COMPREHENSIVE LESSON NOTEBOOK

AQA GCSE Mathematics

HIGHER TIER • 8300

UNIT 2 | ALGEBRA

HIGHER PURPOSE

Develop algebra as a connected language: manipulate exactly, prove generally, interpret functions and graphs, solve non-linear systems and model unfamiliar situations.

Specification map and progression

Lesson	AQA	Coverage
1	A1-A3	Algebraic language, substitution and identities
2	A4	Expanding, factorising and index manipulation
3	A4	Algebraic fractions and advanced manipulation
4	A5-A7	Rearranging formulae, proof and functions
5	A8-A10	Straight-line graphs, gradients and equations
6	A11-A12	Quadratic, cubic, reciprocal and exponential graphs
7	A12-A13	Trigonometric graphs and function transformations
8	A14-A15	Real-life graphs, gradients and areas under curves
9	A16	Circle equations and tangents
10	A17-A18	Linear and quadratic equations
11	A19-A20	Simultaneous equations and iteration
12	A21	Forming equations and algebraic modelling
13	A22	Linear, quadratic and graphical inequalities
14	A23-A24	Recursive, arithmetic, geometric and special sequences
15	A25	Linear and quadratic nth terms

HIGHER BOUNDARY

Includes all prerequisite algebra plus Higher-only algebraic fractions, proof, composite/inverse functions, perpendicular lines, transformations, circle tangents, non-linear rates, linear-quadratic systems, iteration and quadratic inequalities.

Formula and method bank

Structure	Key method
Line	$y=mx+c$; $m=(y_2-y_1)/(x_2-x_1)$
Quadratic	$x=(-b\pm\sqrt{(b^2-4ac)})/(2a)$
Circle at origin	$x^2+y^2=r^2$; radius $\perp$ tangent
Composite function	$fg(x)=f(g(x))$
Quadratic sequence	second difference=2a
Iteration	$x_{n+1}=g(x_n)$; check convergence

LESSON 01 • A1-A3

Algebraic language, substitution and identities

Higher reasoning • representation • method • proof

Specification focus

AQA references A1-A3. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Algebraic language, substitution and identities

expression → equation → identity

Represent → transform → justify → check

Understand the idea

Algebra describes a structure that remains true when numbers change. A term is a product of numbers and variables; a coefficient is its numerical multiplier; an expression has no equality sign; an equation asserts equality for particular value(s).

Powers record repeated factors: x^2 means $x\times x$, while $2x$ means $2\times x$. A fraction bar groups its entire numerator and denominator. Write algebra in conventional order, with numerical coefficients before variables.

Substitution replaces every occurrence of a variable. Insert negative and fractional values with brackets: if $x=-3$, then $x^2=(-3)^2=9$, but $-x^2$ means $-(x^2)=-9$. Follow BIDMAS after substitution.

Build the method

An identity is true for every permitted value, written with $\equiv$; an equation is true only for its solution(s). An expression such as $3(x+2)\equiv 3x+6$ is an identity because expansion proves equivalence.

Scientific and unfamiliar formulae are substitution tasks. Match each value to its symbol, convert units first, preserve calculator brackets, and state the requested accuracy.

Higher questions often ask which statement is always, sometimes or never true. Test examples to explore, then use algebra to justify rather than relying on numerical evidence alone.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Simplify $x \times x \times y \times 3$.

1 **Represent:** Write the algebraic structure and any restrictions.

2 **Transform:** Apply one justified step at a time to obtain $3x^2y$.

3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$3x^2y$

Worked example 2

QUESTION

Evaluate $2a^2-3b$ for $a=-2, b=5$.

1 **Choose:** Select an algebraic, graphical or numerical route.

2 **Execute:** The result is -7.

3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

-7

Guided Higher practice

GUIDED QUESTION 1

Classify $3x+2=17$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Show $4(2x-3)\equiv 8x-12$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Simplify $x\times x\times y\times 3$.	4. Show $4(2x-3)\equiv 8x-12$.
2. Evaluate $2a^2-3b$ for $a=-2,b=5$.	5. Evaluate $(p+q)/(p-q)$ for $p=7,q=3$.
3. Classify $3x+2=17$.	6. Explain why $x/x=1$ needs a restriction.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 02 • A4

Expanding, factorising and index manipulation

Higher reasoning • representation • method • proof

Specification focus

AQA references A4. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Expanding, factorising and index manipulation

expand ⇄ factorise

Represent → transform → justify → check

Understand the idea

Collect only like terms: they must have identical variable parts and powers. $3x^2$ and $5x^2$ combine; $3x^2$ and $5x$ do not. Think of the variable part as the unit being counted.

Expansion uses the distributive law. Every term outside a bracket multiplies every term inside. For two or more binomials, expand systematically and collect only after all products are visible.

Factorising reverses expansion. First remove the greatest common factor. For x^2+bx+c, find two values with product c and sum b. For ax^2+bx+c, split or use a structured product method.

