



GCSE MATHEMATICS
HIGHER TIER
UNIT 1: NUMBER
PRACTICE BOOK

Modules 1-7 | 315 original questions | Formula guides, full methods and marking guidance

FREE WEBSITE EDITION

Student name: _____

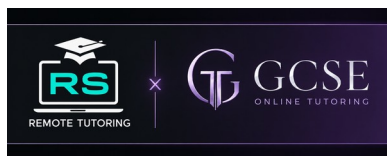
Target grade: _____ Start date: _____ Completion date: _____

gcseonlinetutoring.uk | rsremotetutoring.co.uk

Aligned to the AQA GCSE Mathematics (8300) specification. Independent educational resource - not produced, approved, endorsed or affiliated with AQA.

Copyright and licence note

Free for personal and household educational use. Tutors and schools must obtain the appropriate licence before copying for learners. Resale, paid redistribution and third-party uploading are prohibited.



Unit 1 Contents and Study Plan

MODULES 1-7 • FREE WEBSITE EDITION

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

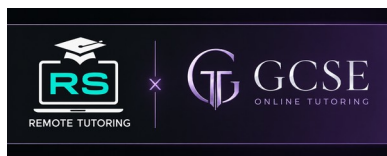
Name: _____ Date: _____ Score: ____ / ____

Begin with the short fluency check, then move through Standard Higher, Multi-step Higher and Grade 7-9 Challenge. Formula guides and all worked answers are collected at the end.

Module	Topic	Specification	Score
1	Calculation, Fractions and Recurring Decimals	N1-N3, N10-N12	____ / ____
2	Factors, Primes and Counting	N4-N5	____ / ____
3	Powers, Roots and Fractional Indices	N6-N7	____ / ____
4	Standard Form	N9	____ / ____
5	Surds and Exact Values	N8, N10	____ / ____
6	Estimation, Rounding and Bounds	N13-N16	____ / ____
7	Mixed Higher Number Problem Solving	N1-N16 review	____ / ____

Book structure

- 8 purposeful fluency questions per module
- 12 Standard Higher questions per module
- 15 multi-step Higher exam questions per module
- 10 Grade 7-9 challenge questions per module
- Formula and method bank for every module
- All answers, formulas and worked methods at the end



Unit 1 Specification Coverage Map

INDEPENDENTLY AUTHORED PRACTICE MAP

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Every question is independently authored. This map identifies the GCSE Mathematics skills covered by each module. It is not a reproduction or adaptation of an AQA question paper, mark scheme, examiner report or other assessment material.

Module	Specification alignment	Design guidance
M1	N1-N3, N10-N12	signed calculation; exact fractions; recurring decimals; contextual interpretation
M2	N4-N5	prime factors; HCF/LCM; square/cube multiples; product rule for counting
M3	N6-N7	negative and fractional indices; roots; common-base equations
M4	N9	standard-form arithmetic; calculator displays; scientific contexts
M5	N8, N10	surds; rationalisation; exact values; recurring decimals
M6	N13-N16	estimation; significant figures; intervals; compound upper/lower bounds
M7	N1-N16	mixed AO2/AO3 problems linking Number strands

Specification references above identify curriculum coverage only. All questions, diagrams, solutions and marking guidance in this resource are independently authored.



How to Use and Check Your Work

STUDENT ROUTINE AND QUALITY STANDARD

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

1 Attempt Work without notes first. Keep fractions, surds and powers exact unless the question requests rounding.

2 Mark Use the answer and method. Award method marks only when the required setup is visible.

3 Code K knowledge; M method; A arithmetic/algebra; R reading; C calculator/rounding; P presentation.

4 Repair Redo the question from a blank start, then complete a similar question.

5 Retest Return after 2-7 days. A correction is secure only when you can reproduce it without support.

Answer standard

- Formula or governing rule where relevant
- Substitution and main processing steps
- Exact result before any requested approximation
- Units, inequality signs and interval endpoints
- A final statement answering the actual question

Publishing note

This book is an independent practice resource. It analyses specification requirements and assessment structures, but it is not an official AQA publication and makes no prediction about future papers.



Calculation, Fractions and Recurring Decimals

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Order -3.7, $-15/4$, -3.68 and -3.705 from smallest to largest. [3 marks]

2. Calculate $-18 + 7 \times (-4)$. [2 marks]

3. Calculate $5/6 - 7/15$. [2 marks]

4. Calculate $2 \frac{1}{4} \div 3/8$. [3 marks]

5. Write 0.0375 as a fraction in simplest form. [2 marks]

6. Write $13/40$ as a decimal and as a percentage. [2 marks]

7. Calculate $7 - [3 - 2(5-8)]$. [3 marks]

8. Find the reciprocal of $-2 \frac{2}{5}$. [2 marks]



Calculation, Fractions and Recurring Decimals

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing rule and exact working.

1. Calculate $\frac{3}{7}$ of $\frac{5}{9}$. [2 marks]

2. Calculate $4.8 \div 0.06$. [2 marks]

3. Write 0.272727... as a fraction. [2 marks]

4. Calculate $(-3)^4 - -3^4$. [3 marks]

5. Insert $<$, $>$ or $=$: $-\frac{5}{8}$ ____ -0.62 . [2 marks]

6. Calculate $1 - (\frac{3}{8} \div \frac{9}{16})$. [3 marks]



Calculation, Fractions and Recurring Decimals

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret the result in context.

7. A number is multiplied by -6 and then 11 is added. The result is 53. Find the number. [3 marks]

8. A number x satisfies $2\frac{1}{3} - x = \frac{5}{8}$. Find x , then write the reciprocal of x in its simplest form. [4 marks]

9. A tank is $\frac{3}{5}$ full. After 44 litres are added it is $\frac{7}{8}$ full. Find the capacity of the tank and the amount of water originally in it. [5 marks]

10. Write $0.1363636\dots$ as a fraction in simplest form. Only 36 repeats. Hence calculate $0.1363636\dots \div \frac{3}{22}$. [5 marks]

11. Without using a calculator, determine which is greater: $\frac{29}{41}$ or $\frac{46}{65}$. State the difference as a fraction. [4 marks]

12. A student evaluates $-3^4 + (-3)^3$ as 108. Explain both sign errors and give the correct value. [4 marks]



Calculation, Fractions and Recurring Decimals

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A bank balance is -£184. A payment of £275 enters the account and two bills of £63.50 leave. Find the new balance. [3 marks]

2. A recipe uses $\frac{7}{12}$ kg of flour per batch. There are $4\frac{3}{8}$ kg available. How many complete batches can be made? [4 marks]

3. Evaluate $(3.6-5.1)^2 \div 0.45$. [3 marks]

4. A student says $\frac{2}{3} + \frac{5}{8} = \frac{7}{11}$. Explain the error and give the correct answer. [3 marks]

5. The temperature changes from -6.8°C to 4.7°C . Find the increase. [2 marks]



Calculation, Fractions and Recurring Decimals

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. Calculate $5/12 \div (3/8 - 1/6)$. [4 marks]

7. Three quarters of a number is 18 less than the number. Find the number. [3 marks]

8. A 6.4 m cable is cut into pieces of length 0.35 m. Find the length left after as many complete pieces as possible are cut. [3 marks]

9. A class has 32 students. $3/8$ study French. Of these, $5/6$ also study Spanish. How many study both? [3 marks]

10. Write the exact value of 2.1 recurring (only the 1 recurs) as a fraction. [3 marks]



Calculation, Fractions and Recurring Decimals

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check exact form, units and accuracy.

11. Calculate $1\frac{5}{6} - 2\frac{3}{4}$ and give the answer as a mixed number or fraction. [3 marks]

12. A lift moves from floor 12 to floor -4, then rises 9 floors. State the total distance travelled and final floor. [3 marks]

13. Find the value of $(\frac{2}{5})^{-1} + (\frac{3}{4})^{-1}$. [3 marks]

14. A number is divided by 0.12 and the result is 45. Find the number. [2 marks]

15. Prove that $0.1454545... = \frac{8}{55}$. [4 marks]



Calculation, Fractions and Recurring Decimals

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan before calculating.

1. Calculate $[2/3 - (5/6 - 3/4)] \div 7/8$. [4 marks]

2. A fraction is claimed to equal $5/7$. Adding 6 to its numerator and 10 to its denominator is also claimed to leave its value unchanged. Show that no such fraction exists. [4 marks]

3. Find the integer n if $2 - 3[4 - (n - 5)] = -28$. [4 marks]

4. A tank is $7/10$ full. After 24 litres are removed it is $11/20$ full. Find the capacity. [4 marks]

5. $x = 0.123123\ldots$. Write x as a fraction in simplest form. [3 marks]



Calculation, Fractions and Recurring Decimals

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. The reciprocal of a number is $1\frac{3}{7}$. Find the number and its square. [3 marks]

7. A positive number is reduced by $\frac{3}{8}$ of itself and the result is 55. Find the number. [3 marks]

8. Without a calculator, decide which is larger: $\frac{17}{29}$ or $\frac{23}{39}$. [3 marks]

9. Find two consecutive integers between which $\sqrt{70}$ lies, and explain. [2 marks]

10. A number rounds to 0.4 to 1 decimal place and is a rational number with denominator 24. List all possible positive values in simplest form. [4 marks]



Factors, Primes and Counting

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Write 84 as a product of prime factors. [3 marks]

2. Write 126 as a product of prime factors. [3 marks]

3. Write 180 as a product of prime factors. [3 marks]

4. Find the HCF of 72 and 120. [2 marks]

5. Find the HCF of 96 and 144. [2 marks]

6. Find the HCF of 150 and 210. [2 marks]

7. Find the LCM of 18 and 24. [2 marks]

8. Find the LCM of 28 and 42. [2 marks]



Factors, Primes and Counting

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Higher-tier application. Show the governing rule and exact working.

1. Find the LCM of 45 and 60. [2 marks]

2. Explain why 437 is not prime. [2 marks]

3. A code uses one of 3 letters followed by one of 2 digits. How many codes are possible? [2 marks]

4. How many outcomes are possible when choosing one of 4 starters, one of 6 mains and one of 3 desserts? [2 marks]

5. Find the smallest integer that must multiply 75 to make a square number. [3 marks]

6. Find the positive square root of the HCF of 288 and 648. [3 marks]



Factors, Primes and Counting

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the Higher set. Interpret the result in context.

7. Find the greatest prime factor of 1386. [3 marks]

8. The HCF of two numbers is 18 and their LCM is 1260. One number is 180. Find the other number and verify the stated HCF. [4 marks]

9. A number has prime factorisation $2^a \times 3^b$. It has 40 factors and is a multiple of 72. Find the least possible number. [5 marks]

10. A code contains three different letters chosen from six letters, followed by a three-digit odd number made from 1,2,3,4 without repetition. How many codes are possible? [5 marks]

11. Find the least positive integer n for which $450n$ is a perfect cube. Give the cube root of the resulting number. [4 marks]

12. Prove that the square of every odd integer leaves remainder 1 when divided by 8. [5 marks]



Factors, Primes and Counting

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Original multi-step exam-style questions. Show every connected stage.

