COMPREHENSIVE LESSON NOTEBOOK

AQA GCSE Mathematics

HIGHER TIER • 8300

UNIT 5 | PROBABILITY

HIGHER PURPOSE

Build probability from explicit models, justify addition and multiplication, represent dependence accurately and calculate conditional probabilities from tables, trees and Venn diagrams.

Specification map and progression

Lesson	AQA	Coverage
1	P1-P3	Experiments, theoretical probability and expected frequency
2	P5	Relative frequency, sample size and convergence
3	P4	Exhaustive events, complements and addition rules
4	P6-P7	Systematic counting and possibility spaces
5	P6	Venn diagrams, set notation and algebra
6	P8	Independent events and multiplication
7	P8	Dependent events and probability without replacement
8	P9	Conditional probability with tables and frequency trees
9	P9	Conditional probability with Venn and tree diagrams
10	P2-P9	Modelling assumptions and multi-stage reasoning

HIGHER PROGRESSION

Includes all P1-P8 prerequisite content and the Higher-only P9 conditional-probability requirement, with modelling assumptions and reverse conditional reasoning.

Probability method bank

Structure	Relationship
Complement	$P(A')=1-P(A)$
General union	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
Independent intersection	$P(A\cap B)=P(A)P(B)$
Conditional	$P(A\mid B)=P(A\cap B)/P(B)$
Expected frequency	probability×number of trials
Tree	multiply along; add disjoint routes

LESSON 01 • P1-P3

Experiments, theoretical probability and expected frequency

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P1-P3. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.

Experiments, theoretical probability and expected frequency

sample space → event → probability → interpretation

check exhaustive • mutually exclusive • equally likely • independent

Understand the idea

Probability models long-run uncertainty on a scale from 0 to 1. Zero means impossible within the model; one means certain; neither statement predicts an isolated random outcome.

Theoretical probability comes from a justified model, often equally likely outcomes. Experimental probability or relative frequency is observed frequency divided by number of trials.

Expected frequency is probability×number of trials. It is a modelled long-run count, not a promise that the experiment will produce exactly that value.

Fairness means specified outcomes have equal long-run chances. Physical symmetry can support fairness, but an experimental check cannot prove perfect fairness.

Build the method

A frequency tree preserves counts: child branches total their parent. Find missing frequencies by subtraction before forming probabilities.

Probabilities may be fractions, decimals or percentages. Keep exact fractions during combined calculations, then convert only if requested.

Compare experimental and theoretical values by both direction and size of difference, while considering sample size.

State assumptions: random trials, stable conditions and accurate recording. Without them, disagreement may reflect bias rather than chance.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

54 successes in 150: relative frequency.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 0.36.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

0.36

Worked example 2

QUESTION

P(success)=0.36 in 800 trials: expected frequency.

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 288.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

288

Guided Higher practice

GUIDED QUESTION 1

Fair die P(number>4).

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Frequency tree parent 120, branches 47 and ?.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. 54 successes in 150: relative frequency.	4. Frequency tree parent 120, branches 47 and ?.
2. P(success)=0.36 in 800 trials: expected frequency.	5. Can expected frequency be non-integer?
3. Fair die P(number>4).	6. Why does fair not mean alternating outcomes?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

LESSON 02 • P5

Relative frequency, sample size and convergence

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P5. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.

Relative frequency, sample size and convergence

sample space → event → probability → interpretation

check exhaustive • mutually exclusive • equally likely • independent

Understand the idea

Relative frequency changes as new trials are added. Early values can fluctuate widely because each result forms a large fraction of a small total.

For a stable unbiased process, relative frequency tends to settle near theoretical probability as trial count grows. It need not move closer after every trial.

A larger sample reduces random sampling variation but does not remove systematic bias, faulty equipment or changing conditions.

Two studies should not be compared only by raw success counts; compare relative frequencies and their sample sizes.

Build the method

Use an earlier experiment to estimate future counts only when the future population and procedure are sufficiently similar.

An unusual run, such as ten heads, is possible in a fair process. Evidence of bias comes from sustained discrepancy over suitable samples, not one striking pattern.

When no theoretical model is known, a large relevant experiment can supply an empirical estimate, with acknowledged uncertainty.

A strong conclusion uses cautious language: suggests, estimates, consistent with—not proves.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

18 successes in 50.