Build the method

Difference of two squares follows $(u+v)(u-v)=u^2-v^2$. It applies only to subtraction of two perfect squares; x^2+25 is not a difference of squares over the real numbers.

Algebraic index laws match numerical index laws because variables represent numbers. Multiplication adds indices, division subtracts them, and a power of a power multiplies indices.

Check factorisation by expanding. This reverse check is fast, detects sign errors and is especially important when the leading coefficient is not 1.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Expand $3x(2x-5)$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $6x^2-15x$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$6x^2-15x$

Worked example 2

QUESTION

Expand $(x+4)(x-7)$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $x^2-3x-28$.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$x^2-3x-28$

Guided Higher practice

GUIDED QUESTION 1

Factorise $x^2-3x-28$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Factorise $6x^2+x-2$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Expand $3x(2x-5)$.	4. Factorise $6x^2+x-2$.
2. Expand $(x+4)(x-7)$.	5. Factorise $25x^2-16$.
3. Factorise $x^2-3x-28$.	6. Simplify $(3x^2y^{-1})^2$.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 03 • A4

Algebraic fractions and advanced manipulation

Higher reasoning • representation • method • proof

Specification focus

AQA references A4. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Algebraic fractions and advanced manipulation

factor first • restrict • cancel

Represent → transform → justify → check

Understand the idea

An algebraic fraction behaves like a numerical fraction, but its denominator creates restrictions. Factor before cancelling: only common factors cancel, never separate terms joined by addition.

For addition and subtraction, use a common denominator. Multiply every term in a numerator by the required factor and keep brackets around a subtracted numerator.

For multiplication, factor all numerators and denominators, state excluded values and cancel common factors. Division means multiply by the reciprocal.

Build the method

A complex fraction can be simplified by multiplying top and bottom by the lowest common denominator of the smaller fractions. This removes fractions without changing value.

Surds may occur inside algebraic expressions. Expand, collect like surds and rationalise denominators using a single surd or conjugate as appropriate.

A cancelled factor still creates an excluded value in the original expression. Simplification changes the appearance, not the original domain.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Simplify $(x^2-9)/(x^2+5x+6)$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $(x-3)/(x+2)$, $x\neq3,-2$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$(x-3)/(x+2)$, $x\neq3,-2$

Worked example 2

QUESTION

Add $2/x + 3/(x+1)$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $(5x+2)/(x(x+1))$.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$(5x+2)/(x(x+1))$

Guided Higher practice

GUIDED QUESTION 1

Simplify $(x^2-4)/(x^2-2x)$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Divide $x/(x-1)$ by $2x/(x^2-1)$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Simplify $(x^2-9)/(x^2+5x+6)$.	4. Divide $x/(x-1)$ by $2x/(x^2-1)$.
2. Add $2/x + 3/(x+1)$.	5. State restrictions for $(x^2-1)/(x-1)$.
3. Simplify $(x^2-4)/(x^2-2x)$.	6. Rationalise $2/(\sqrt{x}+1)$.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 04 • A5-A7

Rearranging formulae, proof and functions

Higher reasoning • representation • method • proof

Specification focus

AQA references A5-A7. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Rearranging formulae, proof and functions

input → function → output → inverse

Represent → transform → justify → check

Understand the idea

Changing the subject isolates a chosen variable while preserving equality. Treat the formula as an equation and perform inverse operations to both sides. Undo outer operations first.

When the subject appears more than once, collect its terms on one side and factorise. When it is squared, isolate the square before taking a root; include ± only when solving for all possible values and the context permits both.

Algebraic proof begins with a general representation: consecutive integers n and $n+1$; even $2n$; odd $2n+1$; a multiple of k as kn. Manipulate to reach the required form.

Build the method

A function maps each permitted input to one output. $f(3)$ means substitute 3 into f, not multiply f by 3. Composite $fg(x)$ means $f(g(x))$: the right-hand function acts first.

An inverse function reverses every operation in reverse order. Write $y=f(x)$, swap x and y, then rearrange for y. Restrict the domain when the original function is not one-to-one.

Proof must cover every permitted case. Checking several numbers can suggest a pattern but cannot establish a universal claim. A single valid counterexample can disprove a universal claim.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Make x subject: $y=(3x-5)/2$.

1 **Represent:** Write the algebraic structure and any restrictions.

2 **Transform:** Apply one justified step at a time to obtain $x=(2y+5)/3$.

3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$x=(2y+5)/3$

Worked example 2

QUESTION

Make r subject: $A=\pi r^2$.

1 **Choose:** Select an algebraic, graphical or numerical route.

2 **Execute:** The result is $r=\sqrt{(A/\pi)}$ for $r\geq 0$.

