GCSE MATHEMATICS

HIGHER TIER

UNIT 3: ALGEBRA
PRACTICE BOOK

Modules 1-10 | 450 original questions | Formula guides, full methods and marking guidance

FREE WEBSITE EDITION

Student name: ____

Target grade: _______ Start date: _________ Completion date: _________

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MODULE 01 / A1-A25

Unit 3 Contents and Study Plan

MODULES 1-10 • FREE WEBSITE EDITION

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Begin with the short fluency check, then move through Standard Higher, Multi-step Higher and Grade 7-9 Challenge. Formula guides and all worked answers are collected at the end.

Book structure

- 8 purposeful fluency questions per module
- 12 Standard Higher questions per module
- 15 multi-step Higher exam questions per module
- 10 Grade 7-9 challenge questions per module
- Formula and method bank for every module
- All answers, formulas and worked methods at the end

Unit 3 Specification Coverage Map

INDEPENDENTLY AUTHORED PRACTICE MAP

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Every question is independently authored. This map identifies the GCSE Mathematics skills covered by each module. It is not a reproduction or adaptation of an AQA question paper, mark scheme, examiner report or other assessment material.

Module	Specification alignment	esign guidance
M1	A1-A4,A6	Algebraic Language, Substitution and Proof
M2	A4,A6	Expansion, Factorisation and Algebraic Fractions
M3	A5,A17,A21	Formulae, Rearrangement, Equations and Modelling
M4	A17-A19,A21	Linear, Quadratic and Simultaneous Equations
M5	A22	Linear and Quadratic Inequalities
M6	A23-A25	Sequences: Quadratic, Geometric and Recursive
M7	A8-A10,A16	Straight Lines, Coordinates and Circles
M8	A11, A12, A14, A15, A18	Graphs and Graphical Interpretation
M9	A7, A13	Functions, Composites and Transformations
M10	A20; selected A1-A25 synthesis	Iteration, Numerical Methods and Mixed Higher Algebra

Specification references above identify curriculum coverage only. All questions, diagrams, solutions and marking guidance in this resource are independently authored.

MODULE 01 / A1-A25

How to Use and Check Your Work

STUDENT ROUTINE AND QUALITY STANDARD

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

1 Attempt Work without notes first. Translate the task into an expression, equation, graph or proof before calculating.

2 Mark Use the worked answer. Credit a method only when its mathematical setup is visible.

3 Repair Identify whether the error was modelling, units, algebra, arithmetic or interpretation; then redo from blank.

4 Retest Repeat a similar question after 2-7 days without support.

5 Present Give exact values where appropriate, sensible accuracy, units and a concluding statement.

Higher answer standard

- Define the variables and governing relationship
- State restrictions, domains and excluded values where required
- Preserve exact values until a question requests rounding
- Distinguish average rates (chords) from instantaneous rates (tangents)
- Check whether a result is physically and mathematically reasonable

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Write $3a^2b \times 4ab^3$ in its simplest form.** [2 marks]

2. **Write $18x^3y^2 \div 6xy^5$ using positive indices.** [2 marks]

3. **For $p=-3$ and $q=2$, evaluate $2p^2q-3q^3$.** [2 marks]

4. **State whether $3(x+2)=3x+6$ is an equation or an identity.** [1 mark]

5. **Write the nth even integer and the next consecutive even integer.** [1 mark]

6. **Simplify $(x\text{^}-2y^3)^2$ using positive indices.** [2 marks]

7. **If $f=3a^2/b$, find f when $a=-4$ and $b=6$.** [2 marks]

8. **Write an expression for the product of three consecutive integers starting with n.** [1 mark]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **For $x=-2$ and $y=5$, evaluate $(3x^2-2xy)/(x-y)$.** [3 marks]

2. **Simplify $(6a^3b^{-2})(-2a^{-5}b^4)$, using positive indices.** [3 marks]

3. **Show that $(n+3)^2-(n-3)^2=12n$.** [3 marks]

4. **A student says $(a+b)^2=a^2+b^2$. Give an algebraic correction and a numerical counterexample.** [4 marks]

5. **Simplify $(4x^2y^3)^3/(8x^4y)$, using positive indices.** [3 marks]

6. **For $a=-1/2$ and $b=3$, evaluate $4a^3-2ab^2$ exactly.** [3 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Prove that the sum of two consecutive odd integers is divisible by 4.** [4 marks]

8. **Prove that the difference between the squares of consecutive integers is odd.** [4 marks]

9. **Write 2/x+3/y as one fraction and state restrictions.** [3 marks]

10. **The expression $kx^2-12x+9$ is a perfect square. Find the positive value of k.** [4 marks]

11. **Show that n(n+1) is even for every integer n.** [3 marks]

12. **Given $A=2\pi r(r+h)$, find A exactly when $r=3\sqrt{2}$ and $h=\sqrt{2}$.** [4 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Original multi-step exam-style questions. Show every connected stage.