1. Three alarms sound every 24, 36 and 54 minutes. They sound together at 09:00. When do they next sound together? [4 marks]

2. A florist has 168 red and 252 white flowers. Identical bouquets use all flowers. Find the greatest number of bouquets and the composition of each. [4 marks]

3. A security code has one of 5 letters, followed by two different digits chosen from 0,1,2,3. How many codes are possible? [3 marks]

4. How many three-digit numbers can be made from 2,4,6,8 without repetition? [3 marks]

5. Find the smallest number greater than 500 divisible by 18 and 24. [3 marks]



Factors, Primes and Counting

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the multi-step set. Explain decisions where requested.

6. The HCF of two numbers is 12 and their LCM is 420. One number is 60. Find the other. [4 marks]

7. A number has prime factorisation $2^a \times 3^2 \times 5$. It has 24 factors. Find a. [4 marks]

8. Find the least positive integer divisible by 8, 15 and 21. [3 marks]

9. Explain why the square of any prime greater than 3 leaves remainder 1 when divided by 3. [4 marks]

10. A menu offers 4 starters and 7 mains. A customer may choose starter only, main only, or both. How many choices? [3 marks]



Factors, Primes and Counting

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Complete the multi-step set. Check exact form, units and accuracy.

11. Find the smallest cube number divisible by 108. [4 marks]

12. Show that there is no integer between 200 and 260 that is divisible by both 6 and 15 and has digit sum 9. [3 marks]

13. Two different books are selected from 8 books. How many unordered pairs are possible? [3 marks]

14. A six-sided die is rolled and a coin is tossed twice. How many combined outcomes? [2 marks]

15. Prove that the product of four consecutive integers is divisible by 24. [4 marks]



Factors, Primes and Counting

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Extended reasoning. Plan before calculating.

- 1. Find the least positive integer n such that $540n$ is both a square and a cube. [5 marks]**

- 2. Two numbers have HCF 18 and LCM 756. Both are greater than 18. Give one possible pair. [4 marks]**

- 3. How many positive factors of 720 are multiples of 12? [5 marks]**

- 4. A code uses three distinct letters from A,B,C,D,E followed by one of 4 digits. How many codes? [3 marks]**

- 5. Find the smallest number which leaves remainder 3 when divided by 8 and remainder 4 when divided by 5. [4 marks]**



Factors, Primes and Counting

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Complete the challenge set. Present a clear mathematical argument.

6. Prove that $n^3 - n$ is divisible by 6 for every integer n . [4 marks]

7. A product contains exactly 72 factors. It has prime form $2^a \times 3^b$ with $a > b > 0$. Find all pairs (a, b) . [5 marks]

8. How many four-digit even numbers can be made from 1,2,3,4,5 without repetition? [4 marks]

9. Find the greatest square number that divides 12600. [4 marks]

10. A school can arrange students in rows of 18, 24 or 30 with none left. Find the least possible number greater than 1000. [3 marks]



Powers, Roots and Fractional Indices

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Simplify $2^5 \times 2^{-3}$. [2 marks]

2. Simplify $x^7 \div x^{-2}$. [2 marks]

3. Simplify $(a^{-3})^4$. [2 marks]

4. Write $1/\sqrt[3]{x^2}$ using a fractional index. [2 marks]

5. Evaluate $81^{(3/4)}$. [3 marks]

6. Evaluate $32^{(2/5)}$. [3 marks]

7. Evaluate $16^{-3/4}$. [3 marks]

8. Simplify $(3x^2y^{-1})(4x^{-5}y^3)$. [3 marks]



Powers, Roots and Fractional Indices

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing rule and exact working.

1. Simplify $(p^{1/2})^6$. [2 marks]

2. Solve $5^n = 1/125$. [2 marks]

3. Estimate $\sqrt{53}$ to one decimal place. [2 marks]

4. Between which consecutive integers does cube root of 150 lie? [2 marks]

5. Simplify $x^{(5/2)} \div x^{(1/2)}$. [2 marks]

6. Write $\sqrt{a} \times \text{cube root of } a$ as one power. [3 marks]



Powers, Roots and Fractional Indices

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret the result in context.

7. Evaluate $27^{-2/3}$. [3 marks]

8. Simplify $(16x^{-8}y^{12})^{3/4}$, giving an answer with positive indices. [4 marks]

9. Solve $4^{(2x-1)}=8^{(x+2)}$. Give the exact value of x . [4 marks]

10. Given $p^{1/2}=3$ and $q^{1/3}=2$, find the exact value of $(p^3q^2)^{1/2}$. [4 marks]

11. Show that $81^{(n+1)} \div 27^{(2n-1)}$ can be written as $3^{(7-2n)}$. Hence find n when the expression equals $1/9$. [5 marks]

12. A cube has volume $2^{15} \times 5^6 \text{ mm}^3$. Write its edge length in exact prime-power form and as an integer. [4 marks]



Powers, Roots and Fractional Indices

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A population is multiplied by 2^n . It increases by a factor of 128. Find n . [2 marks]

2. Simplify $(8x^6)^{(2/3)}$. [3 marks]

3. Simplify $(16a^{-8})^{(1/4)}$. [3 marks]

4. Solve $9^x = 27$. [3 marks]

5. Write (x cube root of x^2) as a single power. [2 marks]



Powers, Roots and Fractional Indices

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. A square has area 5^7 cm^2 . Write its side length as a power of 5. [3 marks]

7. Evaluate $(64/125)^{(2/3)}$. [3 marks]

8. Simplify $(a^3b^{-2})^2 \div (a^{-1}b)$. [3 marks]

9. Find k if $4^k \times 8 = 2^{17}$. [3 marks]

10. Show that $\sqrt[3]{54}/\sqrt[3]{2}$ is an integer. [2 marks]



Powers, Roots and Fractional Indices

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check exact form, units and accuracy.

11. A cube has volume 10^{15} mm^3 . Write its edge in standard form. [2 marks]

12. Write $x^{-1/2}$ without negative indices. [1 mark]

13. Solve $8^{(x+1)} = 4^{(2x-1)}$. [4 marks]

14. Evaluate $125^{(4/3)} \div 25$. [3 marks]

15. Simplify $\sqrt{(x^6)}$ for $x \geq 0$. [2 marks]



Powers, Roots and Fractional Indices

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan before calculating.

1. Solve $2^{(x+1)}=8^{(2x-1)}$. [4 marks]

2. Simplify $[(x^{(2/3)}y^{-1/2})^6]^{(1/2)}$. [4 marks]

3. Find the exact value of $16^{(3/4)}+27^{(2/3)}$. [3 marks]

4. If $a^{(1/3)}=4$ and $b^{(1/2)}=5$, find $(a^2b)^{1/2}$. [4 marks]

5. Prove that $4^n+4^n+4^n+4^n=4^{(n+1)}$. [3 marks]



Powers, Roots and Fractional Indices

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. Solve $9^x = 1/(3\sqrt{3})$. [4 marks]

7. Simplify $(81x^{-4}y^8)^{3/4}$. [4 marks]

8. Find n if $32^n = 8\sqrt{2}$. [4 marks]

9. Explain why $\sqrt{(x^2)} = x$ is not always true. [3 marks]

10. Find the smallest integer n for which $3^n > 5000$. [3 marks]



Standard Form

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Write 0.000072 in standard form. [2 marks]

2. Write 8.04×10^7 as an ordinary number. [2 marks]

3. Calculate $(3 \times 10^5)(4 \times 10^{-2})$. [3 marks]

4. Calculate $(8.4 \times 10^9) \div (2.1 \times 10^3)$. [3 marks]

5. Calculate $5.6 \times 10^7 + 3.2 \times 10^6$. [3 marks]

6. Calculate $9.1 \times 10^{-4} - 2.7 \times 10^{-5}$. [3 marks]

7. Order 7×10^{-6} , 6.9×10^{-5} , 8×10^{-7} . [2 marks]

8. Calculate $(2 \times 10^3)^4$. [3 marks]



Standard Form

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Higher-tier application. Show the governing rule and exact working.

1. A calculator shows $4.75\text{E}-8$. Write this in standard form. [1 mark]

2. Write 63.2 million in standard form. [2 marks]

3. Write 4.09×10^{-3} as an ordinary decimal. [2 marks]

4. Calculate $\sqrt{9 \times 10^{14}}$. [2 marks]

5. Simplify $(6 \times 10^8) / (3 \times 10^{-4})$. [3 marks]

6. Calculate 0.0003×0.00008 in standard form. [3 marks]



Standard Form

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the Higher set. Interpret the result in context.

7. Write $1/250000$ in standard form. [3 marks]

8. A probe travels 7.2×10^{11} m at 3.0×10^4 m/s. Calculate the travel time in days, giving the answer to 3 significant figures. [5 marks]

9. A cube has volume $5.832 \times 10^{-6} \text{ m}^3$. Find its edge length in standard form. [4 marks]

10. The masses of particles A and B are 4.8×10^{-19} kg and 7.5×10^{-22} kg. How many times the mass of B is the mass of A? [3 marks]

11. A population is 6.4×10^5 and grows by a factor of 1.5 each hour. Calculate the population after 6 hours in standard form. [4 marks]

12. Evaluate $[(3 \times 10^{-4})^2(8 \times 10^9)]/(1.2 \times 10^{-3})$, giving the answer in standard form. [5 marks]



Standard Form

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Original multi-step exam-style questions. Show every connected stage.

1. Light travels at 3×10^8 m/s. Find the distance travelled in 2.4×10^{-3} s. [3 marks]

2. A cell has mass 6.5×10^{-12} g. Find the mass of 4×10^8 cells. [3 marks]

3. The distance 1.496×10^{11} m is travelled in 500 s. Find speed in standard form. [4 marks]

4. A data file is 2.5×10^9 bytes. How many complete files fit in 8×10^{11} bytes? [3 marks]

5. The populations of two cities are 3.8×10^6 and 7.5×10^5 . Find the difference. [3 marks]



Standard Form

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the multi-step set. Explain decisions where requested.

6. A nanoparticle is 4×10^{-8} m wide. How many placed end to end make 2 mm? [4 marks]

7. Calculate $(1.2 \times 10^{-3})^2$ and give standard form. [2 marks]

8. A number is 6×10^n . Multiplying by 5×10^4 gives 3×10^{12} . Find n. [3 marks]

9. The area of a square is $4.9 \times 10^{13} \text{ mm}^2$. Find its side in standard form. [3 marks]

10. Calculate $[(6 \times 10^5)(4 \times 10^{-3})]/(8 \times 10^2)$. [4 marks]



Standard Form

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Complete the multi-step set. Check exact form, units and accuracy.

11. A quantity increases from 2×10^5 to 7.4×10^5 . Find the increase in standard form. [2 marks]

12. Write 0.000000000803 in standard form. [2 marks]

13. Calculate $3.6 \times 10^8 \div (1.2 \times 10^{-6})$. [3 marks]

14. A calculator displays $1.23456789 \times 10^{12}$. Give the value to 4 significant figures in standard form. [2 marks]

15. Explain why 0.43×10^7 is not standard form and correct it. [2 marks]



Standard Form

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Extended reasoning. Plan before calculating.

