COMPREHENSIVE LESSON NOTEBOOK

GCSE MATHEMATICS

HIGHER TIER

UNIT 4 | GEOMETRY & MEASURES

HIGHER PURPOSE

Move from precise diagrams and constructions to theorem-based proof, exact mensuration, non-right-triangle trigonometry, similarity and vector arguments.

Specification map and progression

Lesson	AQA	Coverage
1	G1-G2	Geometric language, constructions and loci
2	G3-G4	Angles, polygons and quadrilateral properties
3	G5-G6	Congruence, similarity and geometric proof
4	G7-G8	Transformations and combinations
5	G9-G10	Circle vocabulary and circle theorems
6	G11	Coordinate geometry in shapes
7	G12-G13	Three-dimensional shapes, plans and elevations
8	G14-G15	Bearings, maps and scale drawings
9	G16	Area, perimeter and prisms
10	G17-G18	Circles, arcs and sectors
11	G17	Surface area, volume, pyramids, cones, spheres and frustums
12	G19	Similarity in length, area and volume
13	G20-G21	Pythagoras, right-triangle trigonometry and exact values
14	G22-G23	Sine rule, cosine rule and triangle area
15	G24-G25	Vectors and geometric proof

HIGHER PROGRESSION

Includes the complete G1-G25 pathway, with negative enlargements, combined transformations, circle-theorem proof, frustums, exact trigonometry, sine/cosine rules and vector proof.

Formula and theorem bank

Structure	Relationship
Polygon interior sum	$(n-2)\times180°$

Circle sector	arc=$(\theta/360)2\pi r$; area=$(\theta/360)\pi r^2$
Solids	cone/pyramid=1/3×base area×height
Similarity	LSF k; ASF k^2; VSF k^3
Cosine rule	$a^2=b^2+c^2-2bc\cos A$
Triangle area	$1/2\ ab\sin C$

LESSON 01 • G1-G2

Geometric language, constructions and loci

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 2. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.

Geometric language, constructions and loci

define → diagram → theorem/formula → calculate → justify

mark equalities, right angles, correspondence and units

Understand the idea

Geometry communicates through definitions. Parallel, perpendicular, chord, bisector, tangent and plane each impose precise conditions; a diagram that merely looks correct is not evidence.

Ruler-and-compass constructions work because points on an arc are at a fixed distance from its centre. Intersections of equal-radius arcs create equal sides, which produce perpendicular or angle bisectors.

A perpendicular bisector is the locus of points equidistant from two endpoints. An angle bisector is the locus of points equidistant from the two sides of an angle.

Build the method

The shortest distance from a point to a line is measured along the perpendicular. This fact converts distance conditions into constructible boundary lines.

Loci problems combine regions: “within” creates an interior, “more than” an exterior and “equidistant” a bisector. Draw every boundary before shading the intersection.

Construction arcs must remain visible and sufficiently large to intersect clearly. A measured-looking line without arcs does not demonstrate construction.

Check scale and boundary language: “at least” includes a boundary, while “less than” excludes it.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Construct perpendicular bisector of AB.

1 **Diagram:** Label known values, correspondence and required result.

2 **Apply:** Use the exact theorem or formula to obtain Equal-radius arcs from A and B; join intersections..

3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

Equal-radius arcs from A and B; join intersections.

Worked example 2

QUESTION

Locus equidistant from two intersecting lines.

1 **Select:** Choose construction, similarity, trigonometry or vector route.

2 **Calculate:** The result is the two angle bisectors.

3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

the two angle bisectors

Guided Higher practice

GUIDED QUESTION 1

Shortest route from point P to line l.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Construct 60° from a ray.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Construct perpendicular bisector of AB.	4. Construct 60° from a ray.
2. Locus equidistant from two intersecting lines.	5. Region within 3 cm of A.
3. Shortest route from point P to line l.	6. Why keep arcs visible?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

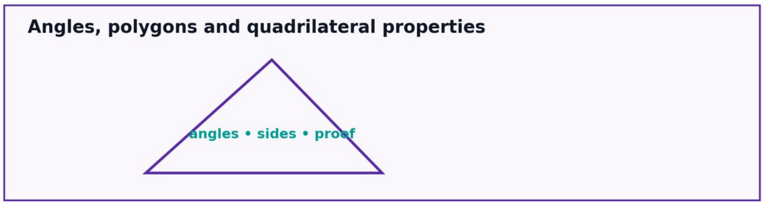
LESSON 02 • G3-G4

Angles, polygons and quadrilateral properties

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 4. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Angles at a point total 360°, on a straight line 180°, and vertically opposite angles are equal. Name the fact at each step in a proof.