1. **The side lengths of a rectangle are $2x+3$ and $x-1$. Its area is $3x^2+7x-6$. Determine whether this is an identity and justify your answer.** [4 marks]

2. **Let n be an integer. Prove that n^2+n+6 is even.** [4 marks]

3. **Prove that the square of an odd integer is 1 more than a multiple of 8.** [5 marks]

4. **Simplify $[(3x^{-2}y^3)^2(2x^5y^{-1})]/(6xy^2)$, using positive indices.** [4 marks]

5. **A sequence has nth term n^2+3n. Show that every term is even.** [5 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. The difference between two consecutive square numbers is 41. Find the numbers algebraically. [4 marks]

7. Prove that the product of three consecutive integers is divisible by 6. [4 marks]

8. For $x \neq 0$, simplify $(x+1/x)^2-(x-1/x)^2$ and explain why the result is independent of x. [4 marks]

9. The expression $(ax+b)(2x-3)$ is identical to $6x^2+x-6$. Find a and b. [5 marks]

10. A student claims the sum of any three consecutive integers is divisible by 6. Test and correct the claim using algebra. [5 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Show that $(a+b+c)^2-(a-b-c)^2=4a(b+c)$.** [4 marks]

12. **If $p+q=7$ and $pq=10$, find p^2+q^2 and p^3+q^3 without solving for p and q.** [5 marks]

13. **Prove that n^4-n^2 is divisible by 12 for every integer n.** [5 marks]

14. **Simplify $(a^2-b^2)/(a^2+2ab+b^2) \div (a-b)/(a+b)$, stating restrictions.** [5 marks]

15. **A formula is $T=(2x-y)^2-(2x+y)^2$. Simplify T, then find all integer x,y for which $T=24$ and $xy<0$.** [5 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **Prove that no square integer can leave remainder 2 when divided by 4.** [5 marks]

2. **Prove that if n^2 is even, then n is even.** [5 marks]

3. **The expression $x^2+kx+36$ is a perfect square for integer k. Find all k.** [4 marks]

4. **Show that the sum of the squares of two consecutive odd integers is 2 more than a multiple of 8.** [5 marks]

5. **Given $x+1/x=5$, find x^2+1/x^2 and x^3+1/x^3 exactly.** [5 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the challenge set. Present a clear mathematical argument.

6. **Prove $\sqrt{3}$ is irrational.** [5 marks]

7. **Find all integers n for which n^2+2n+5 is a perfect square less than 30.** [5 marks]

8. **Prove $(n^2+n+1)^2-(n^2+n)^2$ is always odd.** [4 marks]

9. **If $a+b+c=0$, prove $a^3+b^3+c^3=3abc$.** [5 marks]

10. **Find all integer pairs (x,y) satisfying $x^2-y^2=45$.** [5 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Factorise x^2-25.** [2 marks]

2. **Expand (2x-3)(x+5).** [2 marks]

3. **Factorise $6x^2+9x$.** [2 marks]

4. **Simplify $(x^2-9)/(x+3)$.** [2 marks]

5. **Expand $(x+2)^3$.** [3 marks]

6. **Factorise $2x^2-7x+3$.** [3 marks]

7. **Simplify $\sqrt{48}+\sqrt{75}$.** [2 marks]

8. **Rationalise $4/(\sqrt{5}-1)$.** [3 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. **Factorise $6x^2+x-2$.** **[3 marks]**

2. **Expand $(2x-1)(x+3)(x-4)$.** **[4 marks]**

3. **Simplify $(x^2-4x-12)/(x^2-9)$.** **[3 marks]**

4. **Simplify $2/x+3/(x-1)$.** **[4 marks]**

5. **Simplify $(x^2-16)/(x^2+7x+12)$.** **[4 marks]**

6. **Rationalise $5/(2+\sqrt{3})$.** **[3 marks]**

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Expand and simplify $(\sqrt{3}+x)^2-(x-\sqrt{3})^2$.** [3 marks]

8. **Factorise $12x^2-11x-5$.** [3 marks]

9. **Simplify $(3/x)/(6/x^2)$.** [3 marks]

10. **Write $1/(x+1)-1/(x-1)$ as one fraction.** [4 marks]

11. **Complete the factorisation x^3-4x.** [3 marks]

12. **Show $(x+1)(x^2-x+1)=x^3+1$.** [3 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Original multi-step exam-style questions. Show every connected stage.