1. Calculate $(9 \times 10^{-6})^{(3/2)}$. [4 marks]

2. Find x if $(4 \times 10^x)(7.5 \times 10^{-3}) = 3 \times 10^9$. [4 marks]

3. The radius of a sphere is 3×10^{-4} m. Find its volume using $V = \frac{4\pi r^3}{3}$, in standard form. [4 marks]

4. A value is 2.4×10^a and another is 6×10^b . Their product is 1.44×10^{17} . Give one pair of integers a,b. [4 marks]

5. Calculate $(5 \times 10^{12} + 8 \times 10^{11}) / (2 \times 10^4)$. [4 marks]



Standard Form

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. A bacteria population triples every hour from 2×10^4 . Give the population after 8 hours in standard form. [4 marks]

7. Show that $(2 \times 10^n)^3 = 8 \times 10^{(3n)}$. [3 marks]

8. Calculate the cube root of 6.4×10^7 . [3 marks]

9. A ratio is $(3 \times 10^{-4}) : (7.5 \times 10^{-6})$. Simplify it. [3 marks]

10. If 1 light-year is 9.46×10^{15} m, express 4.2×10^{13} km in light-years to 3 significant figures. [4 marks]



Surds and Exact Values

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Simplify $\sqrt{72}$. [2 marks]

2. Simplify $\sqrt{200}$. [2 marks]

3. Simplify $3\sqrt{12}$. [2 marks]

4. Calculate $5\sqrt{3} + 2\sqrt{3} - \sqrt{3}$. [1 mark]

5. Expand $\sqrt{5}(3 + \sqrt{5})$. [2 marks]

6. Expand $(\sqrt{7} + 2)(\sqrt{7} - 2)$. [2 marks]

7. Rationalise $5/\sqrt{3}$. [2 marks]

8. Rationalise $4/(2 + \sqrt{3})$. [3 marks]



Surds and Exact Values

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Higher-tier application. Show the governing rule and exact working.

1. Calculate exactly $(3+\sqrt{2})^2$. [3 marks]

2. Simplify $\sqrt{18} \times \sqrt{8}$. [2 marks]

3. Write 0.363636... as a fraction. [2 marks]

4. Write 0.1666... as a fraction. [3 marks]

5. Write $\frac{7}{12}$ as a recurring decimal. [2 marks]

6. Simplify $\sqrt{50}/\sqrt{2}$. [2 marks]



Surds and Exact Values

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the Higher set. Interpret the result in context.

7. Show that $\sqrt{45} - \sqrt{5} = 2\sqrt{5}$. [2 marks]

8. Simplify $(3 + \sqrt{5})^2 - (3 - \sqrt{5})^2$ without converting the surd to a decimal. [4 marks]

9. Rationalise and simplify $(5 + \sqrt{3})/(5 - \sqrt{3})$. [5 marks]

10. The exact side length of a square is $4 + \sqrt{7}$ cm. Find its exact area and perimeter. [4 marks]

11. Let $x = 2 + \sqrt{3}$. Show that $1/x = 2 - \sqrt{3}$ and hence find $x + 1/x$ and $x^2 + 1/x^2$. [5 marks]

12. Prove that $\sqrt{18} + \sqrt{8}$ is irrational. [5 marks]



Surds and Exact Values

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A square has area 98 cm^2 . Give its perimeter in simplest surd form. [3 marks]

2. Rationalise and simplify $6/\sqrt{12}$. [3 marks]

3. Write $0.2454545\dots$ as a fraction. [4 marks]

4. Simplify $(2\sqrt{3})(\sqrt{12}-\sqrt{3})$. [3 marks]

5. The hypotenuse of a right triangle has exact length $\sqrt{65}$. One shorter side is 4. Find the other side. [3 marks]



Surds and Exact Values

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. Expand and simplify $(\sqrt{5}+\sqrt{2})^2$. [3 marks]

7. Rationalise $3/(\sqrt{5}-1)$. [3 marks]

8. If $x=2+\sqrt{3}$, find $1/x$ exactly. [3 marks]

9. Show that $(\sqrt{6}+\sqrt{2})/\sqrt{2}$ is an integer plus a surd. [2 marks]

10. Write 0.1272727... as a fraction. [4 marks]



Surds and Exact Values

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Complete the multi-step set. Check exact form, units and accuracy.

11. Simplify $\sqrt{3}(\sqrt{27}-\sqrt{12})$. [3 marks]

12. Find the exact distance between (1,2) and (6,9). [3 marks]

13. A rectangle has sides $3+\sqrt{2}$ and $3-\sqrt{2}$. Find its exact area. [2 marks]

14. Prove that $0.999...=1$. [3 marks]

15. Simplify $1/(\sqrt{3}+\sqrt{2})+1/(\sqrt{3}-\sqrt{2})$. [4 marks]



Surds and Exact Values

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Extended reasoning. Plan before calculating.

1. Solve $(x-\sqrt{5})(x+\sqrt{5})=44$ for x . [3 marks]

2. Simplify $(3+2\sqrt{2})^2$ and hence identify its positive square root structure. [4 marks]

3. Rationalise $1/(\sqrt{7}-\sqrt{5})$. [3 marks]

4. If $a=\sqrt{3}+1$ and $b=\sqrt{3}-1$, find a^2+b^2 exactly. [3 marks]

5. Write $0.1234234234\dots$ as a fraction. [5 marks]



Surds and Exact Values

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. Show that $\sqrt{9+4\sqrt{5}}=\sqrt{5}+2$. [4 marks]

7. Simplify $(\sqrt{18}+\sqrt{8})/\sqrt{2}$. [2 marks]

8. Find exact x if $x/(\sqrt{2})=3-\sqrt{2}$. [2 marks]

9. A cube has volume $16\sqrt{2} \text{ cm}^3$. Find the exact edge length. [4 marks]

10. Prove that the product of $2+\sqrt{3}$ and $2-\sqrt{3}$ is rational. [2 marks]



Estimation, Rounding and Bounds

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Round 0.007856 to 3 significant figures. [2 marks]

2. Write the error interval for $x=4.7$ correct to 1 decimal place. [2 marks]

3. Write the error interval for $n=380$ correct to the nearest 10. [2 marks]

4. A length is 12 cm to the nearest cm. State its bounds. [2 marks]

5. A value is truncated to 6.2 to 1 decimal place. State its interval. [2 marks]

6. Estimate $(39.6 \times 0.204) / 1.98$. [2 marks]

7. Find the upper bound of 8.4×3.2 when both values are correct to 1 decimal place. [3 marks]

8. Find the lower bound of $15.0 \div 2.4$ when both values are correct to 1 decimal place. [3 marks]



Estimation, Rounding and Bounds

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing rule and exact working.

- 1. A mass is 2.50 kg to the nearest 0.01 kg. State the maximum possible mass. [2 marks]**

- 2. A calculator gives 19.99948. Round to 4 significant figures. [2 marks]**

- 3. A crowd is 68500 to the nearest 100. State the lower bound. [1 mark]**

- 4. A time is 3.2 s truncated to 1 decimal place. Could it be 3.29 s? [2 marks]**

- 5. Estimate $\sqrt{82.4}$. [1 mark]**

- 6. Find the absolute error when 7.46 is rounded to 7.5. [2 marks]**



Estimation, Rounding and Bounds

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Continue the Higher set. Interpret the result in context.

7. State the maximum rounding error for a measurement to the nearest 0.1 cm. [1 mark]

8. A rectangle measures 8.4 cm by 5.7 cm, each correct to the nearest 0.1 cm. Find the upper bound for its area and the lower bound for its perimeter. [5 marks]

9. The mass $m=3.20$ kg correct to 3 significant figures and volume $V=0.75$ m³ correct to 2 significant figures. Find the upper bound for density m/V . [5 marks]

10. A journey is 120 km correct to the nearest 5 km and takes 1.8 hours correct to the nearest 0.1 hour. Find bounds for the average speed. [5 marks]

11. A sphere radius is 6.0 cm correct to the nearest 0.1 cm. Find the upper bound for its volume in terms of π . [4 marks]

12. A value x is truncated to 0.048 at 3 decimal places. Find bounds for x and for $1/x$. [5 marks]



Estimation, Rounding and Bounds

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A rectangle is 8.2 cm by 5.6 cm, each to the nearest 0.1 cm. Find the upper bound for area. [4 marks]

2. For the same rectangle, find the lower bound for perimeter. [4 marks]

3. A speed is distance/time. Distance is 420 m nearest 10 m; time is 36 s nearest second. Find the upper bound for speed. [4 marks]

4. A density is mass/volume. Mass is 2.8 kg nearest 0.1 kg and volume 0.65 m³ nearest 0.01 m³. Find the lower bound. [4 marks]

5. A square has side 7.4 cm to 1 decimal place. Find bounds for its area. [4 marks]



Estimation, Rounding and Bounds

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. A value rounds to 3.5 to 1 decimal place and 4 to the nearest integer. Give the combined interval. [3 marks]

7. A bus carries 52 people. Estimate the number of buses for 1247 people, then give the exact minimum. [3 marks]

8. A length 2.4 m is correct to nearest 0.1 m. It is cut into pieces each 0.31 m correct to nearest 0.01 m. Find the greatest guaranteed number of complete pieces. [4 marks]

9. The area of a circle uses $r=5.2$ cm nearest 0.1 cm. Give the upper bound in terms of π . [3 marks]

10. A quotient is $12.8/3.4$ using rounded inputs to 0.1. Find the upper bound. [3 marks]



Estimation, Rounding and Bounds

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check exact form, units and accuracy.

11. A product is reported as 72 to 2 significant figures. Give its error interval. [2 marks]

12. A journey is 180 km nearest 10 km and 2.5 h nearest 0.1 h. Find bounds for average speed. [4 marks]

13. Explain why intermediate calculator values should be retained. [2 marks]

14. A measurement is 0.040 m to 2 significant figures. State its interval. [3 marks]

15. A cube edge is 3.6 cm nearest 0.1 cm. Find upper bound for volume. [3 marks]



Estimation, Rounding and Bounds

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan before calculating.