3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$r=\sqrt{(A/\pi)}$ for $r\geq 0$

Guided Higher practice

GUIDED QUESTION 1

Prove sum of consecutive integers is odd.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

$f(x)=3x-2$; find $f(5)$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Make x subject: $y=(3x-5)/2$.	4. $f(x)=3x-2$; find $f(5)$.
2. Make r subject: $A=\pi r^2$.	5. $g(x)=x^2+1$; find $fg(2)$.
3. Prove sum of consecutive integers is odd.	6. Find inverse of $f(x)=3x-2$.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

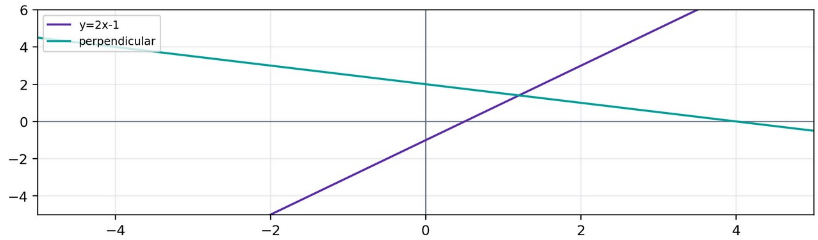
LESSON 05 • A8-A10

Straight-line graphs, gradients and equations

Higher reasoning • representation • method • proof

Specification focus

AQA references A8-A10. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

A coordinate (x,y) records horizontal movement first and vertical movement second. A straight line has constant gradient: change in y divided by change in x.

In y=mx+c, m is the gradient and c is the y-intercept. A positive m rises left to right; a negative m falls; m=0 is horizontal.

Through two points, $m=(y_2-y_1)/(x_2-x_1)$. Substitute one point into y=mx+c to find c, or use $y-y_1=m(x-x_1)$.

Build the method

Parallel lines have equal gradients. Non-vertical perpendicular lines have gradients whose product is -1, so the perpendicular gradient is the negative reciprocal.

Vertical lines have equation x=k and undefined gradient. Horizontal lines have equation y=k and zero gradient; these need separate treatment from y=mx+c.

Check a proposed line by substituting both given points and checking the gradient condition. A sketch reveals impossible signs or intercepts.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Find gradient through (2,5),(6,13).

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 2.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

2

Worked example 2

QUESTION

Find equation with gradient 3 through (2,-1).

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is y=3x-7.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

y=3x-7

Guided Higher practice

GUIDED QUESTION 1

Line parallel to y=-2x+7 through (4,1).

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Perpendicular gradient to 3/4.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Find gradient through (2,5),(6,13).	4. Perpendicular gradient to 3/4.
2. Find equation with gradient 3 through (2,-1).	5. Equation of vertical line through (-2,5).
3. Line parallel to y=-2x+7 through (4,1).	6. Find x-intercept of y=3x-12.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

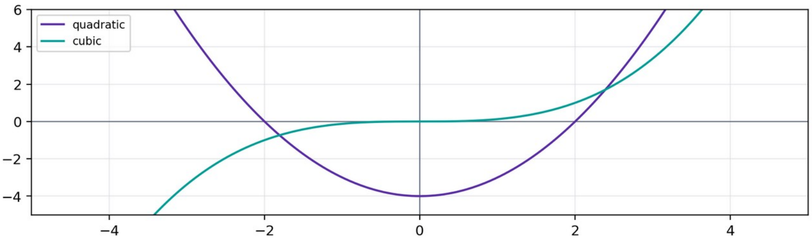
LESSON 06 • A11-A12

Quadratic, cubic, reciprocal and exponential graphs

Higher reasoning • representation • method • proof

Specification focus

AQA references A11-A12. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

A quadratic graph is a parabola. Roots are x-values where $y=0$; the y-intercept occurs at $x=0$; the turning point is the maximum or minimum. The axis of symmetry passes through the turning point.

Factorisation reveals roots. Completing the square reveals the turning point: $x^2+6x+5=(x+3)^2-4$, so the minimum is $(-3,-4)$.

A positive leading coefficient opens upward; a negative one opens downward. Changing the constant translates the graph vertically, while repeated roots make the graph touch the axis.

Build the method

Cubic graphs may have one or three real roots and have opposite end behaviour. The reciprocal graph $y=1/x$ has asymptotes $x=0$ and $y=0$ and is undefined at $x=0$.

For $y=k^x$ with $k>1$, the graph passes through $(0,1)$, stays positive and grows increasingly rapidly. For $0<k<1$ it decays but remains positive.

A sketch should label intercepts, roots, turning points and asymptotes that can be deduced. Do not draw a reciprocal curve through an asymptote.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Find roots of $y=x^2-5x+6$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 2 and 3.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

2 and 3

Worked example 2

QUESTION

Turning point of $y=x^2+6x+5$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is (-3,-4).
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

(-3,-4)

Guided Higher practice

GUIDED QUESTION 1

State y-intercept of $y=-2x^2+3x+4$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Why is $y=1/x$ undefined at $x=0$?