With parallel lines, corresponding and alternate angles are equal; co-interior angles sum to 180°. These results depend on the lines actually being parallel.

Triangle angles total 180°. Splitting an n-gon into n-2 triangles gives interior-angle sum (n-2)×180°.

Build the method

The exterior angles of any convex polygon sum to 360°. For a regular n-gon, each exterior angle is 360°/n and each interior angle is 180°-360°/n.

Quadrilateral definitions form a hierarchy: every square is a rectangle and rhombus, but the converses are false. Use necessary properties, not visual appearance.

Isosceles triangles have equal base angles; the converse also holds. Equilateral triangles have three 60° angles.

For multi-step angle problems, mark equal angles, write an equation and retain exact values until the conclusion.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Interior sum of decagon.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 1440°.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

1440°

Worked example 2

QUESTION

Regular polygon exterior angle 24°: sides?

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 15.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

15

Guided Higher practice

GUIDED QUESTION 1

Interior angle regular 15-gon.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Angles in isosceles triangle apex 38°. Base angle.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Interior sum of decagon.	4. Angles in isosceles triangle apex 38°. Base angle.
2. Regular polygon exterior angle 24°: sides?	5. Why can't alternate angles be used without parallel marks?
3. Interior angle regular 15-gon.	6. One interior angle of parallelogram 67°. Adjacent?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

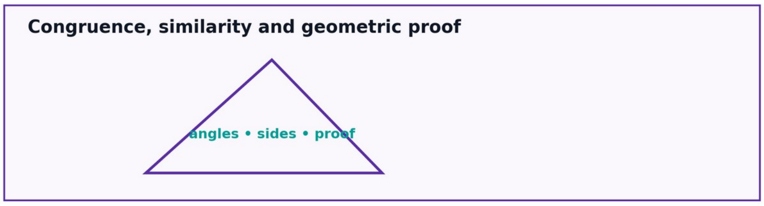
LESSON 03 • G5-G6

Congruence, similarity and geometric proof

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 6. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Congruent figures have identical shape and size. Triangle criteria SSS, SAS, ASA and RHS provide enough information to force one unique triangle.

SSA is not generally a congruence criterion because two different triangles may satisfy the same data. AAA proves similarity, not congruence.

A proof identifies corresponding vertices, states the criterion and then concludes corresponding sides or angles are equal.

Build the method

Similarity preserves angles and scales every corresponding length by one factor. Congruence is similarity with scale factor 1.

Pythagoras can be derived using similar triangles or rearranged areas. In proof questions, distinguish a known theorem from the result being established.

To prove an isosceles result, congruent sub-triangles often reveal equal corresponding angles or sides. Add a construction only if it is justified.

A diagram is not necessarily to scale. Use only stated marks, definitions and derived facts.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Which criterion: three equal sides?

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain SSS.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

SSS

Worked example 2

QUESTION

Why not AAA for congruence?

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is It fixes shape but not size..
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

It fixes shape but not size.

Guided Higher practice

GUIDED QUESTION 1

RHS requires what?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Scale factor 1 implies?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Which criterion: three equal sides?	4. Scale factor 1 implies?
2. Why not AAA for congruence?	5. Two sides and included angle equal: criterion.
3. RHS requires what?	6. How finish after triangles proven congruent?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

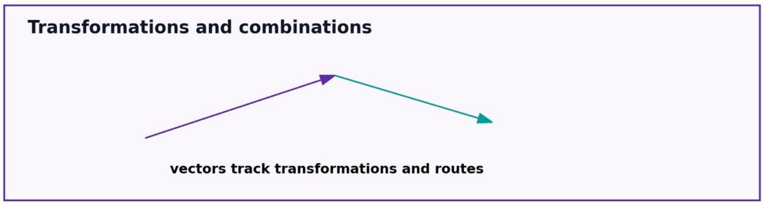
LESSON 04 • G7-G8

Transformations and combinations

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 8. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

A translation moves every point by the same column vector and preserves orientation, length and angles.

A rotation requires centre, angle and direction. Construct images by keeping each point the same distance from the centre and turning through the stated angle.

A reflection requires its mirror line. The line is the perpendicular bisector of each point-image segment.

Build the method

An enlargement requires centre and scale factor. Positive factors keep points on the same ray; negative factors place images on the opposite ray and include a half-turn effect.