1. **Solve $(x^2-9)/(x-3)=8$, stating restrictions.** [4 marks]

2. **Simplify $[(x^2-4)/(x^2+x-2)]\div[(x+2)/(x-1)]$.** [5 marks]

3. **Prove $(a+b)^3+(a-b)^3=2a(a^2+3b^2)$.** [5 marks]

4. **Find k if $x^2+kx+49$ is a perfect square.** [4 marks]

5. **Simplify $(\sqrt{12}+\sqrt{27})/(\sqrt{3})$.** [3 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **Rationalise $3/(\sqrt{7}+\sqrt{2})$.** **[4 marks]**

7. **Solve $x/(x-2)=3/(x+1)$.** **[5 marks]**

8. **Show $x^4-16=(x-2)(x+2)(x^2+4)$.** **[4 marks]**

9. **Simplify $(2x/(x^2-9))+(1/(x+3))$.** **[4 marks]**

10. **Factorise $4x^3-12x^2+9x$.** **[4 marks]**

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Find exact value $(3+\sqrt{5})^2+(3-\sqrt{5})^2$.** [4 marks]

12. **If $x+1/x=4$, find x^2+1/x^2.** [4 marks]

13. **Simplify $(a/b-b/a)/(a/b+b/a)$.** [5 marks]

14. **Solve $(x+2)/(x-1)+(x-2)/(x+1)=3$.** [5 marks]

15. **Prove that x^2+4x+8 is positive for every real x.** [4 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **Find all real x satisfying $(x^2-1)/(x-1)=0$.** [4 marks]

2. **Prove $x^4+4=(x^2-2x+2)(x^2+2x+2)$.** [5 marks]

3. **Simplify $1/(x-1)+1/(x+1)-2x/(x^2-1)$.** [4 marks]

4. **Solve $\sqrt{(x+6)}=x$, giving valid roots only.** [5 marks]

5. **Find exact value $1/(2-\sqrt{3})^3$.** [5 marks]

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the challenge set. Present a clear mathematical argument.

6. **If $a+b=5$ and $ab=3$, find a^4+b^4.** [5 marks]

7. **Factorise x^6-y^6 fully.** [5 marks]

8. **Prove $(x^2+y^2)(a^2+b^2)\geq(ax+by)^2$.** [5 marks]

9. **Solve $(x^2-5x+6)/(x^2-4)=0$.** [4 marks]

10. **Find the minimum value of $x^2+6x+13$.** [4 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Make x the subject of $y=3x-7$.** [2 marks]

2. **Make r the subject of $C=2\pi r$.** [2 marks]

3. **Solve $5(2x-3)=4x+9$.** [2 marks]

4. **Solve $(x+3)/4=5$.** [2 marks]

5. **Make h the subject of $A=bh/2$.** [2 marks]

6. **Solve $3/x=5$, $x\neq 0$.** [2 marks]

7. **Make v the subject of $u=v+at$.** [2 marks]

8. **Solve $2x/3-5=7$.** [2 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. **Make x the subject of y=(3x-5)/4.** **[3 marks]**

2. **Make r the subject of V=πr²h.** **[4 marks]**

3. **Make x the subject of y=5/(x-2).** **[4 marks]**

4. **Solve (2x-1)/3-(x+4)/5=2.** **[4 marks]**

5. **Make b the subject of P=2a+2b.** **[3 marks]**

6. **Solve 4/(x+1)=x, stating excluded values.** **[4 marks]**

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Make t the subject of $v^2=u^2+2at$.** [3 marks]

8. **Make x the subject of $y=ax+b$, where $a\neq0$.** [2 marks]

9. **Solve $3(x-2)/4+(x+1)/2=5$.** [4 marks]

10. **Make m the subject of $E=mc^2+K$.** [3 marks]

11. **Solve $\sqrt{(2x+3)}=5$.** [3 marks]

12. **Make q the subject of $p=(q-r)/(q+r)$.** [5 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Original multi-step exam-style questions. Show every connected stage.

1. **Make x the subject of y=(2x+3)/(x-4).** [5 marks]

2. **Make v the subject of 1/f=1/u+1/v.** [5 marks]

3. **Solve 2/(x-1)+3/(x+1)=1.** [5 marks]

4. **A rectangle has perimeter 62 cm. Its length is 5 cm more than twice its width. Find both dimensions.** [4 marks]

5. **The sum of three consecutive integers is 12 less than their product. Find the integers.** [5 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **A right triangle has shorter sides x and x+7 and hypotenuse x+8. Find x and the side lengths.** [5 marks]

7. **Make x the subject of y=√[(x-a)/b].** [4 marks]

8. **Solve (x+2)/(x-3)=2x/(x+1).** [5 marks]

9. **A taxi fare consists of a fixed charge and a constant price per kilometre. A 7 km journey costs £18.20 and a 12 km journey costs £27.70. Form a formula and find the cost of 20 km.** [5 marks]

10. **Make x the subject of A=x^2+2xy, assuming the required root is non-negative.** [5 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. The area of a rectangle is 96 cm². Its length is 4 cm greater than its width. Find its dimensions exactly. [4 marks]

12. A number is increased by 20%, then 15 is subtracted. The result equals 90% of the original number. Find it. [4 marks]

13. Make h the subject of $S=2\pi r^2+2\pi rh$. [4 marks]

14. Solve $3x+\sqrt{(x+1)}=11$. [5 marks]

15. Two numbers have sum 17 and product 60. Form and solve an equation to find them. [4 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **Make x the subject of y=(ax+b)/(cx+d).** [5 marks]

2. **Solve 1/(x-1)+1/(x+1)=3/4.** [5 marks]

3. **A cuboid has square base side x and height x+3. Its volume is 112. Find x.** [5 marks]