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Find roots of $y=x^2-5x+6$.	4. Why is $y=1/x$ undefined at $x=0$?
2. Turning point of $y=x^2+6x+5$.	5. Point on $y=2^x$ when $x=0$.
3. State y-intercept of $y=-2x^2+3x+4$.	6. Describe end behaviour of $y=x^3$.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

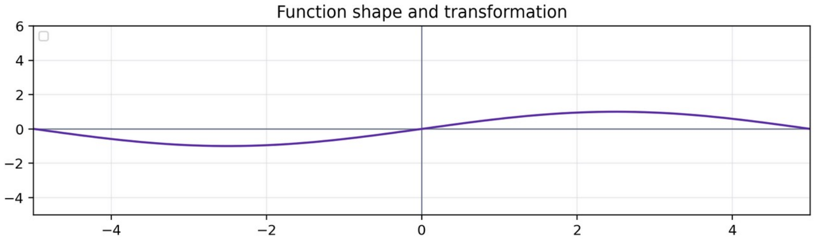
LESSON 07 • A12-A13

Trigonometric graphs and function transformations

Higher reasoning • representation • method • proof

Specification focus

AQA references A12-A13. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

Sine and cosine repeat every 360°; tangent repeats every 180°. In degrees, sin 0=0, cos 0=1 and tan 0=0. Key points determine the sketch between cycles.

The tangent graph has vertical asymptotes at 90°+180°n because cos x=0 there. Curves approach but never cross these asymptotes.

For y=f(x)+a, outputs increase by a: a vertical translation. For y=f(x-a), the same output occurs at an x-value a larger, so the graph translates right by a.

Build the method

The transformation y=-f(x) reflects in the x-axis; y=f(-x) reflects in the y-axis. Distinguish changing outputs from changing inputs.

A point-mapping method is reliable. If (p,q) lies on y=f(x), then (p,q+a) lies on y=f(x)+a and (-p,q) lies on y=f(-x).

Describe transformations completely: type, direction and magnitude. For reflections, name the axis. Use brackets around transformed inputs.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Period of sin x.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 360°.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

360°

Worked example 2

QUESTION

Solve cos $x=0$ for $0 \leq x \leq 360$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is 90°,270°.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

90°,270°

Guided Higher practice

GUIDED QUESTION 1

Translate $y=f(x)$ right 4.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Reflect $y=f(x)$ in x-axis.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Period of sin x.	4. Reflect y=f(x) in x-axis.
2. Solve cos x=0 for $0 \le x \le 360$.	5. Map (2,5) under y=f(x)+3.
3. Translate y=f(x) right 4.	6. Map (2,5) under y=f(-x).

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

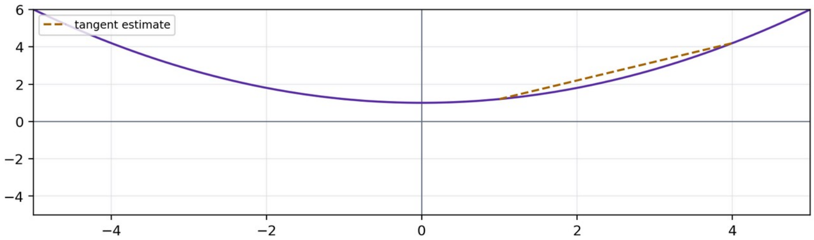
LESSON 08 • A14-A15

Real-life graphs, gradients and areas under curves

Higher reasoning • representation • method • proof

Specification focus

AQA references A14-A15. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

A graph in context has units. Gradient means change in vertical quantity per horizontal quantity: on distance-time it is speed; on velocity-time it is acceleration.

For a curve, draw a tangent at the required point and calculate its gradient using a large triangle. The result is an estimate and must include units.

Area under a velocity-time graph gives displacement because velocity×time has distance units. Areas below the time axis are negative displacement. Total distance uses absolute areas.

Build the method

For non-linear sections, estimate area with trapezia or counting squares. Narrower intervals generally improve the estimate, although the direction of error depends on curvature.

A horizontal distance-time section means stationary; a horizontal velocity-time section means constant velocity. A negative gradient on distance-time would indicate returning toward the reference point, not negative speed.

Interpret the numerical result in the story. State whether it is exact or estimated, include units and avoid reading beyond the plotted domain without justification.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Distance rises 120 m in 15 s: speed.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 8 m/s.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

8 m/s

Worked example 2

QUESTION

Velocity rises 4 to 10 m/s in 3 s: acceleration.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is 2 m/s^2.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

2 m/s^2

Guided Higher practice

GUIDED QUESTION 1

Area under constant 8 m/s for 5 s.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Why use a large tangent triangle?