1 **Represent:** Draw or define the sample space, set, table or tree.

2 **Calculate:** Use the justified probability rule to obtain 0.36.

3 **Interpret:** Check $0 \leq P \leq 1$ and state the event and assumptions.

ANSWER

0.36

Worked example 2

QUESTION

Estimate in next 600.

1 **Restrict:** Identify the correct route or conditional denominator.

2 **Execute:** The result is 216.

3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

216

Guided Higher practice

GUIDED QUESTION 1

Which more reliable: 9/20 or 450/1000?

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Theory 0.4, observed 398/1000: compare.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. 18 successes in 50.	4. Theory 0.4, observed 398/1000: compare.
2. Estimate in next 600.	5. Does 10 heads prove bias?
3. Which more reliable: 9/20 or 450/1000?	6. Why can larger biased sample mislead?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

LESSON 03 • P4

Exhaustive events, complements and addition rules

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P4. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.

Exhaustive events, complements and addition rules

sample space → event → probability → interpretation

check exhaustive • mutually exclusive • equally likely • independent

Understand the idea

A sample space is exhaustive when it contains every possible outcome. Probabilities across an exhaustive set sum to 1.

Mutually exclusive events cannot occur together. For such events, P(A or B)=P(A)+P(B).

The complement A′ means not A, so P(A′)=1-P(A). Complements are automatically mutually exclusive and exhaustive.

Overlapping events cannot be added directly because the intersection would be counted twice. The general rule is $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

Build the method

“Or” in probability is normally inclusive: A or B or both. Read context carefully if exclusive wording is intended.

An impossible intersection does not mean events are independent. Mutually exclusive non-zero events are dependent because occurrence of one prevents the other.

Use bounds as a check: every answer lies from 0 to 1, a union is at least as large as each component, and an intersection is no larger than either.

Draw a Venn diagram when overlap is uncertain; it makes the subtraction term visible.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

$P(A)=0.63$. Find $P(A')$.

1. **Represent:** Draw or define the sample space, set, table or tree.
2. **Calculate:** Use the justified probability rule to obtain 0.37.
3. **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

0.37

Worked example 2

QUESTION

$P(A)=0.55, P(B)=0.42, P(A \cap B)=0.18$. Union.

1. **Restrict:** Identify the correct route or conditional denominator.
2. **Execute:** The result is 0.79.
3. **Verify:** Use complements, margins or total leaf probability.

ANSWER

0.79

Guided Higher practice

GUIDED QUESTION 1

Mutually exclusive 0.28,0.31. Union.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Can non-zero mutually exclusive events be independent?

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. $P(A)=0.63$. Find $P(A')$.	4. Can non-zero mutually exclusive events be independent?
2. $P(A)=0.55, P(B)=0.42, P(A\cap B)=0.18$. Union.	5. If $P(A\cup B)=0.8$ and intersection 0.2, with $P(A)=0.5$, $P(B)$?
3. Mutually exclusive 0.28,0.31. Union.	6. Why subtract intersection?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

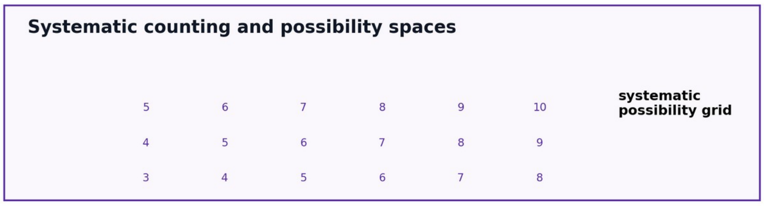
LESSON 04 • P6-P7

Systematic counting and possibility spaces

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P6-P7. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

A possibility space lists every combined outcome. Probability by counting is valid only when listed outcomes are equally likely.

A Cartesian grid is effective for two numerical experiments. Ordered pairs distinguish first and second results, so (2,5) and (5,2) are different unless the problem defines unordered selections.

The product rule multiplies choices across stages. Restrictions such as no repetition reduce later branch counts and must be represented explicitly.

A systematic list needs a fixed order. This provides evidence that nothing is missed or duplicated, which is part of the reasoning mark.