4. **Make x the subject of y=x+1/x.** [5 marks]

5. **The roots of x^2-px+q=0 are consecutive positive integers with product 72. Find p and q.** [5 marks]

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **A rectangle has area numerically equal to its perimeter. Its length is twice its width. Find its dimensions.** [4 marks]

7. **Solve $\sqrt{(x+6)}+\sqrt{x}=6$.** [5 marks]

8. **Make r the subject of $V=4\pi r^3/3$, taking $r\geq 0$.** [4 marks]

9. **A car travels 120 km at x km/h and 180 km at (x+20) km/h. Total time is 5 hours. Find x.** [5 marks]

10. **The equation $kx^2-12x+k=0$ has exactly one real solution. Find k.** [5 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Solve $7x-5=3x+19$.** [2 marks]

2. **Solve $x^2-7x+12=0$.** [2 marks]

3. **Solve $3x^2=27$.** [2 marks]

4. **Solve simultaneously: $x+y=11$ and $x-y=3$.** [2 marks]

5. **Solve $x^2+6x+5=0$.** [2 marks]

6. **Solve $4(2x-1)=3x+11$.** [2 marks]

7. **Write x^2-8x+3 in completed-square form.** [2 marks]

8. **State the discriminant of $2x^2-5x+1=0$.** [2 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **Solve 5/(x-2)=x+1, stating the restriction.** [4 marks]

2. **Solve $3x^2-11x-4=0$.** [3 marks]

3. **Solve $2x^2+7x-3=0$, giving exact answers.** [4 marks]

4. **Solve simultaneously: 3x+2y=16 and 5x-y=15.** [4 marks]

5. **The line y=2x+3 intersects $y=x^2-4$. Find the x-coordinates exactly.** [4 marks]

6. **Solve $x^4-13x^2+36=0$.** [4 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Solve $(x-3)^2=5x+1$.** [4 marks]

8. **Find the values of k for which $x^2+kx+16=0$ has equal roots.** [3 marks]

9. **A rectangle has area 60 cm^2 and length 7 cm greater than width. Find its dimensions.** [4 marks]

10. **Solve simultaneously: $y=x^2-2x-3$ and $y=3x+1$.** [5 marks]

11. **The equation $x^2-6x+m=0$ has no real roots. State the range of m.** [3 marks]

12. **Solve $2/(x+1)-1/(x-2)=1$, stating restrictions.** [5 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **The sum of the squares of two consecutive positive integers is 313. Find the integers.** [4 marks]

2. **Solve simultaneously: $xy=12$ and $y=x+1$.** [4 marks]

3. **A line $y=mx+4$ is tangent to $y=x^2$ at one point. Find all possible m.** [4 marks]

4. **A line $y=mx-4$ is tangent to $y=x^2$. Find m and each point of contact.** [5 marks]

5. **Solve $x+12/x=7$, where $x\neq 0$.** [4 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **A right-angled triangle has legs $x+1$ and $x+8$ and hypotenuse $2x+1$. Find its side lengths.** [5 marks]

7. **Solve simultaneously: $2x+y=7$ and $x^2+y^2=13$.** [5 marks]

8. **For what values of p does $px^2+4x+p=0$ have exactly one real solution?** [5 marks]

9. **The roots of $2x^2-7x+c=0$ differ by 3. Find c.** [5 marks]

10. **Solve $\sqrt{(3x+4)}=x+2$ and check every candidate.** [4 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **The graph $y=x^2+ax+b$ passes through (1,0) and has its turning point on $x=3$. Find a and b.** [4 marks]

12. **A number exceeds its reciprocal by 15/4. Find the number exactly.** [4 marks]

13. **Solve simultaneously: $x+y=5$ and $x^2+xy+y^2=19$.** [5 marks]

14. **A square is enlarged so that its side increases by 3 cm and its area increases by 81 cm^2. Find the original side.** [3 marks]

15. **Find all intersections of $y=x^2-4x+1$ and $y=2-x$.** [5 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **The equation $x^2-(k+1)x+k=0$ has roots whose difference is 5. Find all k.** [5 marks]

2. **Solve $x^4-5x^2+4=0$ and explain why the substitution used loses no real solutions.** [5 marks]

3. **A quadratic $x^2+px+q=0$ has roots α and β. Given $\alpha^2+\beta^2=34$ and $\alpha\beta=3$, find all possible p and q.** [5 marks]

4. **The line $y=2x+c$ is tangent to $y=x^2-4x+7$. Find c and the point of contact.** [5 marks]

5. **Solve simultaneously over the reals: $x^2+y^2=25$ and $xy=12$.** [5 marks]

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **A monic quadratic has integer roots. Their sum equals their product and both roots are greater than 1. Find the roots.** [5 marks]

7. **Find all real values of k for which $x^2-2kx+k+2=0$ has two distinct positive roots.** [5 marks]

8. **Solve $\sqrt{(x+4)}+\sqrt{(9-x)}=5$.** [5 marks]