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Distance rises 120 m in 15 s: speed.	4. Why use a large tangent triangle?
2. Velocity rises 4 to 10 m/s in 3 s: acceleration.	5. Meaning of negative velocity.
3. Area under constant 8 m/s for 5 s.	6. Difference between distance and displacement.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

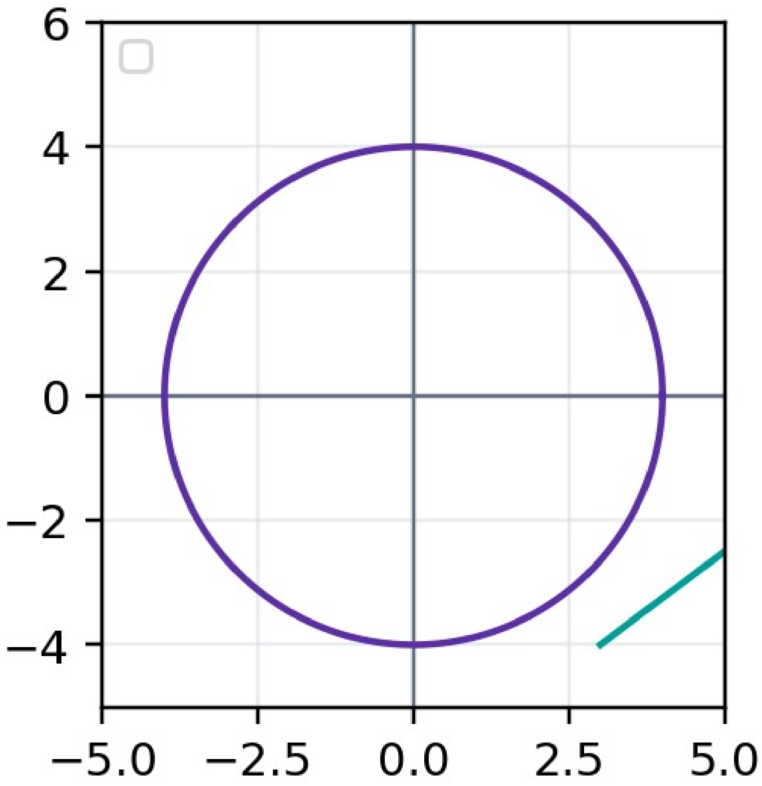
LESSON 09 • A16

Circle equations and tangents

Higher reasoning • representation • method • proof

Specification focus

AQA references A16. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

The circle with centre (0,0) and radius r has equation $x^2+y^2=r^2$, derived from Pythagoras applied to the radius from the origin to (x,y).

A point lies on the circle only if its coordinates satisfy the equation. The intercepts are (±r,0) and (0,±r).

The radius to a point on a circle is perpendicular to the tangent there. Find the radius gradient, take its negative reciprocal, then use the point to find the tangent equation.

Build the method

If the radius is vertical, the tangent is horizontal; if the radius is horizontal, the tangent is vertical. Handle these without dividing by zero.

The equation $x^2+y^2=25$ represents a complete circle, not a function y=f(x), because most x-values correspond to two y-values.

Check tangency by confirming the point lies on the circle and the product of the two finite gradients is -1.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Radius of $x^2+y^2=49$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 7.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

7

Worked example 2

QUESTION

Does (3,4) lie on $x^2+y^2=25$?

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is Yes: 9+16=25..
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

Yes: 9+16=25.

Guided Higher practice

GUIDED QUESTION 1

Radius gradient to (3,4).

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Tangent gradient at (3,4).

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Radius of $x^2+y^2=49$.	4. Tangent gradient at (3,4).
2. Does (3,4) lie on $x^2+y^2=25$?	5. Tangent equation at (3,4).
3. Radius gradient to (3,4).	6. Tangent at (0,5).

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 10 • A17-A18

Linear and quadratic equations

Higher reasoning • representation • method • proof

Specification focus

AQA references A17-A18. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Linear and quadratic equations

difference → coefficient → nth term

Represent → transform → justify → check

Understand the idea

Solving an equation preserves equality. Expand brackets, clear fractions carefully, collect variable terms and use inverse operations. Check by substitution into the original equation.

Quadratics must equal zero before factorising. The zero-product rule then gives a solution from each factor. Some quadratics do not factorise conveniently.

Completing the square rewrites ax^2+bx+c in vertex form and can solve equations or reveal turning points. If $a\neq1$, factor it from the quadratic terms first.

Build the method

The quadratic formula solves $ax^2+bx+c=0$. Identify a,b,c with signs, calculate the discriminant b^2-4ac and use brackets in the calculator.

The discriminant predicts roots: positive gives two real roots, zero one repeated root, negative no real roots. Exact surd answers may be required.

Graphical roots are approximate x-intercepts. Algebraic and graphical methods should agree within plotting accuracy. Reject solutions only for stated domain or contextual reasons.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Solve $5x-7=2x+11$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $x=6$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$x=6$

Worked example 2

QUESTION

Solve $x^2-7x+12=0$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $x=3,4$.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$x=3,4$

Guided Higher practice

GUIDED QUESTION 1

Solve $x^2+6x+2=0$ by completing square.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Solve $2x^2+3x-4=0$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Solve $5x-7=2x+11$.	4. Solve $2x^2+3x-4=0$.
2. Solve $x^2-7x+12=0$.	5. Number of real roots of x^2+4x+4.
3. Solve $x^2+6x+2=0$ by completing square.	6. Why set a quadratic equal to zero?