1. $A=xy/z$. $x=4.2$, $y=7.5$ and $z=2.1$, each to 1 decimal place. Find the upper bound of A. [5 marks]

2. A cylinder volume is $\pi r^2 h$. $r=3.0$ cm and $h=8.0$ cm, each nearest 0.1 cm. Find the upper bound in terms of pi. [4 marks]

3. Two rounded lengths are equal to 12.0 cm nearest 0.1 cm. Find the greatest possible percentage difference between their true values. [5 marks]

4. A number x rounds to 2 significant figures as 0.0056. State its interval. [3 marks]

5. A distance $d=350$ m nearest 10 m and speed $v=12$ m/s nearest 1 m/s. Find the maximum time d/v . [4 marks]



Estimation, Rounding and Bounds

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. The perimeter of a square is 48 cm nearest cm. Find bounds for its area. [5 marks]

7. A value is rounded to 1 decimal place as 7.3, then doubled and reported as 14.6. Find bounds for the doubled true value. [3 marks]

8. A reciprocal $1/x$ is required where $x=0.40$ to 2 significant figures. Find its bounds. [5 marks]

9. A cone radius and height are both measured to nearest mm. Explain which bounds maximise volume. [2 marks]

10. A student uses $5.4/2.7=2$ for rounded inputs and says the exact quotient is 2. Show it need not be. [3 marks]



Mixed Higher Number Problem Solving

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Calculate $7/12 + 5/18$. [2 marks]

2. Write 756 in prime factors. [3 marks]

3. Evaluate $16^{(3/4)}$. [2 marks]

4. Calculate $(4 \times 10^6)(3 \times 10^{-4})$. [2 marks]

5. Simplify $\sqrt{147}$. [2 marks]

6. Write 0.2181818... as a fraction. [3 marks]

7. Bounds for 6.8 nearest 0.1. [2 marks]

8. Find LCM(72,90). [2 marks]



Mixed Higher Number Problem Solving

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing rule and exact working.

1. Simplify $x^{(7/3)} \div x^{(1/3)}$. [2 marks]

2. Rationalise $3/\sqrt{6}$. [2 marks]

3. Calculate $1.2 \times 10^5 + 8.7 \times 10^4$. [2 marks]

4. Find the smallest multiplier making 98 a square. [2 marks]

5. Calculate $(2 \frac{1}{3})^{-1}$. [2 marks]

6. Estimate $\sqrt{99}$. [1 mark]



Mixed Higher Number Problem Solving

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret the result in context.

7. Find upper bound of 4.6×2.3 , inputs nearest 0.1. [3 marks]

8. A square has area $7.2 \times 10^5 \text{ cm}^2$. Express its exact side length as a simplified surd. [4 marks]

9. A value is 4.8×10^3 correct to 2 significant figures. It is divided by 0.036 correct to 2 significant figures. Find the upper bound of the quotient. [5 marks]

10. The number $N = 2^a \times 3^b$ is a square, has 45 factors and is divisible by 144. Find the least possible N. [5 marks]

11. Solve $0.272727... \times 3^x = 81/11$, giving the exact value of x. [5 marks]

12. A cylinder has radius $(2 + \sqrt{3}) \text{ cm}$ and height $(2 - \sqrt{3}) \text{ cm}$. Find its exact volume in the form $a\pi + b\pi\sqrt{3}$. [5 marks]



Mixed Higher Number Problem Solving

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A machine makes 2.4×10^5 parts per day. 0.35% are defective. Find expected defective parts. [3 marks]

2. A square has area $2.88 \times 10^4 \text{ cm}^2$. Give side in simplest form. [4 marks]

3. Two timers repeat every 84 s and 126 s. When next together? [3 marks]

4. A measured rectangle is 12.0 by 8.0 cm, nearest 0.1. Find lower bound area. [3 marks]

5. Solve $4^x = 8$. [3 marks]



Mixed Higher Number Problem Solving

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. Write 0.041666... as a fraction. [3 marks]

7. A code has 4 distinct letters chosen from 6, followed by 3 digits with repetition. How many? [4 marks]

8. Simplify $(\sqrt{12} + \sqrt{27})/\sqrt{3}$. [3 marks]

9. A distance is 6.2×10^7 m and time 4×10^2 s. Find speed. [3 marks]

10. A tank is $\frac{5}{8}$ full. Adding 42 L makes it $\frac{7}{8}$ full. Capacity? [3 marks]



Mixed Higher Number Problem Solving

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check exact form, units and accuracy.

11. A value is 250 nearest 10 and divided by 8.0 nearest 0.1. Find upper bound. [4 marks]

12. Find exact value $(3+\sqrt{5})(3-\sqrt{5})$. [2 marks]

13. Show $2^{-3}+2^{-3}=2^{-2}$. [2 marks]

14. A number has 30 factors and form $2^a \times 3^b$. Give one solution with $a > b$. [3 marks]

15. Order $\frac{5}{8}$, 0.63, 6.2×10^{-1} . [3 marks]



Mixed Higher Number Problem Solving

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan before calculating.

1. Find exact value $(\sqrt{3}+1)^4$. [5 marks]

2. Find least n making $360n$ a sixth power. [5 marks]

3. $A=xy^2/z$. $x=2.5, y=4.0, z=1.2$, all nearest 0.1. Find upper bound. [5 marks]

4. Solve $27^{(x-1)}=9^{(2x+1)}$. [4 marks]

5. Write 0.0123123123... as a fraction. [5 marks]



Mixed Higher Number Problem Solving

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. Rationalise and simplify $4/(\sqrt{6}+\sqrt{2})$. [4 marks]

7. A sphere radius is 2.0 cm nearest 0.1. Find upper bound volume in terms of π . [4 marks]

8. How many five-digit odd numbers from 1,2,3,4,5 without repetition? [3 marks]

9. Prove $\sqrt{2}$ is irrational using the standard even/odd contradiction. [5 marks]

10. A value x rounds to 0.0060 to 2 significant figures. Find bounds for $1/x$. [5 marks]



Calculation, Fractions and Recurring Decimals

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Ordering mixed numbers	Convert to a common form; on the negative side, the value farther left is smaller.
Fraction operations	Add/subtract using a common denominator. Multiply numerators and denominators. Divide by multiplying by the reciprocal.
Recurring decimals	Choose powers of 10 that align repeating digits, subtract the equations, then simplify.
Priority and signs	Brackets, indices, multiplication/division, addition/subtraction. A power applies only to what is inside brackets.
Contextual division	Interpret the remainder: complete groups may require rounding down; capacity questions may require rounding up.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Factors, Primes and Counting

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Prime factorisation	Write each number as a product of prime powers.
HCF and LCM	HCF uses the lowest shared powers; LCM uses the highest powers present.
Number of factors	If $n = p^a q^b \dots$, number of positive factors $= (a+1)(b+1)\dots$
Product rule for counting	Multiply the number of choices at successive independent stages.
Square/cube multiples	Adjust prime exponents to multiples of 2 for a square or 3 for a cube.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Powers, Roots and Fractional Indices

FORMULAS AND METHOD BANK

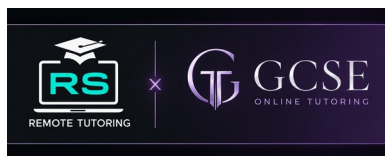
HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Index multiplication/division	$a^m a^n = a^{(m+n)}$; $a^m / a^n = a^{(m-n)}$.
Power of a power	$(a^m)^n = a^{(mn)}$.
Negative index	$a^{-n} = 1/a^n$.
Fractional index	$a^{(1/n)} = \text{nth root of } a$; $a^{(m/n)} = (\text{nth root of } a)^m$.
Common base equations	Rewrite both sides using the same base, then equate exponents.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Standard Form

FORMULAS AND METHOD BANK

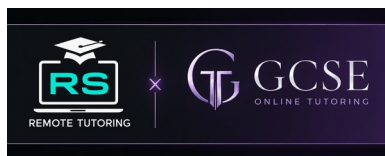
HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Standard form	Write $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.
Multiply/divide	Operate on coefficients and add/subtract powers of 10; renormalise.
Add/subtract	Rewrite with the same power of 10 before combining coefficients.
Powers and roots	Apply the power/root to both coefficient and power of 10.
Units	Convert units before combining values; include the power-of-ten change.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Surds and Exact Values

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Simplifying surds	Factor out the largest square: $\sqrt{ab} = \sqrt{a}\sqrt{b}$.
Like surds	Only coefficients of identical surds may be added/subtracted.
Rationalising one-term denominators	Multiply numerator and denominator by the surd.
Conjugates	For $a \pm \sqrt{b}$, multiply by $a \mp \sqrt{b}$ so the denominator becomes $a^2 - b$.
Recurring decimals	Align the repeating block using powers of 10, subtract, solve and simplify.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Estimation, Rounding and Bounds

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Rounding interval	Nearest unit u gives stated value $-u/2 \leq \text{true value} < \text{stated value} + u/2$.
Truncation interval	Truncated value $\leq \text{true value} < \text{next truncation step}$.
Maximum product	For positive factors, multiply upper bounds.
Maximum quotient	For positive values, numerator upper bound $\div$ denominator lower bound.
Context	Use strict/inclusive boundary notation correctly and round only the final result.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Mixed Higher Number Problem Solving

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Mixed-question routine	Identify the strand, write the governing rule, then calculate and check form/units.
Exact form	Keep fractions, surds and powers exact unless rounding is requested.
Reasonableness	Estimate magnitude and sign before accepting a calculator result.
Bounds	Choose extreme values deliberately; do not simply substitute stated rounded values.
Proof	Use a general argument, not a few examples.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.



Calculation, Fractions and Recurring Decimals

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Convert $-15/4 = -3.75$. Order: $-15/4$, -3.705 , -3.7 , -3.68 .	3	M1 setup; M1 process; A1 conclusion.
2	Use multiplication first: $7 \times (-4) = -28$; $-18 + (-28) = -46$.	2	M1 method; A1 answer.
3	Common denominator 30: $25/30 - 14/30 = 11/30$.	2	M1 method; A1 answer.
4	$2 \frac{1}{4} = 9/4$; $(9/4) \times (8/3) = 6$.	3	M1 setup; M1 process; A1 conclusion.
5	$0.0375 = 375/10000 = 3/80$.	2	M1 method; A1 answer.
6	$13/40 = 0.325 = 32.5\%$.	2	M1 method; A1 answer.
7	$5 - 8 = -3$; $2(-3) = -6$; $[3 - (-6)] = 9$; $7 - 9 = -2$.	3	M1 setup; M1 process; A1 conclusion.
8	$-2 \frac{2}{5} = -12/5$, so the reciprocal is $-5/12$.	2	M1 method; A1 answer.