Fractional scale factors reduce size; the image remains similar. Lengths multiply by k, areas by k^2 and orientations behave according to the sign of k.

A combination is performed in stated order; transformations generally do not commute. Track a labelled vertex through each step.

Invariance means what remains unchanged. Isometries preserve distance; enlargement preserves angle and ratios but not lengths unless $|k|=1$.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Translate by vector (3,-2).

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain right 3, down 2.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

right 3, down 2

Worked example 2

QUESTION

Describe reflection information needed.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is the equation/position of mirror line.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

the equation/position of mirror line

Guided Higher practice

GUIDED QUESTION 1

Enlarge centre O factor -2: meaning.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Do translation then reflection commute?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Translate by vector (3,-2).	4. Do translation then reflection commute?
2. Describe reflection information needed.	5. Area factor when k=-3.
3. Enlarge centre O factor -2: meaning.	6. Which preserve orientation: translation or reflection?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

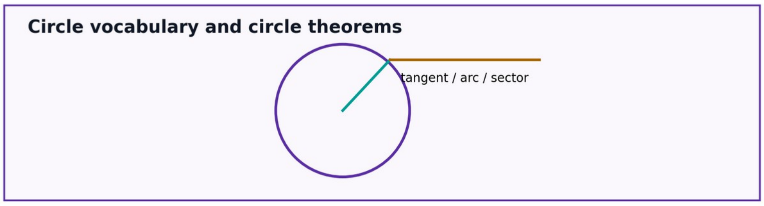
LESSON 05 • G9-G10

Circle vocabulary and circle theorems

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 10. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

A radius joins centre to circumference; diameter is a chord through centre; tangent touches once and is perpendicular to the radius at contact.

The angle at the centre is twice the angle at the circumference subtended by the same arc. Identify the common endpoints of the arc.

An angle in a semicircle is 90°. Angles in the same segment are equal, and opposite angles of a cyclic quadrilateral sum to 180°.

Build the method

Tangents from one external point are equal. The radius-tangent right angles often create congruent right triangles.

The perpendicular from centre to a chord bisects the chord; conversely the line from centre to midpoint of chord is perpendicular.

The alternate segment theorem equates the angle between tangent and chord with the angle in the opposite arc. Mark exactly which chord is involved.

A circle proof chains named theorems. Avoid vague claims such as “angles in a circle”; cite the precise result and arc or tangent.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Centre angle 124°. Same arc circumference angle.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 62°.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

62°

Worked example 2

QUESTION

Angle in semicircle.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 90°.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

90°

Guided Higher practice

GUIDED QUESTION 1

Cyclic quadrilateral one angle 103°. Opposite.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Two tangents PA,PB. Compare lengths.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Centre angle 124°. Same arc circumference angle.	4. Two tangents PA,PB. Compare lengths.
2. Angle in semicircle.	5. Radius to tangent angle.
3. Cyclic quadrilateral one angle 103°. Opposite.	6. Why identify same arc?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

LESSON 06 • G11

Coordinate geometry in shapes

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.

Coordinate geometry in shapes

define → diagram → theorem/formula → calculate → justify

mark equalities, right angles, correspondence and units

Understand the idea

Coordinate geometry converts shape facts into algebra. Gradient identifies parallelism and perpendicularity; distance uses Pythagoras; midpoint averages coordinates.

Distance between (x_1,y_1) and (x_2,y_2) is $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$. Keep exact surds when requested.

The midpoint is $((x_1+x_2)/2,(y_1+y_2)/2)$. Equal midpoints show diagonals bisect, a useful parallelogram test.

Build the method

Parallel segments have equal gradients. Perpendicular non-vertical gradients multiply to -1. Vertical and horizontal cases need separate reasoning.

To classify a quadrilateral, prove sufficient properties: equal opposite vectors, equal gradients and lengths, or diagonal behaviour.

Area can be found by decomposition, base-height or coordinate methods. A sketch helps prevent ordering vertices incorrectly.

Verify each claimed point lies on a stated line by substitution, not appearance.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Distance (1,2) to (7,10).

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 10.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

10

Worked example 2

QUESTION

Midpoint of (-3,5),(9,-1).

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is (3,2).
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

(3,2)

Guided Higher practice

GUIDED QUESTION 1

Gradient through (2,1),(6,9).

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Perpendicular gradient.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Distance (1,2) to (7,10).	4. Perpendicular gradient.
2. Midpoint of (-3,5),(9,-1).	5. How show diagonals bisect?
3. Gradient through (2,1),(6,9).	6. Exact distance of (0,0) to (3,3).