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 11 • A19-A20

Simultaneous equations and iteration

Higher reasoning • representation • method • proof

Specification focus

AQA references A19-A20. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Simultaneous equations and iteration

difference → coefficient → nth term

Represent → transform → justify → check

Understand the idea

Simultaneous solutions satisfy both equations at once. Elimination combines equations to remove one variable; substitution replaces one variable with an equivalent expression.

Scale equations before elimination so coefficients match. Add when coefficients are opposites; subtract when they are equal. Substitute back and verify in both original equations.

For a linear and quadratic pair, substitute the linear expression into the quadratic. The resulting quadratic may give two intersection points, one tangent point or none.

Build the method

Iteration rewrites an equation as $x=g(x)$ and repeatedly calculates $x_{n+1}=g(x_n)$. Record enough decimal places to see convergence and follow the requested starting value.

Convergence is not automatic. A sequence that oscillates or grows may require a different rearrangement. A fixed point satisfies $x=g(x)$.

To justify a root to a stated interval, combine iteration with a sign-change check or show consecutive iterates settle to the required accuracy.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Solve $x+y=9$, $x-y=3$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $x=6, y=3$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$x=6, y=3$

Worked example 2

QUESTION

Solve $2x+3y=13$, $x+y=5$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $x=2, y=3$.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$x=2, y=3$

Guided Higher practice

GUIDED QUESTION 1

Intersections $y=x+1$ and $y=x^2-3$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Use $x_{n+1}=\sqrt{(10-x_n)}$, $x_1=3$: find x_2.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Solve $x+y=9$, $x-y=3$.	4. Use $x_{n+1}=\sqrt{(10-x_n)}$, $x_1=3$: find x_2.
2. Solve $2x+3y=13$, $x+y=5$.	5. What is a fixed point?
3. Intersections $y=x+1$ and $y=x^2-3$.	6. Why verify simultaneous answers?

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 12 • A21

Forming equations and algebraic modelling

Higher reasoning • representation • method • proof

Specification focus

AQA references A21. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Forming equations and algebraic modelling

difference → coefficient → nth term

Represent → transform → justify → check

Understand the idea

Define the unknown before forming an equation. Translate each quantity from the context into an expression using consistent units.

Equality comes from two descriptions of the same quantity: equal lengths, total cost, perimeter, area, time or number of objects. Do not begin solving until the equation represents the story.

Geometry may create a quadratic equation. Draw and label a diagram, use exact expressions for dimensions, then apply the relevant geometric relationship.

Build the method

A context may require two variables and two equations. Choose variables with units, form independent equations and solve simultaneously.

Interpret solutions. Reject negative lengths, non-integer counts or values outside a stated interval, but explain the contextual reason rather than silently discarding them.

Finally substitute the accepted value into the original context, not only the rearranged equation, and answer the actual question with units.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Rectangle length $x+3$,width x, area 40: equation.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $x(x+3)=40$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$x(x+3)=40$

Worked example 2

QUESTION

Solve that model.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $x=5$ (reject -8).
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$x=5$ (reject -8)

Guided Higher practice

GUIDED QUESTION 1

Three adult and two child tickets cost £46; adult £12. Child price?

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Consecutive integers product 72. Form equation.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Rectangle length x+3,width x, area 40: equation.	4. Consecutive integers product 72. Form equation.
2. Solve that model.	5. Why reject some algebraic roots?
3. Three adult and two child tickets cost £46; adult £12. Child price?	6. Perimeter of sides x,x+2,x+5 is 34. Find x.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

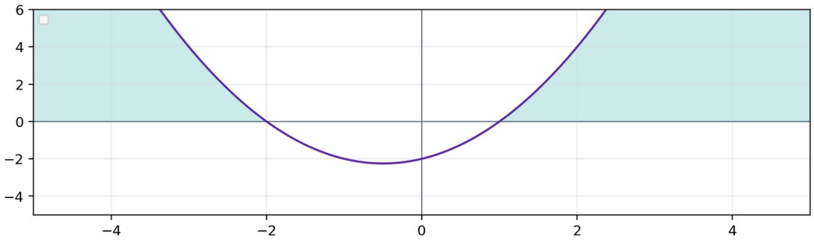
LESSON 13 • A22

Linear, quadratic and graphical inequalities

Higher reasoning • representation • method • proof

Specification focus

AQA references A22. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.



Understand the idea

An inequality describes a set of values. Solve a linear inequality like an equation, but multiplying or dividing by a negative reverses the inequality because number-line order reverses.

Represent strict boundaries with open circles and included boundaries with closed circles. Interval or set notation must match the symbols.

For a quadratic inequality, move everything to one side, find critical roots and use a sign diagram or sketch. The sign can change only at roots.

Build the method

For two-variable inequalities, draw the boundary. Use a solid line for ≤ or ≥ and dashed for < or >. Test a point to determine the correct half-plane.