Calculation, Fractions and Recurring Decimals

STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$(3/7)(5/9)=15/63=5/21$.	2	M1 method; A1 answer.
2	Multiply both by 100: $480 \div 6 = 80$.	2	M1 method; A1 answer.
3	$0.272727... = 27/99 = 3/11$.	2	M1 method; A1 answer.
4	$(-3)^4 = 81$, but $-3^4 = -(81) = -81$; difference 162.	3	M1 setup; M1 process; A1 conclusion.
5	$-5/8 = -0.625$, which is smaller than -0.62 , so $<$.	2	M1 method; A1 answer.
6	$3/8 \times 16/9 = 2/3$; $1 - 2/3 = 1/3$.	3	M1 setup; M1 process; A1 conclusion.
7	$-6x + 11 = 53$; $-6x = 42$; $x = -7$.	3	M1 setup; M1 process; A1 conclusion.
8	$x = 7/3 - 5/8 = 56/24 - 15/24 = 41/24$. Therefore $1/x = 24/41$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Added fraction $= 7/8 - 3/5 = 35/40 - 24/40 = 11/40$. Capacity $= 44 \div (11/40) = 160$ litres. Originally $(3/5) \times 160 = 96$ litres.	5	M1 setup; M2-M4 development; A1 conclusion.
10	Let $x = 0.13636...$; $10x = 1.3636...$ and $1000x = 136.3636...$. Thus $990x = 135$, so $x = 3/22$. The quotient is 1.	5	M1 setup; M2-M4 development; A1 conclusion.
11	Cross-products are $29 \times 65 = 1885$ and $46 \times 41 = 1886$, so $46/65$ is greater. Difference $= (1886 - 1885) / (41 \times 65) = 1/2665$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$-3^4 = -(3^4) = -81$, while $(-3)^3 = -27$. The correct value is $-81 - 27 = -108$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Calculation, Fractions and Recurring Decimals

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$-184 + 275 - 2(63.50) = -184 + 275 - 127 = -£36.$	3	M1 setup; M1 process; A1 conclusion.
2	$4 \frac{3}{8} = \frac{35}{8}. (\frac{35}{8}) \div (\frac{7}{12}) = (\frac{35}{8})(\frac{12}{7}) = 7.5$, so 7 complete batches.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$3.6 - 5.1 = -1.5$; $(-1.5)^2 = 2.25$; $2.25 \div 0.45 = 5.$	3	M1 setup; M1 process; A1 conclusion.
4	Denominators are not added. Common denominator 24: $\frac{16}{24} + \frac{15}{24} = \frac{31}{24} = 1 \frac{7}{24}.$	3	M1 setup; M1 process; A1 conclusion.
5	$4.7 - (-6.8) = 11.5^\circ\text{C}.$	2	M1 method; A1 answer.
6	$\frac{3}{8} - \frac{1}{6} = \frac{9}{24} - \frac{4}{24} = \frac{5}{24}$; $(\frac{5}{12}) \div (\frac{5}{24}) = 2.$	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$x - 3x/4 = x/4 = 18$, so $x = 72.$	3	M1 setup; M1 process; A1 conclusion.
8	18 pieces use $18 \times 0.35 = 6.30$ m; $6.40 - 6.30 = 0.10$ m.	3	M1 setup; M1 process; A1 conclusion.



Calculation, Fractions and Recurring Decimals

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	$32 \times 3/8 = 12$; $12 \times 5/6 = 10$ students.	3	M1 setup; M1 process; A1 conclusion.
10	Let $x = 2.111\dots$; $10x = 21.111\dots$; $9x = 19$; $x = 19/9$.	3	M1 setup; M1 process; A1 conclusion.
11	$11/6 - 11/4 = 22/12 - 33/12 = -11/12$.	3	M1 setup; M1 process; A1 conclusion.
12	Down 16 floors, then up 9: distance 25 floors; final floor 5.	3	M1 setup; M1 process; A1 conclusion.
13	$(2/5)^{-1} = 5/2$ and $(3/4)^{-1} = 4/3$; total $15/6 + 8/6 = 23/6$.	3	M1 setup; M1 process; A1 conclusion.
14	$x/0.12 = 45$, so $x = 45 \times 0.12 = 5.4$.	2	M1 method; A1 answer.
15	Let $x = 0.14545\dots$; $10x = 1.4545\dots$ and $1000x = 145.4545\dots$; $990x = 144$; $x = 144/990 = 8/55$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Calculation, Fractions and Recurring Decimals

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$5/6 - 3/4 = 1/12$; $2/3 - 1/12 = 7/12$; $(7/12) \div (7/8) = 2/3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Let the fraction be $5k/7k$. Then $(5k+6)/(7k+10) = 5/7$ would give $35k+42=35k+50$, so $42=50$. This contradiction proves that no such fraction exists.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$4-(n-5)=9-n$; $2-3(9-n)=-28$; $3n-25=-28$; $3n=-3$; $n=-1$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Removed fraction $7/10 - 11/20 = 3/20$. $3/20$ of capacity = 24, so capacity = 160 litres.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$1000x = 123.123\dots$; $999x = 123$; $x = 123/999 = 41/333$.	3	M1 setup; M1 process; A1 conclusion.
6	$1\frac{3}{7} = 10/7$, so number = $7/10$; square = $49/100$.	3	M1 setup; M1 process; A1 conclusion.
7	Remaining fraction = $5/8$. $(5/8)x = 55$; $x = 88$.	3	M1 setup; M1 process; A1 conclusion.
8	Cross multiply: $17 \times 39 = 663$ and $23 \times 29 = 667$, so $23/39$ is larger.	3	M1 setup; M1 process; A1 conclusion.
9	$8^2 = 64$ and $9^2 = 81$, so $8 < \sqrt{70} < 9$.	2	M1 method; A1 answer.
10	$0.35 \leq x < 0.45$. Multiples of $1/24$: $9/24 = 3/8$ and $10/24 = 5/12$. ($11/24$ is too large.)	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Factors, Primes and Counting

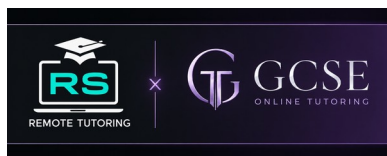
FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$84=2^2 \times 3 \times 7$.	3	M1 setup; M1 process; A1 conclusion.
2	$126=2 \times 3^2 \times 7$.	3	M1 setup; M1 process; A1 conclusion.
3	$180=2^2 \times 3^2 \times 5$.	3	M1 setup; M1 process; A1 conclusion.
4	$\text{HCF}(72, 120)=24$.	2	M1 method; A1 answer.
5	$\text{HCF}(96, 144)=48$.	2	M1 method; A1 answer.
6	$\text{HCF}(150, 210)=30$.	2	M1 method; A1 answer.
7	$\text{LCM}(18, 24)=72$.	2	M1 method; A1 answer.
8	$\text{LCM}(28, 42)=84$.	2	M1 method; A1 answer.



Factors, Primes and Counting

STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$\text{LCM}(45,60)=180$.	2	M1 method; A1 answer.
2	$437=19 \times 23$, so it is composite.	2	M1 method; A1 answer.
3	Product rule: $3 \times 2 = 6$ codes.	2	M1 method; A1 answer.
4	Product rule: $4 \times 6 \times 3 = 72$.	2	M1 method; A1 answer.
5	$75=3 \times 5^2$; multiply by 3 to get $225=15^2$.	3	M1 setup; M1 process; A1 conclusion.
6	$\text{HCF}=72$, so the exact positive root is $\sqrt{72}=6\sqrt{2}$.	3	M1 setup; M1 process; A1 conclusion.
7	$1386=2 \times 3^2 \times 7 \times 11$, so greatest prime factor=11.	3	M1 setup; M1 process; A1 conclusion.
8	$ab=\text{HCF} \times \text{LCM}$, so $b=18 \times 1260/180=126$. $\text{HCF}(180,126)=18$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$(a+1)(b+1)=40$ and $a \geq 3, b \geq 2$. Possibilities include $(a,b)=(7,4)$ and $(4,7)$; least is $2^7 \times 3^4 = 10368$.	5	M1 setup; M2-M4 development; A1 conclusion.
10	Ordered letters: $6 \times 5 \times 4$. Odd final digit: 2 choices; remaining digit positions: 3×2 . Total = $6 \times 5 \times 4 \times 2 \times 3 \times 2 = 1440$.	5	M1 setup; M2-M4 development; A1 conclusion.
11	$450=2 \times 3^2 \times 5^2$. Multiply by $2^2 \times 3 \times 5 = 60$. Then $450n=27000=30^3$, so $n=60$ and cube root=30.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Let an odd integer be $2k+1$. Its square is $4k(k+1)+1$. One of $k, k+1$ is even, so $k(k+1)=2m$; hence the square is $8m+1$.	5	M1 setup; M2-M4 development; A1 conclusion.



Factors, Primes and Counting

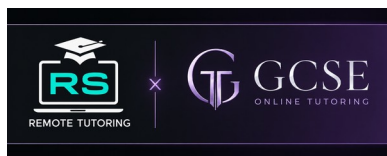
MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$\text{LCM}(24, 36, 54) = 216$ minutes = 3 h 36 min, so 12:36.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$\text{HCF} = 84$ bouquets; each has $168/84 = 2$ red and $252/84 = 3$ white.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$5 \times 4 \times 3 = 60$ codes.	3	M1 setup; M1 process; A1 conclusion.
4	$4 \times 3 \times 2 = 24$.	3	M1 setup; M1 process; A1 conclusion.
5	$\text{LCM} = 72$. Multiples around 500: $72 \times 7 = 504$, so 504.	3	M1 setup; M1 process; A1 conclusion.
6	For two numbers, $\text{product} = \text{HCF} \times \text{LCM}$. $\text{Other} = (12 \times 420)/60 = 84$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Number of factors = $(a+1)(2+1)(1+1) = 6(a+1) = 24$, so $a = 3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$\text{LCM} = 2^3 \times 3 \times 5 \times 7 = 840$.	3	M1 setup; M1 process; A1 conclusion.



Factors, Primes and Counting

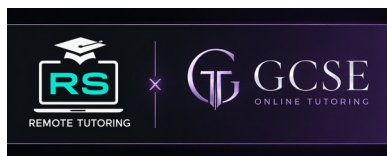
MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	A prime >3 is $3k \pm 1$. Squaring gives $9k^2 \pm 6k + 1$, which is 1 mod 3.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$4 + 7 + (4 \times 7) = 39$ choices.	3	M1 setup; M1 process; A1 conclusion.
11	$108 = 2^2 \times 3^3$. For a cube, powers are multiples of 3: $2^3 \times 3^3 = 216$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$\text{LCM}(6, 15) = 30$. The only multiples of 30 strictly between 200 and 260 are 210 and 240. Their digit sums are 3 and 6, so neither has digit sum 9. Therefore no such integer exists.	3	M1 setup; M1 process; A1 conclusion.
13	$8 \times 7 / 2 = 28$ pairs.	3	M1 setup; M1 process; A1 conclusion.
14	$6 \times 2 \times 2 = 24$.	2	M1 method; A1 answer.
15	Among four consecutive integers: one is divisible by 4, another is even, and one is divisible by 3. Product contains $4 \times 2 \times 3 = 24$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Factors, Primes and Counting

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$540 = 2^2 \times 3^3 \times 5$. Powers must be multiples of 6: multiply by $2^4 \times 3^3 \times 5^5 = 1,350,000$.	5	M1 setup; M2-M4 development; A1 conclusion.
2	Let numbers $18a, 18b$ with coprime a, b and $ab = 756/18 = 42$. For example $a=6, b=7$ gives 108 and 126.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$720 = 2^4 \times 3^2 \times 5$. A multiple of 12 needs $2^2 \times 3$. Choices: power of 2 is 2-4 (3), power of 3 is 1-2 (2), power of 5 is 0-1 (2): 12.	5	M1 setup; M2-M4 development; A1 conclusion.
4	$5 \times 4 \times 3 \times 4 = 240$.	3	M1 setup; M1 process; A1 conclusion.
5	Numbers 3 mod 8: 3, 11, 19, 27...; 19 leaves remainder 4 mod 5. Answer 19.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$n^3 - n = n(n-1)(n+1)$, product of three consecutive integers, so divisible by 2 and 3, hence 6.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$(a+1)(b+1) = 72$. With $a > b > 0$: $(a, b) = (35, 1), (23, 2), (17, 3), (11, 5), (8, 7)$.	5	M1 setup; M2-M4 development; A1 conclusion.
8	Last digit 2 or 4 (2 choices); first then 4 choices; middle two 3×2 : $2 \times 4 \times 3 \times 2 = 48$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$12600 = 2^3 \times 3^2 \times 5^2 \times 7$. Use even powers: $2^2 \times 3^2 \times 5^2 = 900$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	LCM=360. Least multiple > 1000 is 1080.	3	M1 setup; M1 process; A1 conclusion.