Build the method

If elementary outcomes are not equally likely, attach probabilities rather than counting cells. Equal-looking sectors or branches may hide unequal chance.

Events such as a sum, product or difference are calculated by marking all qualifying cells, including boundary values.

Use symmetry as a check: with two fair dice, sums 2 and 12 each have one route; sum 7 has six.

For without-replacement selection, combinations may be counted directly only if each combination is genuinely equally likely under the procedure.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

Two dice: number outcomes.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 36.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

36

Worked example 2

QUESTION

P(sum 7).

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 1/6.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

1/6

Guided Higher practice

GUIDED QUESTION 1

Two coins sample space.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

3-letter code from 5 letters no repeat.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Two dice: number outcomes.	4. 3-letter code from 5 letters no repeat.
2. P(sum 7).	5. Why not count colours on unequal spinner?
3. Two coins sample space.	6. P(product even) for two fair dice.

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

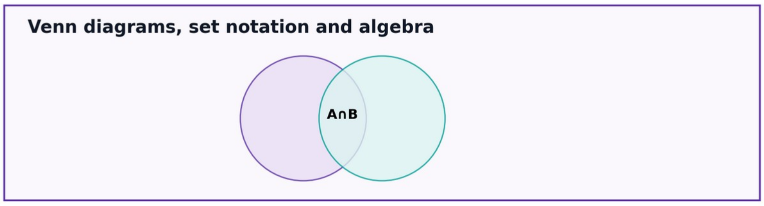
LESSON 05 • P6

Venn diagrams, set notation and algebra

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P6. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

A Venn diagram partitions a universal set into disjoint regions. Fill the intersection first, then exclusive parts, then the outside region.

A∩B means both; A∪B means in A or B or both; A′ means outside A. The universal set ξ contains every object under discussion.

Set totals overlap. Therefore n(A∪B)=n(A)+n(B)-n(A∩B). This is the counting version of the general addition rule.

Three-set diagrams require starting at the central triple intersection, then pairwise-only regions, then single-set-only regions.

Build the method

Conditional statements restrict the denominator to a region. Among A, the probability of B is n(A∩B)/n(A).

Unknown region values can be represented algebraically. Use set totals and the universal total to form equations, then reject negative or impossible values.

“Neither” means outside the union, not merely outside the intersection. “Exactly one” means exclusive single-set regions only.

Check every region is non-negative and totals reconstruct each given set and the universal set.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

30 students: $n(A)=18, n(B)=12$, both$=5$. A only.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 13.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

13

Worked example 2

QUESTION

B only.

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 7.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

7

Guided Higher practice

GUIDED QUESTION 1

Union.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Neither if total 34.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. 30 students: $n(A)=18, n(B)=12$, both=5. A only.	4. Neither if total 34.
2. B only.	5. $P(B\mid A)$.
3. Union.	6. Exactly one of A,B.

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

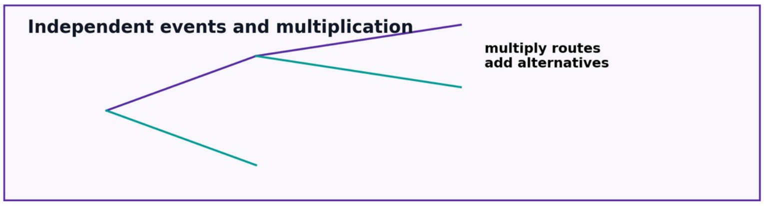
LESSON 06 • P8

Independent events and multiplication

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P8. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

Independent events do not change one another's probabilities: $P(B|A)=P(B)$. With replacement often creates independence, provided draws are random.

For independent events, $P(A\cap B)=P(A)P(B)$. Multiplication represents travelling along a route where all stated events occur.

Alternative complete routes that cannot occur together are added. Thus exactly one success in two trials is success-failure plus failure-success.

A tree diagram writes branch probabilities at each node. Branches from a node sum to 1; route probabilities are products.

Build the method

Repeated independent trials can be handled with trees, systematic counting or binomial reasoning when appropriate, but every arrangement must be counted.

At least one is often easiest by complement: 1-P(no successes). This avoids listing many separate successful routes.

Independence is an assumption to justify, not something inferred from a diagram. Replacement alone is insufficient if selection is not random.