A feasible region satisfies every inequality, so shade or identify the intersection of all permitted half-planes. Check vertices against all constraints.

Do not infer the solution region from the direction of a curved graph without testing intervals. State whether endpoints are included.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Solve $3x-5>10$.

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $x>5$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$x>5$

Worked example 2

QUESTION

Solve $-2x\leq 8$.

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is $x\geq -4$.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

$x\geq -4$

Guided Higher practice

GUIDED QUESTION 1

Solve $(x-2)(x+5)>0$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Boundary for $y<2x+1$.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Solve $3x-5>10$.	4. Boundary for $y<2x+1$.
2. Solve $-2x\leq 8$.	5. Does (0,0) satisfy $y\geq x-3$?
3. Solve $(x-2)(x+5)>0$.	6. Why reverse sign after dividing by -2?

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 14 • A23-A24

Recursive, arithmetic, geometric and special sequences

Higher reasoning • representation • method • proof

Specification focus

AQA references A23-A24. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Recursive, arithmetic, geometric and special sequences

difference → coefficient → nth term

Represent → transform → justify → check

Understand the idea

A position-to-term rule gives any term directly from n. A term-to-term rule generates each term from previous terms and needs a starting value.

An arithmetic sequence has constant first difference and nth term dn+(first term-d). A geometric sequence has constant multiplier and can be expressed using powers of its ratio.

Fibonacci-type sequences define each term from earlier terms. Read the recursion and initial values carefully; different starting terms create different sequences.

Build the method

Square, cube and triangular numbers should be recognised. Their difference patterns help identify the type of sequence and connect diagrams to algebra.

A geometric ratio may be fractional or surd at Higher tier. Negative ratios alternate signs. Check that the proposed formula produces the given first few terms.

Recursive processes can converge, diverge, oscillate or cycle. Describe observed behaviour cautiously; a few terms alone do not prove the long-term result.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

Next terms: 5,9,13,…

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain 17,21.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

17,21

Worked example 2

QUESTION

nth term of 5,9,13,…

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is 4n+1.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

4n+1

Guided Higher practice

GUIDED QUESTION 1

Next two: 3,6,12,…

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Rule $u_{n+1}=u_n+u_{n-1}$, $u_1=1,u_2=3$: u_4?

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Next terms: 5,9,13,...	4. Rule $u_{n+1}=u_n+u_{n-1}$, $u_1=1,u_2=3$: u_4?
2. nth term of 5,9,13,...	5. Is 49 square, cube or neither?
3. Next two: 3,6,12,...	6. Sequence with first 2 ratio √2: third term.

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

LESSON 15 • A25

Linear and quadratic nth terms

Higher reasoning • representation • method • proof

Specification focus

AQA references A25. The lesson develops accurate notation, a justified method, graphical or algebraic interpretation and Higher-tier reasoning.

Linear and quadratic nth terms

difference → coefficient → nth term

Represent → transform → justify → check

Understand the idea

For a linear sequence, constant first difference d gives dn plus an adjustment. Substitute n=1 to find the adjustment or compare dn with the actual terms.

A quadratic sequence has constant second difference. If nth term is an^2+bn+c, the constant second difference is 2a, so a is half the second difference.

Subtract an^2 from the original sequence. The remainder is linear, so find bn+c using first differences or simultaneous equations.

Build the method

Verify the formula with at least three positions. A formula matching only the first two terms is not enough to establish the quadratic pattern.

To find whether a value appears, set the nth-term expression equal to that value and solve. Accept only positive integer n.

Diagram sequences often produce quadratic rules. Count a stable core and the parts added at stage n, then simplify and compare with the difference method.

EXPLAIN IT ALOUD

Name the structure, state why the operation preserves equivalence, and connect the symbolic result to a graph, domain or context.

COMMON TRAP

Do not apply a remembered procedure without checking its conditions. Preserve brackets, restrictions, inequality direction, domains and exact form.

Worked examples

Worked example 1

QUESTION

nth term of 7,12,17,...

1 **Represent:** Write the algebraic structure and any restrictions.
2 **Transform:** Apply one justified step at a time to obtain $5n+2$.
3 **Verify:** Reverse the operation, substitute, expand or read the graph to check.

ANSWER

$5n+2$

Worked example 2

QUESTION

nth term of 3,8,15,24,...

1 **Choose:** Select an algebraic, graphical or numerical route.
2 **Execute:** The result is n^2+2n.
3 **Interpret:** State roots, coordinates, units, domain or contextual meaning as required.

ANSWER

n^2+2n

Guided Higher practice

GUIDED QUESTION 1

Find 10th term of n^2+2n.

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

GUIDED QUESTION 2

Is 80 in sequence n^2+3n?

STRATEGIC HINT

Expose the structure first, maintain exact values and include a line that checks or interprets the result.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. nth term of 7,12,17,...	4. Is 80 in sequence n^2+3n?
2. nth term of 3,8,15,24,...	5. Second difference 6: coefficient of n^2?
3. Find 10th term of n^2+2n.	6. Why require positive integer n?