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

LESSON 07 • G12-G13

Three-dimensional shapes, plans and elevations

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 13. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.

Three-dimensional shapes, plans and elevations

define → diagram → theorem/formula → calculate → justify

mark equalities, right angles, correspondence and units

Understand the idea

Faces are 2D surfaces, edges are line segments where faces meet and vertices are corner points. Curved surfaces require careful vocabulary.

A plan is the view from above; front and side elevations are orthographic projections. Depth may disappear in one view.

Constructing views means projecting significant edges perpendicular to the viewing direction and preserving scale. Hidden detail is included only if conventions require it.

Build the method

Different solids can share the same plan and front elevation, so multiple views may be necessary to determine the object.

A net must preserve face adjacency and avoid overlap when folded. Use edge lengths and face shapes to test validity.

Euler’s relationship V-E+F=2 holds for convex polyhedra and provides a check, but not for cylinders or cones with curved surfaces in the same elementary counting model.

For 3D length problems, locate a right triangle on a face first, then another using the face diagonal and depth.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Cube: faces,edges,vertices.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 6,12,8.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

6,12,8

Worked example 2

QUESTION

Plan of cylinder standing upright.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is a circle.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

a circle

Guided Higher practice

GUIDED QUESTION 1

Can two solids share same plan?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Cuboid 3,4,12: space diagonal.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Cube: faces,edges,vertices.	4. Cuboid 3,4,12: space diagonal.
2. Plan of cylinder standing upright.	5. Euler check cube.
3. Can two solids share same plan?	6. Why use multiple elevations?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

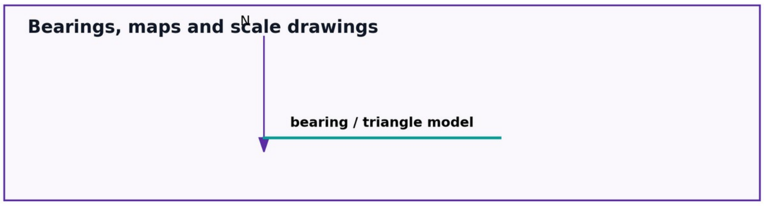
LESSON 08 • G14-G15

Bearings, maps and scale drawings

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 15. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Bearings are measured clockwise from north and written with three digits. Draw a north line at every relevant point.

A reverse bearing differs by 180°: add 180° if below 180°, subtract 180° otherwise.

Scale drawings require consistent units. A scale 1:n multiplies drawing lengths by n for reality and divides real lengths by n for the drawing.

Build the method

In a bearing triangle, interior angles may be found using parallel north lines and alternate angles before sine or cosine rules are applied.

Measurement from a drawing is approximate; distinguish measured data from exact calculated data and state appropriate accuracy.

The shortest route or perpendicular distance may not follow a labelled edge. Interpret the geometry before measuring.

Check a bearing against the quadrant: a south-east direction must lie between 090° and 180°.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Write bearing 7° correctly.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 007°.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

007°

Worked example 2

QUESTION

Reverse of 065°.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 245°.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

245°

Guided Higher practice

GUIDED QUESTION 1

Reverse of 238°.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Scale 1:50000, 4.8 cm real distance.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Write bearing 7° correctly.	4. Scale 1:50000, 4.8 cm real distance.
2. Reverse of 065°.	5. Direction of bearing 315°.
3. Reverse of 238°.	6. Why draw north at both points?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

LESSON 09 • G16

Area, perimeter and prisms

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.

Area, perimeter and prisms

define → diagram → theorem/formula → calculate → justify

mark equalities, right angles, correspondence and units

Understand the idea

Perimeter measures boundary length; area measures surface coverage. Units reveal which quantity has been found.

Triangle area is 1/2×base×perpendicular height. The height need not be a side and may lie outside an obtuse triangle.

Parallelogram area is base×perpendicular height; trapezium area is 1/2(a+b)h for parallel sides a,b.

Build the method

A prism has constant cross-section. Volume=cross-sectional area×length, and all dimensions must use consistent units.

Composite areas are decomposed into known shapes. Mark shared boundaries so internal edges are not included in perimeter.

Rearrangement questions isolate a dimension after substituting known area or volume. Consider whether multiple algebraic roots are physically valid.