9. **The curves $y=x^2$ and $y=6x-k$ meet at two points whose x-coordinates differ by 2. Find k.** [5 marks]

10. **A fraction has denominator 2 more than its numerator. Adding 3 to both makes the fraction 4/5. Find the original fraction.** [3 marks]

MODULE 05 / A22

Linear and Quadratic Inequalities

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Solve $3x+5<20$.** [2 marks]

2. **Solve $-2x\le 8$.** [2 marks]

3. **Solve $4\le 2x+6<12$.** [2 marks]

4. **List the integer values satisfying $-2<x\le 3$.** [2 marks]

5. **Solve $x^2-9>0$.** [2 marks]

6. **Solve $x^2-5x\le 0$.** [2 marks]

7. **Solve $7-3x>1$.** [2 marks]

8. **Write $x\in[-4,2)$ as inequalities.** [1 mark]

MODULE 05 / A22

Linear and Quadratic Inequalities

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **Solve $5-2(3x-1)\geq 4x+7$.** **[3 marks]**

2. **Solve $(2x-1)/3>(x+4)/2$.** **[3 marks]**

3. **Solve $-3<2-5x\leq 12$.** **[4 marks]**

4. **Solve $x^2+x-12<0$.** **[3 marks]**

5. **Solve $2x^2-7x+3\geq 0$.** **[3 marks]**

6. **Solve $x^2+4x+3\leq 0$.** **[3 marks]**

MODULE 05 / A22

Linear and Quadratic Inequalities

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Find the integers satisfying $x^2-2x-8<0$.** [3 marks]

8. **Solve $(x-1)(x+5)>0$.** [3 marks]

9. **Solve $3x^2+2x-1\leq 0$.** [3 marks]

10. **A phone plan costs £12 plus £0.08 per text. Write and solve an inequality for at most £20.** [3 marks]

11. **Solve $x^2+2x-7>0$, giving exact boundary values.** [4 marks]

12. **Solve $4-(x-1)^2\geq 0$.** [3 marks]

MODULE 05 / A22

Linear and Quadratic Inequalities

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **Solve $(x+2)/(x-1)>0$.** [4 marks]

2. **Solve $(2x-3)/(x+4)\leq 0$.** [4 marks]

3. **Solve $x^2-6x+5<4$.** [4 marks]

4. **Find all k for which x^2-4x+k is positive for every real x.** [4 marks]

5. **Solve simultaneously $x+y\leq 7$, $x\geq 2$ and $y\geq 3$ for integer x,y. List all pairs with $x\leq 4$.** [4 marks]

MODULE 05 / A22

Linear and Quadratic Inequalities

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________ Date: __________ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. **A rectangle has width x cm and length $x+5$ cm. Its area is at most 84 cm². Given $x>0$, find the range of x.** [4 marks]

7. **Solve $(x-2)^2 \geq 3x+4$.** [4 marks]

8. **Solve $x^3-4x>0$.** [4 marks]

9. **Solve $(x^2-9)/(x-1) \geq 0$.** [5 marks]

10. **The equation $x^2+(k-1)x+k=0$ has two distinct negative roots. Find the range of k.** [5 marks]

MODULE 05 / A22

Linear and Quadratic Inequalities

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Solve $|2x-5|<7$.** **[3 marks]**

12. **Solve $|3x+1|\geq 8$.** **[3 marks]**

13. **Find the values of x for which $y=x^2-2x-8$ lies below the line $y=x+2$.** **[4 marks]**

14. **A ball is modelled by $h=-5t^2+20t+1$. Find the times for which $h\geq 16$.** **[4 marks]**

15. **Solve $1/(x-2)<1$, stating restrictions.** **[5 marks]**

MODULE 05 / A22

Linear and Quadratic Inequalities

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **Find all k for which $x^2-2kx+k^2-9\le 0$ has solution set of length 8.** [4 marks]

2. **Solve $(x^2-4)/(x^2-1)>0$.** [5 marks]

3. **Find all real k for which $x^2+(k-2)x+k>0$ for every real x.** [5 marks]

4. **Solve $|x-2|\le|2x+1|$.** [5 marks]

5. **The quadratic $x^2-px+12$ is negative exactly when $3<x<4$. Find p.** [4 marks]

MODULE 05 / A22

Linear and Quadratic Inequalities

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the challenge set. Present a clear mathematical argument.