Powers, Roots and Fractional Indices

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$2^{(5-3)}=2^2=4.$	2	M1 method; A1 answer.
2	$x^{(7-(-2))}=x^9.$	2	M1 method; A1 answer.
3	$a^{-12}=1/a^{12}.$	2	M1 method; A1 answer.
4	$x^{(-2/3)}.$	2	M1 method; A1 answer.
5	Fourth root of 81 is 3; $3^3=27.$	3	M1 setup; M1 process; A1 conclusion.
6	Fifth root of 32 is 2; $2^2=4.$	3	M1 setup; M1 process; A1 conclusion.
7	$16^{(3/4)}=8$, so reciprocal= $1/8.$	3	M1 setup; M1 process; A1 conclusion.
8	$12x^{-3}y^2=12y^2/x^3.$	3	M1 setup; M1 process; A1 conclusion.



Powers, Roots and Fractional Indices

STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	p^3 .	2	M1 method; A1 answer.
2	$1/125=5^{-3}$, so $n=-3$.	2	M1 method; A1 answer.
3	$7.2^2=51.84$ and $7.3^2=53.29$, so $\sqrt{53}\approx 7.3$.	2	M1 method; A1 answer.
4	$5^3=125$ and $6^3=216$, so $5<\sqrt[3]{150}<6$.	2	M1 method; A1 answer.
5	x^2 .	2	M1 method; A1 answer.
6	$a^{(1/2+1/3)}=a^{(5/6)}$.	3	M1 setup; M1 process; A1 conclusion.
7	Cube root $27=3$; $3^{-2}=1/9$.	3	M1 setup; M1 process; A1 conclusion.
8	$16^{(3/4)}=8$, x^{-6} and y^9 . Result= $8y^9/x^6$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$2^{(4x-2)}=2^{(3x+6)}$, so $4x-2=3x+6$ and $x=8$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$p=9$ and $q=8$. The value is $\sqrt{(9^3 \times 8^2)}=\sqrt{(729 \times 64)}=27 \times 8=216$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$81^{(n+1)}=3^{(4n+4)}$, $27^{(2n-1)}=3^{(6n-3)}$; division gives $3^{(7-2n)}$. Set $7-2n=-2$, so $n=9/2$.	5	M1 setup; M2-M4 development; A1 conclusion.
12	Cube root gives $2^5 \times 5^2=32 \times 25=800$ mm.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Powers, Roots and Fractional Indices

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$128=2^7$, so $n=7$.	2	M1 method; A1 answer.
2	$8^{(2/3)}x^4=4x^4$.	3	M1 setup; M1 process; A1 conclusion.
3	$2a^{-2}=2/a^2$.	3	M1 setup; M1 process; A1 conclusion.
4	$(3^2)^x=3^3$, so $2x=3$ and $x=3/2$.	3	M1 setup; M1 process; A1 conclusion.
5	$x \times x^{(2/3)}=x^{(5/3)}$.	2	M1 method; A1 answer.
6	Side= $(5^7)^{(1/2)}=5^{(7/2)}$ cm.	3	M1 setup; M1 process; A1 conclusion.
7	Cube root= $4/5$; square= $16/25$.	3	M1 setup; M1 process; A1 conclusion.
8	$a^6b^{-4} \times a b^{-1}=a^7/b^5$.	3	M1 setup; M1 process; A1 conclusion.



Powers, Roots and Fractional Indices

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	$2^{(2k)} \times 2^3 = 2^{17}$; $2k+3=17$; $k=7$.	3	M1 setup; M1 process; A1 conclusion.
10	Cube roots divide: $\sqrt[3]{(54/2)} = \sqrt[3]{27} = 3$.	2	M1 method; A1 answer.
11	Edge = $(10^{15})^{(1/3)} = 10^5$ mm.	2	M1 method; A1 answer.
12	$x^{-1/2} = 1/\sqrt{x}$.	1	B1: correct result or statement.
13	$2^{(3x+3)} = 2^{(4x-2)}$; $3x+3=4x-2$; $x=5$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	$125^{(4/3)} = 5^4 = 625$; $625/25 = 25$.	3	M1 setup; M1 process; A1 conclusion.
15	$\sqrt{(x^6)} = x^3$ because $x \geq 0$.	2	M1 method; A1 answer.



Powers, Roots and Fractional Indices

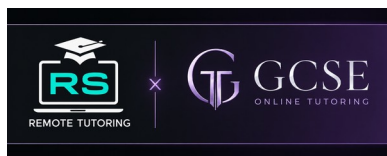
GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$2^{(x+1)}=2^{(6x-3)}$; $x+1=6x-3$; $x=4/5$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Inside power 6 gives x^4y^{-3} ; square root gives $x^2y^{-3/2}=x^2/y^{(3/2)}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$16^{(3/4)}=8$ and $27^{(2/3)}=9$; total 17.	3	M1 setup; M1 process; A1 conclusion.
4	$a=64, b=25$. $\sqrt{(64^2 \times 25)}=64 \times 5=320$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Four equal terms give $4 \times 4^n=4^1 \times 4^n=4^{(n+1)}$.	3	M1 setup; M1 process; A1 conclusion.
6	Right side $=3^{-3/2}$; $9^x=3^{(2x)}$; $2x=-3/2$; $x=-3/4$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$81^{(3/4)}=27$; x^{-3} ; y^6 . Result $27y^6/x^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$32^n=2^{(5n)}$; $8\sqrt{2}=2^{(7/2)}$; $5n=7/2$; $n=7/10$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$\sqrt{(x^2)}= x $. For example $x=-3$ gives $\sqrt{9}=3$, not -3.	3	M1 setup; M1 process; A1 conclusion.
10	$3^7=2187$ and $3^8=6561$, so $n=8$.	3	M1 setup; M1 process; A1 conclusion.



Standard Form

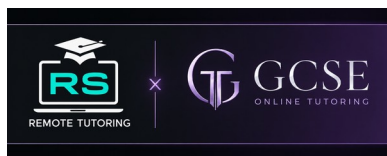
FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	7.2×10^{-5} .	2	M1 method; A1 answer.
2	80,400,000.	2	M1 method; A1 answer.
3	$12 \times 10^3 = 1.2 \times 10^4$.	3	M1 setup; M1 process; A1 conclusion.
4	$(8.4/2.1) \times 10^6 = 4 \times 10^6$.	3	M1 setup; M1 process; A1 conclusion.
5	$5.6 \times 10^7 + 0.32 \times 10^7 = 5.92 \times 10^7$.	3	M1 setup; M1 process; A1 conclusion.
6	$9.1 \times 10^{-4} - 0.27 \times 10^{-4} = 8.83 \times 10^{-4}$.	3	M1 setup; M1 process; A1 conclusion.
7	8×10^{-7} , 7×10^{-6} , 6.9×10^{-5} .	2	M1 method; A1 answer.
8	$2^4 \times 10^{12} = 16 \times 10^{12} = 1.6 \times 10^{13}$.	3	M1 setup; M1 process; A1 conclusion.



Standard Form

STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	4.75×10^{-8} .	1	B1: correct result or statement.
2	6.32×10^7 .	2	M1 method; A1 answer.
3	0.00409.	2	M1 method; A1 answer.
4	3×10^7 .	2	M1 method; A1 answer.
5	2×10^{12} .	3	M1 setup; M1 process; A1 conclusion.
6	$(3 \times 10^{-4})(8 \times 10^{-5}) = 24 \times 10^{-9} = 2.4 \times 10^{-8}$.	3	M1 setup; M1 process; A1 conclusion.
7	$1/(2.5 \times 10^5) = 0.4 \times 10^{-5} = 4 \times 10^{-6}$.	3	M1 setup; M1 process; A1 conclusion.
8	Time = 2.4×10^7 s. Divide by 86400 to get 277.777..., so 278 days.	5	M1 setup; M2-M4 development; A1 conclusion.
9	$5.832 \times 10^{-6} = (1.8 \times 10^{-2})^3$, so edge = 1.8×10^{-2} m.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Ratio = $(4.8/7.5) \times 10^3 = 0.64 \times 10^3 = 640$.	3	M1 setup; M1 process; A1 conclusion.
11	$6.4 \times 10^5 \times 1.5^6 = 7.29 \times 10^6$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$(9 \times 10^{-8} \times 8 \times 10^9)/(1.2 \times 10^{-3}) = 720/(1.2 \times 10^{-3}) = 6 \times 10^5$.	5	M1 setup; M2-M4 development; A1 conclusion.



Standard Form

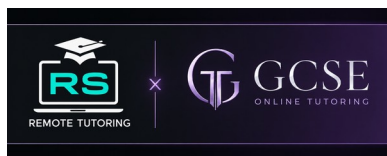
MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Distance = $(3 \times 10^8)(2.4 \times 10^{-3}) = 7.2 \times 10^5$ m.	3	M1 setup; M1 process; A1 conclusion.
2	$(6.5 \times 4) \times 10^{-4} = 26 \times 10^{-4} = 2.6 \times 10^{-3}$ g.	3	M1 setup; M1 process; A1 conclusion.
3	$1.496 \times 10^{11} / 5 \times 10^2 = 0.2992 \times 10^9 = 2.992 \times 10^8$ m/s.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$(8 \times 10^{11}) / (2.5 \times 10^9) = 3.2 \times 10^2 = 320$ files.	3	M1 setup; M1 process; A1 conclusion.
5	$3.8 \times 10^6 - 0.75 \times 10^6 = 3.05 \times 10^6$.	3	M1 setup; M1 process; A1 conclusion.
6	2 mm = 2×10^{-3} m. Number = $(2 \times 10^{-3}) / (4 \times 10^{-8}) = 0.5 \times 10^5 = 5 \times 10^4$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	1.44×10^{-6} .	2	M1 method; A1 answer.
8	$30 \times 10^{(n+4)} = 3 \times 10^{(n+5)} = 3 \times 10^{12}$, so $n=7$.	3	M1 setup; M1 process; A1 conclusion.