Exact answers may contain π or surds. Round only at the final line when requested.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Triangle b=12,h=7 area.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 42.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

42

Worked example 2

QUESTION

Trapezium parallel 8,14 height 5.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 55.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

55

Guided Higher practice

GUIDED QUESTION 1

Prism cross-section 32 cm^2 length 18.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Find length volume 900 area 45.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Triangle b=12,h=7 area.	4. Find length volume 900 area 45.
2. Trapezium parallel 8,14 height 5.	5. Why perpendicular height?
3. Prism cross-section 32 cm^2 length 18.	6. Area units after using metres?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

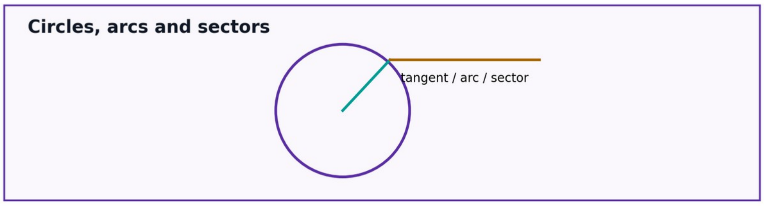
LESSON 10 • G17-G18

Circles, arcs and sectors

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 18. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Circumference $2\pi r$ and area πr^2 come from a full circle. Keep π exact unless a decimal is requested.

A sector is a fraction $\theta/360$ of a circle, so arc length=$(\theta/360)2\pi r$ and sector area=$(\theta/360)\pi r^2$.

Sector perimeter includes two radii as well as the arc. A segment area is sector area minus the appropriate triangle area.

Build the method

Radians are not required for this GCSE specification; use degrees and the 360° fraction unless a question supplies another convention.

For reverse problems, form an equation from arc length or sector area and solve for radius or angle.

Composite circle problems may involve semicircles, holes or overlapping sectors. Sketch and label included and excluded regions.

Check dimensional scaling: doubling radius doubles circumference but quadruples area.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Arc r=9, angle 80°.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 4π.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

4π

Worked example 2

QUESTION

Sector area same data.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 18π.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

18π

Guided Higher practice

GUIDED QUESTION 1

Perimeter of sector.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Angle if arc=5π,r=10.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Arc $r=9$, angle $80°$.	4. Angle if arc$=5\pi$, $r=10$.
2. Sector area same data.	5. Area semicircle $r=6$.
3. Perimeter of sector.	6. Why area scales with r^2?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

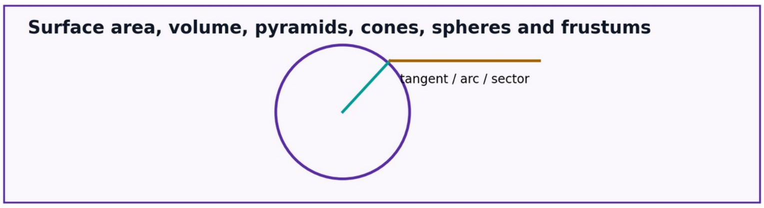
LESSON 11 • G17

Surface area, volume, pyramids, cones, spheres and frustums

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

A right prism has volume cross-section×length. A cylinder is a circular prism: $\pi r^2 h$.

A pyramid has volume 1/3×base area×perpendicular height; a cone has $1/3\pi r^2 h$. The one-third is part of the formula, not a scale factor.

Sphere surface area is $4\pi r^2$ and volume is $4/3\pi r^3$. Dimensional units distinguish the two.

Build the method

Cone curved surface area is πrl where l is slant height; total surface area also includes the circular base. Use Pythagoras if l is not given.

A frustum can be found as large cone minus similar small cone. Use similarity to determine missing heights or radii before subtracting volumes.

Composite solids require attention to joined faces: internal contact surfaces are not part of external surface area.

Retain exact π and surds through intermediate stages; round once after the complete composite calculation.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Cylinder r=3,h=8 volume.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 72π.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

72π

Worked example 2

QUESTION

Cone r=4,h=9 volume.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 48π.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

48π

Guided Higher practice

GUIDED QUESTION 1

Sphere r=5 surface area.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Sphere r=3 volume.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Cylinder r=3,h=8 volume.	4. Sphere r=3 volume.
2. Cone r=4,h=9 volume.	5. Cone r=6,h=8 slant height.
3. Sphere r=5 surface area.	6. Why subtract cones for frustum?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

LESSON 12 • G19

Similarity in length, area and volume

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.

Similarity in length, area and volume

define → diagram → theorem/formula → calculate → justify

mark equalities, right angles, correspondence and units

Understand the idea

Similar figures have equal corresponding angles and proportional corresponding lengths. State the correspondence before using a scale factor.