6. **Solve $(x-1)/(x+2) \geq (x+3)/(x-4)$.** [5 marks]

7. **Find the integer values n such that $n^2-11n+24<0$ and $2n+1 \leq 17$.** [4 marks]

8. **For which values of a does $ax^2-4x+1 \geq 0$ for every real x?** [5 marks]

9. **Solve $x^4-5x^2+4 \leq 0$.** [5 marks]

10. **A company sells x items. Revenue is $R=40x-x^2$ and cost is $C=4x+60$. Find the integer values of x for which profit is positive.** [5 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **Find the nth term of 7, 12, 17, 22, ...** [2 marks]

2. **Find the 20th term of the sequence 3n-8.** [1 mark]

3. **Is 97 a term of 4n+1?** [2 marks]

4. **Write the next two terms of 5, 10, 20, 40, ...** [1 mark]

5. **For $u_n=3u_{n-1}-1$ with $u_1=2$, find u_2 and u_3.** [2 marks]

6. **Find the first difference and second difference of 2, 7, 14, 23, ...** [2 marks]

7. **Write the next term of 2, 3, 5, 8, 13, ...** [1 mark]

8. **Find the common ratio of 81, 27, 9, 3, ...** [1 mark]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. **Find the nth term of 4, 11, 22, 37, 56, ...** [4 marks]

2. **Find the 15th term of n^2-4n+7.** [2 marks]

3. **Determine whether 250 is a term of n^2+n.** [3 marks]

4. **A geometric sequence has first term 6 and common ratio 3. Find its nth term.** [2 marks]

5. **The third term of a geometric sequence is 20 and the fourth is 10. Find its first term.** [3 marks]

6. **Given $u_1=4$ and $u_{n+1}=2u_n+3$, find u_4.** [3 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **The sequence begins 1, 4, 10, 22, ... Find a recurrence relation.** [3 marks]

8. **Find the nth term of 8, 15, 24, 35, 48, ...** [4 marks]

9. **The nth term is $2n^2-3n$. Find the first term greater than 100.** [3 marks]

10. **A sequence is defined by $u_n=5(-2)^{(n-1)}$. Write its first four terms.** [2 marks]

11. **For the Fibonacci-type sequence $u_1=3$, $u_2=7$ and $u_n=u_{n-1}+u_{n-2}$, find u_6.** [3 marks]

12. **A sequence has nth term an^2+bn and begins 5, 14, 27. Find a and b.** [4 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Original multi-step exam-style questions. Show every connected stage.

1. **The nth term of a quadratic sequence is n^2+kn+6. Its fifth term is 46. Find k and the tenth term.** [4 marks]

2. **Find the positions at which the sequence $n^2-9n+14$ has value 14.** [3 marks]

3. **The consecutive terms 12, x, 27 belong to a geometric sequence with positive ratio. Find x and the ratio.** [4 marks]

4. **A geometric sequence has second term 12 and fifth term 324. Find both possible real values of its first term and ratio.** [4 marks]

5. **The sequence $u_1=10$, $u_{n+1}=ku_n-2$ has $u_3=22$. Find all possible k.** [5 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **A quadratic sequence has first differences 7, 13, 19, ... and first term 2. Find its nth term.** [4 marks]

7. **Two sequences are $u_n=n^2+4$ and $v_n=7n-2$. Find every positive integer n for which $u_n=v_n$.** [4 marks]

8. **A theatre has 18 seats in row 1, 22 in row 2 and 26 in row 3. It has 25 rows. Find the total number of seats.** [4 marks]

9. **The first three terms of a quadratic sequence are p, 3p+1, 8p-2. The second difference is 10. Find p.** [4 marks]

10. **A geometric sequence has positive terms. Its first and third terms sum to 50, and its second term is 20. Find the first three terms.** [5 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **The recurrence $u_1=1$, $u_{n+1}=\sqrt{6u_n+4}$ approaches a positive limit L. Find L.** [4 marks]

12. **Show that the sum of the first n odd numbers is n^2.** [4 marks]

13. **The nth term of a sequence is n^2-5n+3. Find its minimum term for positive integer n.** [4 marks]

14. **A population is modelled by $P_n=800\times1.06^n$, where n is whole years after measurement. Find the first n for which P_n exceeds 1200.** [5 marks]

15. **Let $a=1+\sqrt{2}$. A geometric sequence has first term 1 and ratio a. Find its fourth term in the form $p+q\sqrt{2}$.** [4 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **A quadratic sequence begins 2, 6, 12, 20, ... Prove that no term after the first is prime.** [4 marks]

2. **Find all positive integer positions at which $n^2-7n+17$ is a perfect square less than 10.** [5 marks]

3. **A geometric sequence has first term a and ratio r. Its first three terms sum to 21 and their product is 216. Find all real triples of consecutive terms.** [5 marks]

4. **The sequence $u_1=2$, $u_{n+1}=u_n^2-2$ has a repeated value. Determine all its terms.** [3 marks]

5. **Prove that every term of the sequence n^3-n is divisible by 6.** [5 marks]

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ____________ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **A quadratic sequence has nth term n^2+bn+c. Its 3rd term is 2 and its 8th term is 42. Find b and c, then find its zero terms.** [5 marks]

7. **Let $u_1=\sqrt{2}$ and $u_{n+1}=2/u_n$. Prove that the sequence is constant.** [4 marks]

8. **In a Fibonacci sequence with $u_{n+2}=u_{n+1}+u_n$, the fifth and sixth terms are 29 and 47. Find the first two terms.** [4 marks]

9. **A sequence is defined by $u_n=(1+\sqrt{5})^n+(1-\sqrt{5})^n$. Prove u_n is an integer for every positive integer n.** [5 marks]