Standard Form

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	$\sqrt{4.9 \times 10^{13}} = 7 \times 10^6$ mm because $(7 \times 10^6)^2 = 4.9 \times 10^{13}$.	3	M1 setup; M1 process; A1 conclusion.
10	Numerator = 2.4×10^3 ; divide by 8×10^2 gives 3.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	5.4×10^5 .	2	M1 method; A1 answer.
12	8.03×10^{-10} .	2	M1 method; A1 answer.
13	3×10^{14} .	3	M1 setup; M1 process; A1 conclusion.
14	1.235×10^{12} .	2	M1 method; A1 answer.
15	Coefficient must be $1 \leq a < 10$. $0.43 \times 10^7 = 4.3 \times 10^6$.	2	M1 method; A1 answer.



Standard Form

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$(\sqrt{(9 \times 10^{-6})})^3 = (3 \times 10^{-3})^3 = 27 \times 10^{-9} = 2.7 \times 10^{-8}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$30 \times 10^{(x-3)} = 3 \times 10^{(x-2)} = 3 \times 10^9$, so $x=11$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$V = 4\pi/3 \times 27 \times 10^{-12} = 36\pi \times 10^{-12} = 3.6\pi \times 10^{-11} \text{ m}^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Product $= 14.4 \times 10^{(a+b)} = 1.44 \times 10^{(a+b+1)}$, so $a+b+1=17$. Any integers summing 16, e.g. $a=8, b=8$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$5.8 \times 10^{12} / (2 \times 10^4) = 2.9 \times 10^8$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$2 \times 10^4 \times 3^8 = 2 \times 6561 \times 10^4 = 1.3122 \times 10^8$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Cubing gives $2^3 \times (10^n)^3 = 8 \times 10^{(3n)}$.	3	M1 setup; M1 process; A1 conclusion.
8	$6.4 \times 10^7 = 64 \times 10^6$; cube root $= 4 \times 10^2 = 400$.	3	M1 setup; M1 process; A1 conclusion.
9	Divide: $(3/7.5) \times 10^2 = 0.4 \times 100 = 40$, so ratio 40:1.	3	M1 setup; M1 process; A1 conclusion.
10	$4.2 \times 10^{13} \text{ km} = 4.2 \times 10^{16} \text{ m}$. Divide by $9.46 \times 10^{15} = 4.439\dots$, so 4.44 light-years.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Surds and Exact Values

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$.	2	M1 method; A1 answer.
2	$\sqrt{200} = \sqrt{(100 \times 2)} = 10\sqrt{2}$.	2	M1 method; A1 answer.
3	$3 \times 2\sqrt{3} = 6\sqrt{3}$.	2	M1 method; A1 answer.
4	$6\sqrt{3}$.	1	B1: correct result or statement.
5	$3\sqrt{5} + 5$.	2	M1 method; A1 answer.
6	$7 - 4 = 3$.	2	M1 method; A1 answer.
7	$5\sqrt{3}/3$.	2	M1 method; A1 answer.
8	Multiply by $2 - \sqrt{3}$: $4(2 - \sqrt{3})/(4 - 3) = 8 - 4\sqrt{3}$.	3	M1 setup; M1 process; A1 conclusion.



Surds and Exact Values

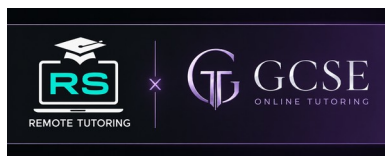
STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$9+6\sqrt{2}+2=11+6\sqrt{2}$.	3	M1 setup; M1 process; A1 conclusion.
2	$\sqrt{144}=12$.	2	M1 method; A1 answer.
3	$36/99=4/11$.	2	M1 method; A1 answer.
4	Let $x=0.1666\dots$; $10x=1.666\dots$ and $100x=16.666\dots$; $90x=15$; $x=1/6$.	3	M1 setup; M1 process; A1 conclusion.
5	$7\div 12=0.58333\dots$	2	M1 method; A1 answer.
6	$\sqrt{25}=5$.	2	M1 method; A1 answer.
7	$\sqrt{45}=3\sqrt{5}$, so $3\sqrt{5}-\sqrt{5}=2\sqrt{5}$.	2	M1 method; A1 answer.
8	Using $(a+b)^2-(a-b)^2=4ab$ gives $4\times 3\times\sqrt{5}=12\sqrt{5}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Multiply by $(5+\sqrt{3})$: $(5+\sqrt{3})^2/(25-3)=(28+10\sqrt{3})/22=(14+5\sqrt{3})/11$.	5	M1 setup; M2-M4 development; A1 conclusion.
10	Area= $(4+\sqrt{7})^2=23+8\sqrt{7}$ cm ² . Perimeter= $16+4\sqrt{7}$ cm.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$1/x=(2-\sqrt{3})/(4-3)=2-\sqrt{3}$. Thus $x+1/x=4$. Squaring: $x^2+2+1/x^2=16$, so $x^2+1/x^2=14$.	5	M1 setup; M2-M4 development; A1 conclusion.
12	$\sqrt{18}+\sqrt{8}=3\sqrt{2}+2\sqrt{2}=5\sqrt{2}$. If this were rational, division by nonzero rational 5 would make $\sqrt{2}$ rational, a contradiction.	5	M1 setup; M2-M4 development; A1 conclusion.



Surds and Exact Values

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Side= $\sqrt{98}=7\sqrt{2}$; perimeter= $28\sqrt{2}$ cm.	3	M1 setup; M1 process; A1 conclusion.
2	$\sqrt{12}=2\sqrt{3}$; $6/(2\sqrt{3})=3/\sqrt{3}=\sqrt{3}$.	3	M1 setup; M1 process; A1 conclusion.
3	Let $x=0.24545\dots$; $10x=2.4545\dots$, $1000x=245.4545\dots$; $990x=243$; $x=27/110$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$\sqrt{12}=2\sqrt{3}$; bracket= $\sqrt{3}$; product=6.	3	M1 setup; M1 process; A1 conclusion.
5	Other ² = $65-16=49$, so other side=7.	3	M1 setup; M1 process; A1 conclusion.
6	$5+2\sqrt{10}+2=7+2\sqrt{10}$.	3	M1 setup; M1 process; A1 conclusion.
7	Multiply by $\sqrt{5}+1$: $3(\sqrt{5}+1)/(5-1)=3(\sqrt{5}+1)/4$.	3	M1 setup; M1 process; A1 conclusion.
8	$1/(2+\sqrt{3})=(2-\sqrt{3})/(4-3)=2-\sqrt{3}$.	3	M1 setup; M1 process; A1 conclusion.



Surds and Exact Values

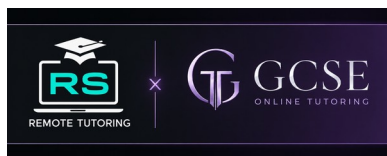
MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	$\sqrt{6}/\sqrt{2}+1=\sqrt{3}+1.$	2	M1 method; A1 answer.
10	Let $x=0.12727\dots$; $10x=1.2727\dots$; $1000x=127.2727\dots$; $990x=126$; $x=7/55.$	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$\sqrt{27}=3\sqrt{3}$, $\sqrt{12}=2\sqrt{3}$; difference $\sqrt{3}$; product=3.	3	M1 setup; M1 process; A1 conclusion.
12	$\sqrt{[(5)^2+(7)^2]}=\sqrt{74}.$	3	M1 setup; M1 process; A1 conclusion.
13	Difference of squares: $9-2=7.$	2	M1 method; A1 answer.
14	Let $x=0.999\dots$; $10x=9.999\dots$; $9x=9$; $x=1.$	3	M1 setup; M1 process; A1 conclusion.
15	Rationalised terms are $\sqrt{3}-\sqrt{2}$ and $\sqrt{3}+\sqrt{2}$; sum= $2\sqrt{3}.$	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Surds and Exact Values

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$x^2-5=44$; $x^2=49$; $x=\pm 7$.	3	M1 setup; M1 process; A1 conclusion.
2	Square $=9+12\sqrt{2}+8=17+12\sqrt{2}$. Therefore $\sqrt{(17+12\sqrt{2})}=3+2\sqrt{2}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Multiply by $\sqrt{7}+\sqrt{5}$: $(\sqrt{7}+\sqrt{5})/(7-5)=(\sqrt{7}+\sqrt{5})/2$.	3	M1 setup; M1 process; A1 conclusion.
4	$a^2=4+2\sqrt{3}$, $b^2=4-2\sqrt{3}$; sum $=8$.	3	M1 setup; M1 process; A1 conclusion.
5	Let $x=0.1234234\dots$; $10x=1.234234\dots$, $10000x=1234.234234\dots$; $9990x=1233$; $x=1233/9990=137/1110$.	5	M1 setup; M2-M4 development; A1 conclusion.
6	$(\sqrt{5}+2)^2=5+4\sqrt{5}+4=9+4\sqrt{5}$; positive roots agree.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$(3\sqrt{2}+2\sqrt{2})/\sqrt{2}=5$.	2	M1 method; A1 answer.
8	$x=\sqrt{2}(3-\sqrt{2})=3\sqrt{2}-2$.	2	M1 method; A1 answer.
9	$16\sqrt{2}=2^4 \times 2^{1/2}=2^{(9/2)}$; cube root $=2^{(3/2)}=2\sqrt{2}$ cm.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Product $=4-3=1$, which is rational.	2	M1 method; A1 answer.



Estimation, Rounding and Bounds

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	0.00786.	2	M1 method; A1 answer.
2	$4.65 \leq x < 4.75$.	2	M1 method; A1 answer.
3	$375 \leq n < 385$.	2	M1 method; A1 answer.
4	$11.5 \leq L < 12.5$ cm.	2	M1 method; A1 answer.
5	$6.2 \leq x < 6.3$.	2	M1 method; A1 answer.
6	Round to $40 \times 0.2 / 2 = 4$.	2	M1 method; A1 answer.
7	$8.45 \times 3.25 = 27.4625$.	3	M1 setup; M1 process; A1 conclusion.
8	For minimum quotient use $14.95 / 2.45 = 6.102040... \dots$	3	M1 setup; M1 process; A1 conclusion.



Estimation, Rounding and Bounds

STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Upper bound is 2.505 kg, not included.	2	M1 method; A1 answer.
2	20.00 (4 significant figures).	2	M1 method; A1 answer.
3	68450.	1	B1: correct result or statement.
4	Yes, because $3.2 \leq t < 3.3$.	2	M1 method; A1 answer.
5	Using 81, $\sqrt{82.4} \approx 9$.	1	B1: correct result or statement.
6	$ 7.5 - 7.46 = 0.04$.	2	M1 method; A1 answer.
7	0.05 cm.	1	B1: correct result or statement.
8	Area upper bound = $8.45 \times 5.75 = 48.5875 \text{ cm}^2$. Perimeter lower bound = $2(8.35 + 5.65) = 28.0 \text{ cm}$.	5	M1 setup; M2-M4 development; A1 conclusion.
9	$3.195 \leq m < 3.205$ and $0.745 \leq V < 0.755$. Density upper bound = $3.205/0.745 = 4.302013... \text{ kg/m}^3$.	5	M1 setup; M2-M4 development; A1 conclusion.
10	$117.5 \leq d < 122.5$; $1.75 \leq t < 1.85$. Lower speed = $117.5/1.85 = 63.5135...$; upper speed = $122.5/1.75 = 70$. Thus $63.5... < v < 70 \text{ km/h}$ (endpoint convention follows the input bounds).	5	M1 setup; M2-M4 development; A1 conclusion.
11	$r < 6.05$. $V < (4/3)\pi(6.05)^3 = 295.240166... \pi \text{ cm}^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$0.048 \leq x < 0.049$. Since $x > 0$, $1/0.049 < 1/x \leq 1/0.048$, so $20.408... < 1/x \leq 20.833...$	5	M1 setup; M2-M4 development; A1 conclusion.



