculate $(3\times10^{7})(4\times10^{-3})$.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 1.2×10^{5}.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

1.2×10^{5}

Guided Higher practice

GUIDED QUESTION 1

Calculate $(8\times10^{5})/(2\times10^{-4})$.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Calculate $6.2\times10^{6}+8\times10^{5}$.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Write 0.0000725 in standard form.	4. Calculate $6.2\times10^{6}+8\times10^{5}$.
2. Calculate $(3\times10^{7})(4\times10^{-3})$.	5. Interpret 1.43E-7.
3. Calculate $(8\times10^{5})/(2\times10^{-4})$.	6. A cell is 2.5×10^{-6} m. How many in 0.4 m?

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 07 • N10

Fractions and recurring decimals

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N10. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Fractions and recurring decimals

x=0.2727...   100x=27.2727...   99x=27

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Exact fraction arithmetic avoids premature rounding. Use a common denominator for addition/subtraction; multiply numerators and denominators; divide by multiplying by the reciprocal.

A recurring decimal is rational. Let x equal the decimal, multiply by a power of ten that shifts one full repeating block, subtract to cancel the recurring tail, then solve for x.

Distinguish 0.1 recurring from 0.01 recurring and from 0.10 recurring. Dot or bar notation must show exactly which digits repeat.

Build the method

Exact fraction arithmetic

A common denominator creates equal-sized parts before addition or subtraction. Cancelling before multiplication reduces arithmetic without changing value. Division by a fraction asks how many groups fit, which is why multiplying by the reciprocal works.

Recurring-decimal proof method

Let x=0.272727…. Because the repeating block has two digits, multiply by 100: 100x=27.272727…. Subtract the original equation to eliminate the infinite tail: 99x=27, so x=27/99=3/11.

Notation and validation

A dot or bar must mark the entire repeating block. Convert the resulting fraction back to a decimal as a check. If the original decimal is between 0 and 1, a final fraction greater than 1 exposes an algebra or place-value error.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Calculate 5/6-7/15.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain 11/30.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

11/30

Worked example 2

QUESTION

Convert 0.27 recurring (both digits) to a fraction.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 3/11.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

3/11

Guided Higher practice

GUIDED QUESTION 1

Convert 0.1 recurring to a fraction.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Convert 7/12 to a recurring decimal.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Calculate 5/6-7/15.	4. Convert 7/12 to a recurring decimal.
2. Convert 0.27 recurring (both digits) to a fraction.	5. Calculate 2 1/3 ÷ 1 3/4.
3. Convert 0.1 recurring to a fraction.	6. Explain why every recurring decimal is rational.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 08 • N11-N12

Fractions, percentages and reverse change

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N11-N12. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Fractions, percentages and reverse change

new value = original × multiplier

Represent → calculate exactly → check magnitude → interpret

Understand the idea

A fraction or percentage can act as an operator. Percentage change is change÷original×100%; always identify the original quantity in the denominator.

A multiplier combines the whole and the change: increase $r\%$ uses $1+r/100$; decrease uses $1-r/100$. Repeated change means repeated multiplication, not adding percentages.

For reverse percentages, divide by the multiplier because the known value is the result after change. This is different from simply adding back the stated percentage.

Build the method

Multipliers unify percentage change

An increase of 12% keeps the original 100% and adds 12%, so the multiplier is 112%=1.12. A decrease of 18% leaves 82%, giving 0.82. This single multiplication is safer than separately finding and adding or subtracting the change.

Reverse percentages

If £621 is the value after a 15% rise, then 621 represents 115% of the original. Divide by 1.15 to recover 100%. Subtracting 15% of £621 is wrong because that 15% would be calculated from the new value, not the original.

Repeated and compound change

Each new percentage acts on the latest amount, so multiply once per period. A rise of 20% followed by a fall of 20% gives $1.2\times0.8=0.96$, a 4% overall decrease. Equal percentage changes in opposite directions do not cancel.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Increase £480 by 12%.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain £537.60.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

£537.60

Worked example 2

QUESTION

Decrease 850 by 18%.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 697.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

697

Guided Higher practice

GUIDED QUESTION 1

After a 15% rise a price is £621. Original?