A length scale factor k gives area factor k^2 and volume factor k^3. This follows because every independent length dimension is scaled.

Reverse from area using a square root and from volume using a cube root. Physical scale factors are positive magnitudes.

Build the method

For similar solids of the same material, mass scales like volume. Surface area scales differently, affecting heat loss and strength-to-weight comparisons.

Similarity can be proved by equal angles in triangles, including parallel-line angle facts. Once proved, corresponding side ratios may be used.

Do not mix linear and area ratios in one equation. Label LSF, ASF and VSF.

A missing length may be found through a chain of similar triangles; keep the same correspondence order in every ratio.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

LSF 3. Area factor.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 9.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

9

Worked example 2

QUESTION

Volume factor.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 27.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

27

Guided Higher practice

GUIDED QUESTION 1

Area ratio 25:49. Length ratio.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Volume ratio 64:125. Length ratio.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. LSF 3. Area factor.	4. Volume ratio 64:125. Length ratio.
2. Volume factor.	5. Mass 120 g, LSF 2.5. New mass.
3. Area ratio 25:49. Length ratio.	6. Why equal angles matter?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

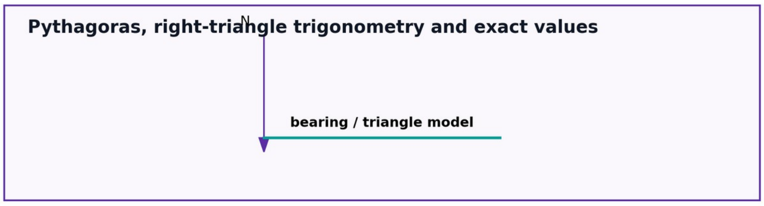
LESSON 13 • G20-G21

Pythagoras, right-triangle trigonometry and exact values

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 21. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Pythagoras $a^2+b^2=c^2$ applies only to right triangles, with c opposite the right angle. It compares squared lengths.

SOHCAHTOA selects a ratio from the known and required sides relative to the marked angle. Label opposite, adjacent and hypotenuse before choosing.

For an angle, use inverse trigonometric functions and include degree mode. For a length, rearrange the ratio algebraically before calculating.

Build the method

In 3D, identify a right triangle on a face, calculate a diagonal, then use a second right triangle. Draw the extracted triangle separately.

Exact values arise from 30-60-90 and 45-45-90 triangles. Know sin, cos for 0°,30°,45°,60°,90° and tan for 0°,30°,45°,60°.

Leave exact answers in surd or fractional form when requested; do not replace $\sqrt{3}/2$ by a rounded decimal.

Check reasonableness: a hypotenuse must be longest, and larger angles face longer sides.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Sides 7,24: hypotenuse.

1 **Diagram:** Label known values, correspondence and required result.
2 **Apply:** Use the exact theorem or formula to obtain 25.
3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

25

Worked example 2

QUESTION

sin θ=3/5 acute θ.

1 **Select:** Choose construction, similarity, trigonometry or vector route.
2 **Calculate:** The result is 36.9°.
3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

36.9°

Guided Higher practice

GUIDED QUESTION 1

Opposite 8, angle 35°, hypotenuse.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

Exact sin 30°.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Sides 7,24: hypotenuse.	4. Exact sin 30°.
2. sin θ=3/5 acute θ.	5. Exact tan 60°.
3. Opposite 8, angle 35°, hypotenuse.	6. Cuboid 3,4,12 space diagonal.

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

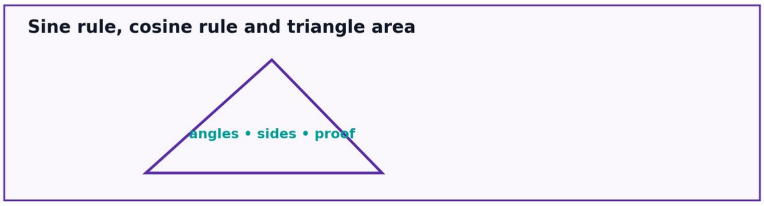
LESSON 14 • G22-G23

Sine rule, cosine rule and triangle area

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 23. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

Use the sine rule when a side-angle opposite pair is known. Maintain matching positions: a/sin A=b/sin B=c/sin C.

Use the cosine rule for SSS to find an angle or SAS to find the third side. The included angle lies between the two known sides.

Area=1/2 ab sin C uses two sides and their included angle. It generalises base×perpendicular height because b sin C is the height.