Check that final mutually exclusive route probabilities sum to the probability of the target event and never exceed 1.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

$P(A)=0.3, P(B)=0.5$ independent. Both.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 0.15.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

0.15

Worked example 2

QUESTION

Two fair coins exactly one head.

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 1/2.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

1/2

Guided Higher practice

GUIDED QUESTION 1

Three trials $p=0.2$: no successes.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

At least one success.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. $P(A)=0.3, P(B)=0.5$ independent. Both.	4. At least one success.
2. Two fair coins exactly one head.	5. Exactly two successes in three trials.
3. Three trials $p=0.2$: no successes.	6. How show replacement on tree?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

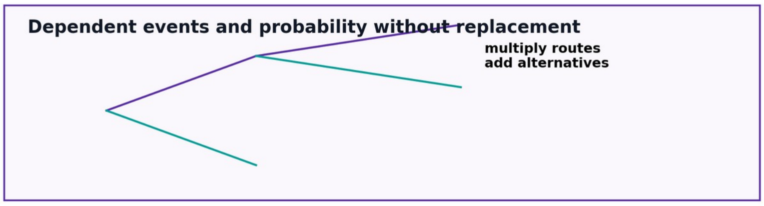
LESSON 07 • P8

Dependent events and probability without replacement

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P8. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

Events are dependent when an earlier outcome changes later probabilities. Without replacement changes both favourable count and total count.

Update every second-stage numerator and denominator from the actual first branch. Different first outcomes usually produce different second probabilities.

Multiply along a complete ordered route. If the event can occur through alternative orders, calculate each route and add.

For one of each colour, red-blue and blue-red are distinct routes. They may have equal probabilities, but this must be shown.

Build the method

A probability tree encodes conditional probabilities: the second red branch after red is $P(R_2 | R_1)$.

Counts and combinations can provide a shorter method, but tree logic remains useful for interpreting dependence and ordered information.

With replacement restores the composition, often making repeated draws independent. Compare the model explicitly.

Check route totals: all final outcomes from the root should sum to 1.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

Bag 4R,3B: P(RR) no replacement.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain $4/7\times3/6=2/7$.
3 **Interpret:** Check $0\leq P\leq 1$ and state the event and assumptions.

ANSWER

$4/7\times3/6=2/7$

Worked example 2

QUESTION

P(RB).

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is $4/7\times3/6=2/7$.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

$4/7\times3/6=2/7$

Guided Higher practice

GUIDED QUESTION 1

P(one of each).

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

P(BB).

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Bag 4R,3B: P(RR) no replacement.	4. P(BB).
2. P(RB).	5. With replacement P(RR).
3. P(one of each).	6. Why denominator changes?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

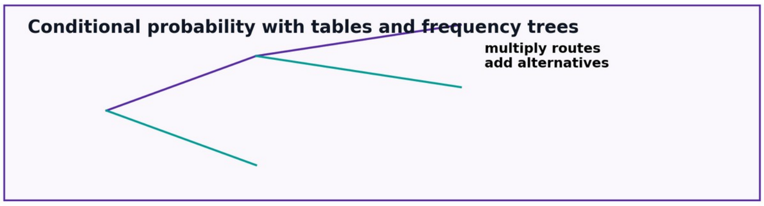
LESSON 08 • P9

Conditional probability with tables and frequency trees

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P9. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

Conditional probability asks for probability within a restricted group. The phrase “given B” changes the denominator from the universal total to the B total.

$P(A|B)=P(A\cap B)/P(B)$, provided $P(B)>0$. In frequencies, use intersection count divided by B count.

A two-way table makes intersections, margins and grand total explicit. Complete missing counts before calculating conditional probabilities.

A frequency tree separates sequential categories using counts. The second split totals its parent, making conditional proportions visible.

Build the method

$P(A|B)$ and $P(B|A)$ usually differ because their denominators differ, even though the intersection count is the same.

Expected frequencies can translate probabilities into an imagined population, often 1000 or a common denominator, simplifying a conditional comparison.

Independence can be tested by checking whether $P(A|B)=P(A)$, or equivalently $P(A\cap B)=P(A)P(B)$.

State the conditioning group in words to prevent denominator errors: “among those who are B…”

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

120 people, 48 B, 18 both A and B. P(A | B).