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

£900 depreciates 8% yearly for 3 years.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Increase £480 by 12%.	4. £900 depreciates 8% yearly for 3 years.
2. Decrease 850 by 18%.	5. A value rises 20% then falls 20%. Overall change?
3. After a 15% rise a price is £621. Original?	6. Express a 35% profit as multiplier on cost.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 09 • N13

Units, compound measures and dimensional thinking

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N13. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Units, compound measures and dimensional thinking

speed=distance/time **density=mass/volume**

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Convert units before substituting. Squared and cubed conversions must also be squared or cubed: 1 m^2=10,000 cm^2 and 1 m^3=1,000,000 cm^3.

Compound measures combine quantities, such as speed=distance/time, density=mass/volume and pressure=force/area. Rearrange the relationship before substituting when the unknown is not the subject.

Dimensional checks expose errors: a speed answer needs distance per time; density needs mass per volume. Use consistent units and state them in the final answer.

Build the method

Scale factors in two and three dimensions

One metre is 100 centimetres. A square metre contains 100 rows of 100 square centimetres, so the area factor is 100^2. A cubic metre contains 100×100×100 cubic centimetres, so the volume factor is 100^3.

Building compound measures

Write the relationship before inserting numbers. Convert 2 h 24 min to 2.4 h, not 2.24 h, because 24 minutes is 24/60=0.4 hours. For density, convert kilograms to grams if volume is in cm^3 and the requested unit is g/cm^3.

Rearranging and checking dimensions

From density=mass/volume, multiplication gives mass=density×volume. Units provide a check: $(g/cm^3)\times cm^3=g$. If units do not cancel to the required final unit, revisit the conversion or rearrangement.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Convert 3.6 m^2 to cm^2.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.

2 **Transform:** Apply the exact rule step by step to obtain 36000 cm^2.

3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

36000 cm^2

Worked example 2

QUESTION

A car travels 156 km in 2 h 24 min. Average speed?

1 **Plan:** Identify every operation and preserve the correct order.

2 **Calculate:** The exact result is 65 km/h.

3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

65 km/h

Guided Higher practice

GUIDED QUESTION 1

Mass 1.62 kg, volume 600 cm^3. Density in g/cm^3?

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Find mass when density=7.8 g/cm^3 and volume=45 cm^3.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Convert 3.6 m^2 to cm^2.	4. Find mass when density=7.8 g/cm^3 and volume=45 cm^3.
2. A car travels 156 km in 2 h 24 min. Average speed?	5. Convert 72 km/h to m/s.
3. Mass 1.62 kg, volume 600 cm^3. Density in g/cm^3?	6. Why is 1 m^3 not 100 cm^3?

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 10 • N14-N15

Estimation, rounding and error intervals

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N14-N15. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Estimation, rounding and error intervals

rounded value ± half the rounding unit

Represent → calculate exactly → check magnitude → interpret

Understand the idea

An estimate replaces values with convenient nearby numbers and must use the same operation structure. State approximations and compare the result with the calculator answer for reasonableness.

Decimal places count digits after the decimal point; significant figures begin at the first non-zero digit. Context may require rounding up, down or to the nearest whole unit.

If x rounds to a to the nearest unit u, then $a-u/2 \le x < a+u/2$. The lower bound is included; the upper bound is excluded because it would round to the next value.

Build the method

Estimation as a reasoning tool

An estimate should make every number convenient while preserving scale. In $(19.8\times0.503)/4.96$, use $(20\times0.5)/5=2$. This confirms that a calculator answer near 2 is sensible and exposes a misplaced decimal.

Choosing an accuracy

Decimal places control a fixed place-value position; significant figures control the number of meaningful digits from the first non-zero digit. Context can override ordinary rounding: 410 travellers in 52-seat buses requires 8 buses because 7.88 buses is impossible.