10. **The positive sequence $x_1,x_2,\dots$ satisfies $x_{n+1}=1+1/x_n$ and approaches a limit L. Find L exactly and explain why the negative root is rejected.** [5 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Find the gradient of the line through (2,3) and (8,15). [2 marks]

2. State the gradient and y-intercept of $y=-3x+7$. [1 mark]

3. Find the midpoint of (-4,5) and (6,-1). [2 marks]

4. Find the length of the line segment joining (1,2) to (4,6). [2 marks]

5. Write the equation of the line with gradient 4 and y-intercept -5. [1 mark]

6. State the centre and radius of $x^2+y^2=49$. [1 mark]

7. Find the gradient of a line perpendicular to $y=2x+1$. [1 mark]

8. Does (3,4) lie on $x^2+y^2=25$? [1 mark]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ________________________________ Date: ______________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **Find the equation of the line through (3,-2) with gradient 5.** [3 marks]

2. **Find the equation of the line through (-2,7) and (4,-5).** [4 marks]

3. **Find the equation of the line perpendicular to $y=3x-4$ and passing through (6,2).** [3 marks]

4. **The line $2y-6x=9$ is parallel to $y=mx+1$. Find m.** [2 marks]

5. **Find the coordinates where $y=2x+5$ meets $y=-x-1$.** [3 marks]

6. **Find the equation of the perpendicular bisector of the segment joining (2,1) and (8,5).** [4 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Write $x^2+y^2-8x+6y-11=0$ in centre-radius form.** [4 marks]

8. **Find the centre and radius of $(x+2)^2+(y-5)^2=13$.** [2 marks]

9. **Find the equation of the tangent to $x^2+y^2=25$ at (3,4).** [4 marks]

10. **The points A(1,2), B(5,10) and C(7,14) are given. Show they are collinear.** [3 marks]

11. **Find the exact distance from (-3,4) to (5,-2).** [2 marks]

12. **A line has equation $ax+by=12$ and passes through (2,3). Given $a=3$, find b and its gradient.** [3 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **The line L passes through (4,-1) and is perpendicular to 2x-3y=12. Find its equation in the form ax+by=c.** [4 marks]

2. **Find the coordinates of the point that divides A(-2,5) to B(10,-1) internally in the ratio 1:2.** [3 marks]

3. **The points P(k,5), Q(2,-1) and R(8,11) are collinear. Find k.** [4 marks]

4. **Find the equation of the circle with diameter endpoints (-2,1) and (6,7).** [4 marks]

5. **The line y=x+1 intersects $x^2+y^2=25$. Find the intersection points exactly.** [4 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **Find the equations of both tangents to $x^2+y^2=25$ at the points where $x=3$.** [5 marks]

7. **A triangle has vertices A(-1,2), B(5,4), C(3,10). Show it is right-angled and state the right angle.** [4 marks]

8. **Find the area of the triangle with vertices (1,1), (7,3), and (4,8).** [4 marks]

9. **A circle has centre (4,-2) and passes through (-1,10). Find its equation.** [3 marks]

10. **The tangent to $x^2+y^2=65$ at P(1,8) meets the x-axis at A. Find A.** [5 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Find the shortest distance from P(5,-1) to the line y=2x+3.** [5 marks]

12. **Lines L_1:y=3x-2 and L_2:y=-x+10 meet at P. Find the equation of the line through P and the origin.** [4 marks]

13. **A circle $x^2+y^2=100$ and the vertical line x=6 enclose a chord. Find its exact length.** [3 marks]

14. **The line y=mx+5 is tangent to $x^2+y^2=25$. Find m.** [5 marks]

15. **The vertices A(2,1), B(8,5), C(6,8) form three vertices of a parallelogram ABCD in order. Find D and the equation of diagonal BD.** [4 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **Find all values of k for which $y=kx+6$ is tangent to $x^2+y^2=20$.** [5 marks]

2. **A circle has equation $x^2+y^2-6x+4y-12=0$. Find the tangent at (6,2).** [5 marks]

3. **Prove that the diagonals of the quadrilateral A(-2,1), B(4,3), C(6,9), D(0,7) bisect each other.** [4 marks]

4. **The line through P(4,7) is tangent to the circle $x^2+y^2=65$. Find its equation using an algebraic tangency condition.** [5 marks]

5. **A variable point P(x,y) is equidistant from A(-2,3) and B(6,-1). Find the locus equation and interpret it.** [5 marks]

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Complete the challenge set. Present a clear mathematical argument.