Estimation, Rounding and Bounds

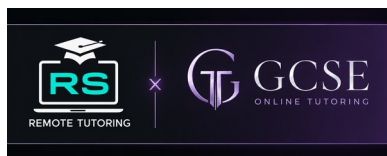
MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$8.25 \times 5.65 = 46.6125 \text{ cm}^2$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$2(8.15 + 5.55) = 27.4 \text{ cm}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$425/35.5 = 11.9718... \text{ m/s}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$2.75/0.655 = 4.19847... \text{ kg/m}^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Lower = $7.35^2 = 54.0225$; upper = $7.45^2 = 55.5025$, so $54.0225 \leq A < 55.5025 \text{ cm}^2$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$3.45 \leq x < 3.55$ (already within $3.5 \leq x < 4.5$).	3	M1 setup; M1 process; A1 conclusion.
7	Estimate $1250/50 = 25$. Exact $1247/52 = 23.98...$, so 24 buses.	3	M1 setup; M1 process; A1 conclusion.
8	Use least total/longest piece: $2.35/0.315 = 7.46...$, so 7 guaranteed.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.



Estimation, Rounding and Bounds

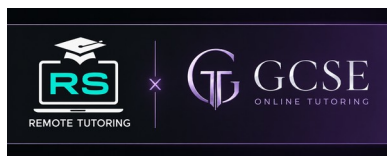
MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	$r < 5.25$, so $A < 5.25^2\pi = 27.5625\pi \text{ cm}^2$.	3	M1 setup; M1 process; A1 conclusion.
10	$12.85/3.35 = 3.83582\dots$	3	M1 setup; M1 process; A1 conclusion.
11	$71.5 \leq x < 72.5$.	2	M1 method; A1 answer.
12	Lower = $175/2.55 = 68.627\dots$; upper = $185/2.45 = 75.510\dots$ km/h.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	Premature rounding accumulates error and may change the final rounded answer.	2	M1 method; A1 answer.
14	$0.0395 \leq x < 0.0405 \text{ m}$.	3	M1 setup; M1 process; A1 conclusion.
15	$3.65^3 = 48.627125 \text{ cm}^3$.	3	M1 setup; M1 process; A1 conclusion.



Estimation, Rounding and Bounds

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Use $x=4.25, y=7.55, z=2.05$: $A < 4.25 \times 7.55 / 2.05 = 15.6524 \dots$	5	M1 setup; M2-M4 development; A1 conclusion.
2	$V = \pi(3.05)^2(8.05) = 74.885125\pi \text{ cm}^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Max difference = $12.05 - 11.95 = 0.10$. Relative to smaller: $0.10 / 11.95 \times 100 = 0.8368 \dots \%$.	5	M1 setup; M2-M4 development; A1 conclusion.
4	$0.00555 \leq x < 0.00565$.	3	M1 setup; M1 process; A1 conclusion.
5	Maximum time = $355 / 11.5 = 30.8696 \dots \text{ s}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Side bounds $47.5/4 = 11.875$ and $48.5/4 = 12.125$. Area: $141.015625 \leq A < 147.015625 \text{ cm}^2$.	5	M1 setup; M2-M4 development; A1 conclusion.
7	$7.25 \leq x < 7.35$, so $14.5 \leq 2x < 14.7$.	3	M1 setup; M1 process; A1 conclusion.
8	$0.395 \leq x < 0.405$. Reciprocal bounds: $1/0.405 < 1/x \leq 1/0.395$, i.e. $2.4691 \dots < 1/x \leq 2.5316 \dots$	5	M1 setup; M2-M4 development; A1 conclusion.
9	$V = (1/3)\pi r^2 h$ increases with positive r and h , so use both upper bounds.	2	M1 method; A1 answer.
10	For example true values 5.35 and 2.75 are allowed; quotient = $1.945 \dots$, not 2.	3	M1 setup; M1 process; A1 conclusion.



Mixed Higher Number Problem Solving

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Common denominator 36: $21/36 + 10/36 = 31/36$.	2	M1 method; A1 answer.
2	$756 = 2^2 \times 3^3 \times 7$.	3	M1 setup; M1 process; A1 conclusion.
3	8.	2	M1 method; A1 answer.
4	1.2×10^3 .	2	M1 method; A1 answer.
5	$7\sqrt{3}$.	2	M1 method; A1 answer.
6	Let $x = 0.21818\dots$; $10x = 2.1818\dots$; $1000x = 218.1818\dots$; $990x = 216$; $x = 12/55$.	3	M1 setup; M1 process; A1 conclusion.
7	$6.75 \leq x < 6.85$.	2	M1 method; A1 answer.
8	360.	2	M1 method; A1 answer.



Mixed Higher Number Problem Solving

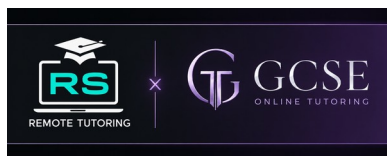
STANDARD HIGHER - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	x^2 .	2	M1 method; A1 answer.
2	$3\sqrt{6}/6 = \sqrt{6}/2$.	2	M1 method; A1 answer.
3	2.07×10^5 .	2	M1 method; A1 answer.
4	$98 = 2 \times 7^2$; multiply by 2 to get 196.	2	M1 method; A1 answer.
5	$3/7$.	2	M1 method; A1 answer.
6	≈ 10 .	1	B1: correct result or statement.
7	$4.65 \times 2.35 = 10.9275$.	3	M1 setup; M1 process; A1 conclusion.
8	Side $= \sqrt{720000} = \sqrt{(360000 \times 2)} = 600\sqrt{2}$ cm.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$4750 \leq A < 4850$ and $0.0355 \leq B < 0.0365$. Upper bound $= 4850/0.0355 = 136619.718\dots$	5	M1 setup; M2-M4 development; A1 conclusion.
10	$(a+1)(b+1) = 45$ with even a, b and $a \geq 4, b \geq 2$. Valid pairs are $(a, b) = (4, 8)$ and $(8, 4)$. The smaller number is $2^8 \times 3^4 = 20736$.	5	M1 setup; M2-M4 development; A1 conclusion.
11	$0.272727\dots = 3/11$. Then $(3/11)3^x = 81/11$, so $3^{x+1} = 3^4$ and $x = 3$.	5	M1 setup; M2-M4 development; A1 conclusion.
12	$V = \pi r^2 h$. Since $(2 + \sqrt{3})(2 - \sqrt{3}) = 1$, $r^2 h = r(rh) = 2 + \sqrt{3}$. Hence $V = (2 + \sqrt{3})\pi \text{ cm}^3$.	5	M1 setup; M2-M4 development; A1 conclusion.



Mixed Higher Number Problem Solving

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	$0.0035 \times 2.4 \times 10^5 = 840.$	3	M1 setup; M1 process; A1 conclusion.
2	Side = $\sqrt{28800} = 120\sqrt{2}$ cm.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	LCM = 252 s = 4 min 12 s.	3	M1 setup; M1 process; A1 conclusion.
4	$11.95 \times 7.95 = 95.0025$ cm ² .	3	M1 setup; M1 process; A1 conclusion.
5	$2^{(2x)} = 2^3$; $x = 3/2.$	3	M1 setup; M1 process; A1 conclusion.
6	$0.041666... = 1/24.$	3	M1 setup; M1 process; A1 conclusion.
7	$6 \times 5 \times 4 \times 3 \times 3 \times 3 = 3240.$	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$(2\sqrt{3} + 3\sqrt{3})/\sqrt{3} = 5.$	3	M1 setup; M1 process; A1 conclusion.



Mixed Higher Number Problem Solving

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: ____ / ____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
9	1.55×10^5 m/s.	3	M1 setup; M1 process; A1 conclusion.
10	Increase $\frac{1}{4}$ capacity=42, so capacity=168 L.	3	M1 setup; M1 process; A1 conclusion.
11	$255/7.95=32.07547\dots$	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$9-5=4$.	2	M1 method; A1 answer.
13	$\frac{1}{8} + \frac{1}{8} = \frac{1}{4} = 2^{-2}$.	2	M1 method; A1 answer.
14	$(a+1)(b+1)=30$. Example $a=9, b=2$ gives $10 \times 3 = 30$.	3	M1 setup; M1 process; A1 conclusion.
15	$6.2 \times 10^{-1} = 0.62$; $\frac{5}{8} = 0.625$; order 0.62, 0.625, 0.63.	3	M1 setup; M1 process; A1 conclusion.



Mixed Higher Number Problem Solving

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Mark s	Check
1	Square= $4+2\sqrt{3}$; square again= $16+16\sqrt{3}+12=28+16\sqrt{3}$.	5	M1 setup; M2-M4 development; A1 conclusion.
2	$360=2^3 \times 3^2 \times 5$. Need exponents 6: $n=2^3 \times 3^4 \times 5^5=2,025,000$.	5	M1 setup; M2-M4 development; A1 conclusion.
3	$2.55 \times 4.05^2 / 1.15 = 36.370760869\dots$	5	M1 setup; M2-M4 development; A1 conclusion.
4	$3^{(3x-3)} = 3^{(4x+2)}$; $x=-5$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$1000x=12.312312\dots$, subtract $x: 999x=12.3=123/10$; $x=123/9990=41/3330$.	5	M1 setup; M2-M4 development; A1 conclusion.
6	$4(\sqrt{6}-\sqrt{2})/(6-2)=\sqrt{6}-\sqrt{2}$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$(4/3)\pi(2.05)^3=11.486833\dots\pi \text{ cm}^3$.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Last digit 1,3,5: 3 choices; remaining positions $4!=24$; total 72.	3	M1 setup; M1 process; A1 conclusion.
9	Assume $\sqrt{2}=a/b$ in lowest terms. Then $a^2=2b^2$, so a even; write $a=2k$. Then $b^2=2k^2$, so b even, contradicting lowest terms.	5	M1 setup; M2-M4 development; A1 conclusion.
10	$0.00595 \leq x < 0.00605$. Thus $1/0.00605 < 1/x \leq 1/0.00595$, i.e. $165.289\dots < 1/x \leq 168.067\dots$	5	M1 setup; M2-M4 development; A1 conclusion.