Constructing an error interval

Rounding to the nearest 5 places boundaries 2.5 either side. Therefore 275 represents values from 272.5 inclusive up to 277.5 exclusive. A value of exactly 277.5 would round to 280, which explains the strict upper inequality.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

Estimate (19.8×0.503)/4.96.

1 **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2 **Transform:** Apply the exact rule step by step to obtain about 2.
3 **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

about 2

Worked example 2

QUESTION

Round 0.004786 to 2 significant figures.

1 **Plan:** Identify every operation and preserve the correct order.
2 **Calculate:** The exact result is 0.0048.
3 **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

0.0048

Guided Higher practice

GUIDED QUESTION 1

275 correct to nearest 5: write error interval.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

A bus seats 52 and 410 travel. How many buses?

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. Estimate (19.8×0.503)/4.96.	4. A bus seats 52 and 410 travel. How many buses?
2. Round 0.004786 to 2 significant figures.	5. Explain why intermediate rounding is risky.
3. 275 correct to nearest 5: write error interval.	6. Truncate 8.946 to 2 decimal places.

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

LESSON 11 • N16

Upper and lower bounds in calculations

Higher reasoning • exact method • proof • interpretation

Specification focus

AQA references N16. This lesson combines prerequisite fluency with Higher-tier exact calculation, justification and multi-step application.

Upper and lower bounds in calculations

quotient UB = numerator UB ÷ denominator LB

Represent → calculate exactly → check magnitude → interpret

Understand the idea

Bounds produce the greatest and least possible result consistent with rounded inputs. Label every input LB or UB before choosing substitutions.

For positive quantities: sum UB uses UBs; difference UB uses first UB minus second LB; product UB uses UBs; quotient UB uses numerator UB divided by denominator LB. Reverse choices for the lower bound.

A bound is not normally attained at an excluded upper endpoint, but examination answers report the limiting value. Keep full precision and round only as instructed.

Build the method

Thinking about the extreme result

Bounds questions are optimisation problems. Do not automatically insert all upper bounds. Decide which change makes the expression larger or smaller. For positive a/b, making a larger increases the result while making b larger decreases it.

Operation-by-operation choices

For a-b, the greatest result uses the greatest a and smallest b. For a product of positive measurements, the greatest product uses both upper bounds. If negative values are possible, test endpoint combinations because the simple positive rules may fail.

Precision and interpretation

Keep the bound calculation unrounded until the end. A calculated upper bound is a limiting value even if an input upper endpoint is excluded. State the requested accuracy and unit, and distinguish an upper bound from an ordinary rounded estimate.

EXPLAIN IT ALOUD

Say what each symbol or value represents, why the chosen rule is valid, and how you know the final form is exact, simplified and reasonable.

HIGHER EXAMINER HABIT

Write exact forms during working, show the transformation used, and round only the final answer unless the question explicitly requests an approximation.

COMMON TRAP

A familiar-looking rule may not apply: addition does not combine indices, unlike surds do not collect, repeated percentages do not add, and bounds require deliberate LB/UB choices.

Worked examples

Worked example 1

QUESTION

$a=8.2$ nearest 0.1. State bounds.

1. **Represent:** Write definitions, factorisation, multiplier or bounds explicitly.
2. **Transform:** Apply the exact rule step by step to obtain $8.15 \le a < 8.25$.
3. **Check:** Test sign, size, units and whether the form is fully simplified.

ANSWER

$8.15 \le a < 8.25$

Worked example 2

QUESTION

$b=3.4$ nearest 0.1. Upper bound of $a+b$.

1. **Plan:** Identify every operation and preserve the correct order.
2. **Calculate:** The exact result is 11.70.
3. **Justify:** Name the rule used and explain why the result fits the conditions.

ANSWER

11.70

Guided Higher practice

GUIDED QUESTION 1

Upper bound of a-b.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

GUIDED QUESTION 2

Upper bound of a/b.

STRATEGIC HINT

Represent the structure first. Keep exact values, show the key transformation and include a checking statement.