REASONING EXTENSION

Compare a second method, identify a necessary restriction or assumption, and explain how the symbolic and graphical representations agree.

Complete answer and reasoning guide

Lesson 1: Algebraic language, substitution and identities

1. $3x^2y$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. -7

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. equation

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. Expansion gives $8x-12$ on both sides.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. 5/2

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. x cannot be 0 because division by zero is undefined.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 2: Expanding, factorising and index manipulation

1. $6x^2-15x$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $x^2-3x-28$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $(x-7)(x+4)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. $(3x+2)(2x-1)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. $(5x-4)(5x+4)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. $9x^4/y^2$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 3: Algebraic fractions and advanced manipulation

1. $(x-3)/(x+2)$, $x\neq 3,-2$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $(5x+2)/(x(x+1))$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $(x+2)/x$, $x\neq 0,2$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. $(x+1)/(2x)$, $x\neq -1,0,1$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. $x\neq 1$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. $2(\sqrt{x}-1)/(x-1)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 4: Rearranging formulae, proof and functions

1. $x=(2y+5)/3$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $r=\sqrt{(A/\pi)}$ for $r\geq 0$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $n+(n+1)=2n+1$, an odd integer.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. 13

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. $f(5)=13$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. $f^{-1}(x)=(x+2)/3$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 5: Straight-line graphs, gradients and equations

1. 2

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. y=3x-7

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. y=-2x+9

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. -4/3

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. x=-2

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. (4,0)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 6: Quadratic, cubic, reciprocal and exponential graphs

1. 2 and 3

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. (-3,-4)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. (0,4)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. Division by zero is undefined.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. (0,1)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 7: Trigonometric graphs and function transformations

1. 360°

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. 90°,270°

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $y=f(x-4)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. $y=-f(x)$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. (2,8)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. (-2,5)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 8: Real-life graphs, gradients and areas under curves

1. 8 m/s

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. 2 m/s^2

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. 40 m

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. It reduces proportional reading error.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. Motion in the chosen negative direction.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. Distance is total path length; displacement includes direction.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 9: Circle equations and tangents

1. 7

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. Yes: 9+16=25.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. 4/3

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. -3/4

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. y=-3x/4+25/4

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. y=5

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 10: Linear and quadratic equations

1. x=6

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. x=3,4

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. x=-3±√7

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. (-3±√41)/4

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. one repeated root

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. The zero-product rule applies to a product equal to zero.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 11: Simultaneous equations and iteration

1. $x=6, y=3$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $x=2, y=3$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $x=(1\pm\sqrt{17})/2$ with $y=x+1$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. $\sqrt{7}\approx 2.646$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. A value satisfying $x=g(x)$.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. An algebra or substitution error may satisfy only one equation.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 12: Forming equations and algebraic modelling

1. $x(x+3)=40$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $x=5$ (reject -8)

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. £5

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. $n(n+1)=72$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. They may violate physical, integer or domain constraints.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. $x=9$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 13: Linear, quadratic and graphical inequalities

1. $x>5$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $x\geq-4$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. $x<-5$ or $x>2$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. dashed line $y=2x+1$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. Yes

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. Division by a negative reverses order on the number line.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 14: Recursive, arithmetic, geometric and special sequences

1. 17,21

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. $4n+1$

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. 24,48

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. 7

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. square

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. 4

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Lesson 15: Linear and quadratic nth terms

1. 5n+2

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

2. n^2+2n

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

3. 120

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

4. Yes, n=8

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

5. 3

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

6. n represents a term position.

Check the governing rule, preserve domain restrictions and exact form, then verify by substitution, expansion, a graph feature or contextual interpretation.

Unit 2 mastery record

AQA	I can demonstrate...	Secure?
A1-A3	Algebraic language, substitution and identities	☐
A4	Expanding, factorising and index manipulation	☐
A4	Algebraic fractions and advanced manipulation	☐
A5-A7	Rearranging formulae, proof and functions	☐
A8-A10	Straight-line graphs, gradients and equations	☐
A11-A12	Quadratic, cubic, reciprocal and exponential graphs	☐
A12-A13	Trigonometric graphs and function transformations	☐
A14-A15	Real-life graphs, gradients and areas under curves	☐
A16	Circle equations and tangents	☐
A17-A18	Linear and quadratic equations	☐
A19-A20	Simultaneous equations and iteration	☐
A21	Forming equations and algebraic modelling	☐
A22	Linear, quadratic and graphical inequalities	☐
A23-A24	Recursive, arithmetic, geometric and special sequences	☐
A25	Linear and quadratic nth terms	☐

Official specification source

AQA GCSE Mathematics (8300), Subject Content 3.2 Algebra: https://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300/specification/subject-content/3.2-algebra

Independently written and not endorsed by AQA or any other examination board. All explanations, examples and diagrams are original and do not reproduce the supplied commercial guide or past-paper questions.