Build the method

The sine rule ambiguous case can produce two angles with the same sine. Check whether 180°-θ also forms a valid triangle with the remaining angle positive.

For a cosine-rule angle, rearrange to cos $A=(b^2+c^2-a^2)/(2bc)$ before inverse cosine. The largest side must face the largest angle.

Choose the method from available information, not from the appearance of the diagram: RHS uses right-triangle methods; non-right triangles use sine/cosine rules.

Keep full precision in intermediate sides and angles, especially in multi-stage bearings and 3D problems.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

$a=8, A=40°, B=65°$. Find b.

1 **Diagram:** Label known values, correspondence and required result.

2 **Apply:** Use the exact theorem or formula to obtain $8\sin 65°/\sin 40° \approx 11.28$.

3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

$8\sin 65°/\sin 40° \approx 11.28$

Worked example 2

QUESTION

$b=7, c=10, A=52°$. Find a.

1 **Select:** Choose construction, similarity, trigonometry or vector route.

2 **Calculate:** The result is $\sqrt{(7^2+10^2-140\cos 52°)} \approx 7.93$.

3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

$\sqrt{(7^2+10^2-140\cos 52°)} \approx 7.93$

Guided Higher practice

GUIDED QUESTION 1

Sides 9,12 included 38°: area.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

SSS method?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. $a=8, A=40°, B=65°$. Find b.	4. SSS method?
2. $b=7, c=10, A=52°$. Find a.	5. Known opposite pair plus another side: method?
3. Sides 9,12 included 38°: area.	6. Why check ambiguous sine case?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

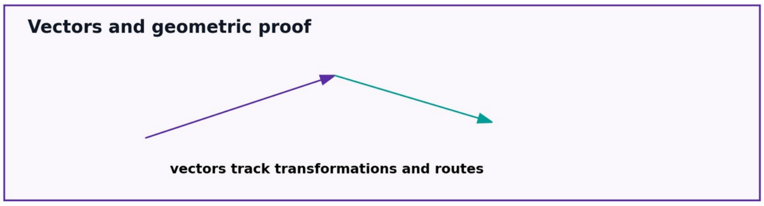
LESSON 15 • G24-G25

Vectors and geometric proof

Higher geometry • construction • calculation • proof

SPECIFICATION ALIGNMENT

AQA 8300 alignment: 25. This lesson develops precise representation, a theorem or construction, a justified calculation and a proof-quality conclusion.



Understand the idea

A vector describes displacement with magnitude and direction. Column vectors add component-wise; subtraction adds the opposite vector.

Multiplying by a scalar changes magnitude and may reverse direction. Parallel vectors are scalar multiples.

Position vectors measure from a common origin. The vector from A to B is b-a when a and b are their position vectors.

Build the method

A route can be replaced by any route with the same start and finish. This path-independence creates vector equations in geometric diagrams.

To prove points are collinear, show one displacement vector is a scalar multiple of another and they share the relevant point.

To prove parallelism, show direction vectors are scalar multiples. To prove a midpoint, show equal displacement vectors or average position vectors.

A vector proof should define base vectors, express every required route consistently, simplify and finish with the precise geometric conclusion.

PROOF HABIT

Mark the diagram, name every theorem or congruence criterion and make each conclusion follow from a stated fact.

COMMON TRAP

Never trust visual appearance. Match corresponding vertices, use perpendicular heights, distinguish length/area/volume scaling and keep calculator precision until the final answer.

Worked examples

Worked example 1

QUESTION

Add (3,-2)+(5,7).

1 **Diagram:** Label known values, correspondence and required result.

2 **Apply:** Use the exact theorem or formula to obtain (8,5).

3 **Justify:** Name the fact and check units, scale or angle range.

ANSWER

(8,5)

Worked example 2

QUESTION

Vector AB if a=(2,4),b=(9,-1).

1 **Select:** Choose construction, similarity, trigonometry or vector route.

2 **Calculate:** The result is (7,-5).

3 **Verify:** Use a reverse calculation or independent geometric property.

ANSWER

(7,-5)

Guided Higher practice

GUIDED QUESTION 1

Meaning of 3a.

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

GUIDED QUESTION 2

How prove parallel lines?

STRATEGIC HINT

Extract the relevant triangle or region, state the theorem before calculation and retain exact values through multi-stage work.

Independent and Grade 7–9 reasoning

Questions 1–3	Questions 4–6
1. Add (3,-2)+(5,7).	4. How prove parallel lines?
2. Vector AB if a=(2,4),b=(9,-1).	5. Midpoint position of a,b.
3. Meaning of 3a.	6. Why routes may be changed?