Independent and reasoning practice

Questions 1–3	Questions 4–6
1. a=8.2 nearest 0.1. State bounds.	4. Upper bound of a/b.
2. b=3.4 nearest 0.1. Upper bound of a+b.	5. Rectangle 12.4 cm by 7.8 cm, each nearest 0.1. Upper area.
3. Upper bound of a-b.	6. Why use denominator LB for quotient UB?

GRADE 7–9 REASONING

Create a second method or counterexample, identify the assumption behind the method, and explain how an apparently plausible error changes the result.

Complete answer and reasoning guide

Lesson 1: Number systems, ordering and calculation

1. √49 is rational (7); √7 is irrational.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 0.6 recurring, 2/3, 0.67

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 9

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. Its digits do not terminate or repeat in a fixed block.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. $-3/4 < -0.7$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. -2/5

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 2: Prime factors, HCF and LCM

1. $2^2 \times 3^3 \times 7$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 36

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 7560

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 72 seconds

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. A prime has exactly two positive factors; 1 has only one.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. No integer square lies between 1 and 4, while 2 lies between them.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 3: Systematic listing and the product rule

1. 28

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 100000

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. $5\times4\times3 = 60$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 6

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. It makes omissions and duplicates visible.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. $(3\times2)^2 = 36$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 4: Powers, roots and fractional indices

1. x^4

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 1/8

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 9

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. $x=2$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. 4 and 5

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. a^6/b^4

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 5: Exact calculation with surds

1. $6\sqrt{5}$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. $8\sqrt{5}$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. $1+\sqrt{3}$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. $5\sqrt{3}/3$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. $3-\sqrt{5}$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. $3+2\sqrt{2}$, which contains non-zero irrational part $2\sqrt{2}$.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 6: Standard form and calculator interpretation

1. 7.25×10^{-5}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 1.2×10^{5}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 4×10^{9}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 7×10^{6}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. 1.43×10^{-7}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. 1.6×10^{5}

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 7: Fractions and recurring decimals

1. 11/30

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 3/11

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 1/9

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 0.58 recurring on 3 only

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. 4/3

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. Subtraction of shifted copies produces a ratio of integers.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 8: Fractions, percentages and reverse change

1. £537.60

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 697

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. £540

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. £700.77 (nearest penny)

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. 4% decrease

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. 1.35

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 9: Units, compound measures and dimensional thinking

1. 36000 cm^2

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 65 km/h

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 2.7 g/cm^3

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 351 g

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. 20 m/s

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. Length scale factor 100 is applied in three dimensions: 100^3.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 10: Estimation, rounding and error intervals

1. about 2

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 0.0048

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. $272.5 \le x < 277.5$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. 8

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. Rounding errors can accumulate and change the final rounded answer.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. 8.94

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Lesson 11: Upper and lower bounds in calculations

1. $8.15 \le a < 8.25$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

2. 11.70

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

3. 4.90

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

4. $8.25/3.35 \approx 2.4627$

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

5. $12.45 \times 7.85 = 97.7325$ cm^2

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

6. A smaller positive denominator makes the quotient larger.

Check: show the governing rule, preserve exact values through intermediate stages, and verify magnitude, sign, units or bounds as appropriate.

Unit 1 mastery record

AQA	I can demonstrate...	Secure?
N1-N3	Number systems, ordering and calculation	☐
N4	Prime factors, HCF and LCM	☐
N5	Systematic listing and the product rule	☐
N6-N7	Powers, roots and fractional indices	☐
N8	Exact calculation with surds	☐
N9	Standard form and calculator interpretation	☐
N10	Fractions and recurring decimals	☐
N11-N12	Fractions, percentages and reverse change	☐
N13	Units, compound measures and dimensional thinking	☐
N14-N15	Estimation, rounding and error intervals	☐
N16	Upper and lower bounds in calculations	☐

Official specification source

AQA GCSE Mathematics (8300), Subject Content 3.1 Number: https://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300/specification/subject-content/3.1-number

Independently written and not endorsed by AQA or any other examination board. Explanations, examples and diagrams are original and do not reproduce the supplied commercial guide or past-paper questions.