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 3/8.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

3/8

Worked example 2

QUESTION

P(B | A) if 30 are A.

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 3/5.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

3/5

Guided Higher practice

GUIDED QUESTION 1

If P(A)=0.4,P(B)=0.5, independent intersection.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Test independence if intersection 0.18.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. 120 people, 48 B, 18 both A and B. $P(A\|B)$.	4. Test independence if intersection 0.18.
2. $P(B\|A)$ if 30 are A.	5. Why complete table margins first?
3. If $P(A)=0.4, P(B)=0.5$, independent intersection.	6. Can $P(A\|B)>P(A)$?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

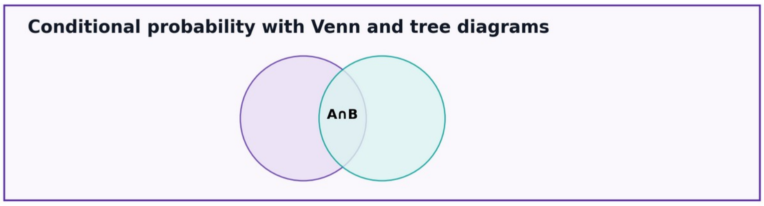
LESSON 09 • P9

Conditional probability with Venn and tree diagrams

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P9. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.



Understand the idea

In a Venn diagram, conditioning on B means ignore everything outside circle B. The denominator becomes the whole B circle.

On a tree, a later branch probability is already conditional on the route reaching that node. Label $P(B|A)$, not simply $P(B)$, when dependence exists.

Bayesian-style reverse questions can be solved without a named formula by using route probabilities or expected frequencies and then restricting to the observed outcome.

For example, if two machines have different production shares and fault rates, multiply to find faulty output from each machine, then divide one faulty route by total faulty probability.

Build the method

Expected-frequency trees avoid tiny decimals: imagine a suitable total, split by first probabilities, then by conditional rates, and condition using final counts.

A conditional probability may be larger or smaller than the unconditional probability. The comparison describes association, not necessarily causation.

Tree branch probabilities from a node sum to 1; final route probabilities across all leaves sum to 1. Use both checks.

Explain the conditioning event and assumptions, especially when probabilities are estimated from data.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

$P(A)=0.6, P(B|A)=0.2$. Route A and B.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain 0.12.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

0.12

Worked example 2

QUESTION

$P(A')=?$

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is 0.4.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

0.4

Guided Higher practice

GUIDED QUESTION 1

Machine X makes 70%, fault 2%; Y 30%, fault 5%. P(fault).

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

Given faulty, probability from Y.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. $P(A)=0.6, P(B \mid A)=0.2$. Route A and B.	4. Given faulty, probability from Y.
2. $P(A')=?$	5. Why use expected 10000 products?
3. Machine X makes 70%, fault 2%; Y 30%, fault 5%. P(fault).	6. Does association prove cause?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

LESSON 10 • P2-P9

Modelling assumptions and multi-stage reasoning

Higher probability • model • representation • conditional reasoning

Specification focus

AQA references P2-P9. This lesson defines the probability model, selects a representation, derives the calculation and interprets the result with its assumptions.

Modelling assumptions and multi-stage reasoning

sample space → event → probability → interpretation

check exhaustive • mutually exclusive • equally likely • independent

Understand the idea

A probability answer depends on a model: sample space, equality of outcomes, random selection, replacement, stability and independence. List relevant assumptions before calculating.

Words determine operations. “Both” suggests an intersection or route product; “either” a union; “at least one” often a complement; “given” a restricted denominator.

Representations are interchangeable but not identical in convenience. Grids suit paired equally likely numbers; Venn diagrams suit overlapping sets; trees suit stages and conditional branches; tables suit classified data.

A multi-stage solution should define events, choose the representation, label probabilities or frequencies, calculate, then interpret and check.

Build the method

Simulation can estimate a difficult probability, but random digits must map fairly to outcomes and rejected values must not introduce bias.

Comparing strategies requires both probability of success and payoff or cost. Expected value may guide repeated play but does not guarantee an individual result.

Data-based probabilities carry sampling uncertainty and may change across populations or time. Avoid presenting them as universal constants.

A complete Higher response explains why addition or multiplication is valid and why the denominator represents the correct sample space.