REASONING EXTENSION

Prove the result using named facts, compare another valid route and explain which features remain invariant under the construction or transformation.

Complete answer and reasoning guide

Lesson 1: Geometric language, constructions and loci

1. Equal-radius arcs from A and B; join intersections.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. the two angle bisectors

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. the perpendicular from P to l

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. construct an equilateral triangle

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. inside/on circle centre A radius 3 cm

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. They justify the ruler-and-compass construction.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 2: Angles, polygons and quadrilateral properties

1. 1440°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 15

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 156°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 71°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. The equality theorem requires parallel lines.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. 113°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 3: Congruence, similarity and geometric proof

1. SSS

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. It fixes shape but not size.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. right angle, equal hypotenuse and one equal side

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. congruence

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. SAS

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. Use corresponding parts of congruent triangles.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 4: Transformations and combinations

1. right 3, down 2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. the equation/position of mirror line

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. opposite ray, twice distance

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. not generally

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 9

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. translation

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 5: Circle vocabulary and circle theorems

1. 62°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 90°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 77°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. PA=PB

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 90°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. Circle-angle theorems depend on subtending the same endpoints.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 6: Coordinate geometry in shapes

1. 10

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. (3,2)

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. -1/2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. Show they have the same midpoint.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. 3√2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 7: Three-dimensional shapes, plans and elevations

1. 6,12,8

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. a circle

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. Yes

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 13

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 8-12+6=2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. One projection can hide depth or internal arrangement.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 8: Bearings, maps and scale drawings

1. 007°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 245°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 058°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 2.4 km

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. north-west

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. Parallel north lines help transfer angles accurately.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 9: Area, perimeter and prisms

1. 42

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 55

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 576 cm^3

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 20

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. Area measures perpendicular separation of parallel base lines.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. m^2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 10: Circles, arcs and sectors

1. 4π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 18π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 18+4π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 90°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 18π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. Area has two length dimensions.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 11: Surface area, volume, pyramids, cones, spheres and frustums

1. 72π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 48π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 100π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 36π

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 10

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. A frustum is the remainder after removing a similar smaller cone.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 12: Similarity in length, area and volume

1. 9

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 27

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. 5:7

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 4:5

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. 1875 g

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. They establish shape similarity and matching vertices.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 13: Pythagoras, right-triangle trigonometry and exact values

1. 25

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. 36.9°

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. $8/\sin 35° \approx 13.95$

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. 1/2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. $\sqrt{3}$

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. 13

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 14: Sine rule, cosine rule and triangle area

1. $8\sin 65°/\sin 40° \approx 11.28$

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. $\sqrt{(7^2+10^2-140\cos 52°)} \approx 7.93$

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. $54\sin 38° \approx 33.25$

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. cosine rule

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. sine rule

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. $\sin\theta=\sin(180°-\theta)$, so two triangles may fit.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Lesson 15: Vectors and geometric proof

1. (8,5)

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

2. (7,-5)

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

3. same direction triple magnitude

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

4. Show direction vectors are scalar multiples.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

5. (a+b)/2

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

6. A displacement depends only on start and finish.

Check the diagram, theorem conditions, corresponding order, exact working and units; a geometrical conclusion must follow from stated facts rather than appearance.

Unit 4 mastery record

AQA	I can demonstrate...	Secure?
G1-G2	Geometric language, constructions and loci	☐
G3-G4	Angles, polygons and quadrilateral properties	☐
G5-G6	Congruence, similarity and geometric proof	☐
G7-G8	Transformations and combinations	☐
G9-G10	Circle vocabulary and circle theorems	☐
G11	Coordinate geometry in shapes	☐
G12-G13	Three-dimensional shapes, plans and elevations	☐
G14-G15	Bearings, maps and scale drawings	☐
G16	Area, perimeter and prisms	☐
G17-G18	Circles, arcs and sectors	☐
G17	Surface area, volume, pyramids, cones, spheres and frustums	☐
G19	Similarity in length, area and volume	☐
G20-G21	Pythagoras, right-triangle trigonometry and exact values	☐
G22-G23	Sine rule, cosine rule and triangle area	☐
G24-G25	Vectors and geometric proof	☐

Official specification source

AQA GCSE Mathematics (8300), Subject Content 3.4 Geometry and measures: https://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300/specification/subject-content/3.4-geometry-and-measures