MODEL CHECK

State whether outcomes are exhaustive, mutually exclusive, equally likely or independent, and identify the denominator before calculating.

COMMON TRAP

Do not multiply merely because events occur in stages or add merely because two events are named. Follow complete routes, correct overlaps and condition on the stated group.

Worked examples

Worked example 1

QUESTION

Best method for two dice sum.

1 **Represent:** Draw or define the sample space, set, table or tree.
2 **Calculate:** Use the justified probability rule to obtain possibility grid.
3 **Interpret:** Check $0 \le P \le 1$ and state the event and assumptions.

ANSWER

possibility grid

Worked example 2

QUESTION

Best for overlapping clubs.

1 **Restrict:** Identify the correct route or conditional denominator.
2 **Execute:** The result is Venn diagram.
3 **Verify:** Use complements, margins or total leaf probability.

ANSWER

Venn diagram

Guided Higher practice

GUIDED QUESTION 1

Best for successive draws.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

GUIDED QUESTION 2

At least one success shortcut.

STRATEGIC HINT

Label the denominator or every branch before calculating; write why routes are multiplied and why alternatives are added.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Best method for two dice sum.	4. At least one success shortcut.
2. Best for overlapping clubs.	5. What must branches from node total?
3. Best for successive draws.	6. Why state assumptions?

REASONING EXTENSION

Compare two representations, test independence and explain how changing a model assumption alters branch probabilities or the conditional denominator.

Complete answer and reasoning guide

Lesson 1: Experiments, theoretical probability and expected frequency

1. 0.36

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 288

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 1/3

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 73

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. Yes; it is a modelled mean count.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. Random order is not fixed even when long-run probabilities are equal.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 2: Relative frequency, sample size and convergence

1. 0.36

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 216

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 450/1000

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 0.398 is very close to 0.4

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. No

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. It estimates the wrong distribution more precisely.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 3: Exhaustive events, complements and addition rules

1. 0.37

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 0.79

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 0.59

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. No

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. 0.5

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. It was included in both individual totals.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 4: Systematic counting and possibility spaces

1. 36

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 1/6

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. HH,HT,TH,TT

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 60

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. Sector probabilities may differ.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. 27/36=3/4

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 5: Venn diagrams, set notation and algebra

1. 13

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 7

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 25

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 9

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. 5/18

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. 20

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 6: Independent events and multiplication

1. 0.15

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 1/2

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. $0.8^3=0.512$

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 0.488

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. $3(0.2^2)(0.8)=0.096$

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. Repeat the same branch probabilities at each stage.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 7: Dependent events and probability without replacement

1. 4/7×3/6=2/7

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 4/7×3/6=2/7

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 4/7

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 3/7×2/6=1/7

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. 16/49

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. One object has been removed.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 8: Conditional probability with tables and frequency trees

1. 3/8

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 3/5

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 0.2

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. No, since 0.18≠0.2

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. They define the correct restricted denominators.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. Yes

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 9: Conditional probability with Venn and tree diagrams

1. 0.12

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. 0.4

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. 0.029

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 0.015/0.029≈0.517

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. It converts probabilities into clear counts without changing ratios.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. No

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Lesson 10: Modelling assumptions and multi-stage reasoning

1. possibility grid

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

2. Venn diagram

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

3. tree diagram

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

4. 1-P(no successes)

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

5. 1

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

6. The calculation is valid only within the chosen probability model.

Check the sample space, denominator, overlap and dependence assumptions; verify branch or region totals and interpret the answer in the stated condition.

Unit 5 mastery record

AQA	I can demonstrate...	Secure?
P1-P3	Experiments, theoretical probability and expected frequency	☐
P5	Relative frequency, sample size and convergence	☐
P4	Exhaustive events, complements and addition rules	☐
P6-P7	Systematic counting and possibility spaces	☐
P6	Venn diagrams, set notation and algebra	☐
P8	Independent events and multiplication	☐
P8	Dependent events and probability without replacement	☐
P9	Conditional probability with tables and frequency trees	☐
P9	Conditional probability with Venn and tree diagrams	☐
P2-P9	Modelling assumptions and multi-stage reasoning	☐

Official specification source

AQA GCSE Mathematics (8300), Subject Content 3.5 Probability: https://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300/specification/subject-content/3.5-probability