6. **The circles $x^2+y^2=25$ and $(x-6)^2+y^2=25$ intersect. Find their intersection points exactly.** [4 marks]

7. **Find the equation of the circle through (0,0), (6,0), and (0,8).** [5 marks]

8. **The lines $y=2x+1$, $y=-x+7$ and $y=k$ form a triangle of area 27/2. Find all possible k.** [5 marks]

9. **A line through (10,0) is tangent to $x^2+y^2=36$. Find the tangent points exactly.** [5 marks]

10. **The point P lies on $y=x^2$ and is equidistant from A(0,1) and B(4,1). Find P.** [4 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. **For y=x^2-5x+6, state the y-intercept.** [1 mark]

2. **Solve x^2-9=0 by reading the roots of y=x^2-9.** [1 mark]

3. **State the vertical and horizontal asymptotes of y=4/x.** [2 marks]

4. **State the y-intercept of y=3^x.** [1 mark]

5. **The curve y=x^3 is translated 2 units upwards. Write its equation.** [2 marks]

6. **Find the gradient of the chord joining (1,2) and (5,14).** [2 marks]

7. **A velocity-time graph is horizontal at 12 m/s for 7 seconds. Find the distance.** [2 marks]

8. **For y=(x-4)^2-7, state the turning point.** [1 mark]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **Find the roots and turning point of y=x^2-6x+5.** [4 marks]

2. **A quadratic has roots -2 and 7 and passes through (0,-28). Find its equation.** [4 marks]

3. **Solve x^3-4x=0 and identify all x-intercepts of the graph.** [3 marks]

4. **The graphs y=x^2+1 and y=4x-2 intersect. Find their x-coordinates.** [4 marks]

5. **Describe fully the transformation from y=1/x to y=1/(x-3)+2.** [3 marks]

6. **Solve 2^x=11 using a graph or logarithms, giving x to 3 significant figures.** [3 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the Higher set. Interpret each result in context.

7. **Estimate the gradient of y=x^2 at x=4 using the points with x=3.9 and x=4.1.** [4 marks]

8. **A tangent to a curve passes through (2,11) and (8,-4). Estimate the instantaneous gradient.** [3 marks]

9. **A velocity rises uniformly from 6 to 18 m/s in 8 s. Find the distance travelled.** [3 marks]

10. **A graph of y=x^2+px+12 has roots -3 and -4. Find p.** [3 marks]

11. **State the range of y=2(x+1)^2-5.** [3 marks]

12. **The curve y=k/x passes through (6,-4). Find k and the value of y when x=-3.** [3 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **The line y=2x+3 intersects y=x^2-4x+7. Find both points of intersection.** **[5 marks]**

2. **A quadratic y=x^2+bx+c has turning point (3,-7). Find b and c.** **[4 marks]**

3. **The graph y=x^3-5x^2+2x+8 crosses the x-axis at x=2. Factorise fully.** **[5 marks]**

4. **The reciprocal curve y=12/(x-1)-3 meets the x-axis and y-axis. Find both intercepts and its asymptotes.** **[5 marks]**

5. **The graphs y=3^x and y=20-x intersect once between x=2 and x=3. Show this and estimate the root to 1 decimal place.** **[5 marks]**

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **A particle has velocity v=18-3t for 0<=t<=8. Find its displacement and total distance travelled.** [5 marks]

7. **A curve passes through (1,6),(3,2),(7,10). Compare the two average rates of change and explain what they imply.** [4 marks]

8. **For y=x^2, calculate the average gradients between x=3 and x=3.1, and between x=3 and x=3.01. Use the results to estimate the tangent gradient at x=3.** [5 marks]

9. **A rectangle has perimeter 24 cm. If one side is x, form an area graph and use its turning point to find the maximum area.** [5 marks]

10. **A ball has height h=2+20t-5t^2. Find its maximum height and when it hits the ground.** [5 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. The area under a speed-time graph is 390 m. Speed rises linearly from 10 to 25 m/s in 12 s, then remains at 25 m/s for T seconds. Find T. [4 marks]

12. A cubic has roots -1, 2 and 5 and y-intercept 20. Find its equation. [4 marks]

13. The graphs y=8/x and y=x+2 intersect. Find the coordinates exactly. [4 marks]

14. The population model P=500(1.08)^t is plotted. Find the average annual increase from t=2 to t=7. [5 marks]

15. A tangent at x=3 to a curve is y=7x-11. State the point of contact and use it to write the normal equation. [5 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. A quadratic y=ax^2+bx+c passes through (0,6) and has roots 2 and k. Its turning point has x-coordinate 5. Find a,b,c and k. [5 marks]

2. The line y=mx+1 is tangent to y=x^2-4x+8. Find the possible values of m. [5 marks]

3. Prove that the average gradient of y=1/x from x=a to x=b is -1/(ab), where a,b are non-zero and unequal. [5 marks]

4. A curve y=x^3+px^2+qx has x-intercepts -2,0,5. Find p and q, then find the average gradient from x=-2 to x=3. [5 marks]

5. A velocity-time curve has values v=8,3,0,-1,0,3,8,15 at times t=0,1,...,7. Use seven trapezia of width 1 to estimate displacement and total distance. [5 marks]

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **The curve y=2^x is transformed to y=-3(2^(x+1))+4. Describe the transformations and state the horizontal asymptote.** [5 marks]

7. **A quadratic has integer coefficients, roots 3+√ (5) and 3-√ (5), and leading coefficient 2. Find it.** [5 marks]

8. **The curves y=x^2 and y=12/x enclose a first-quadrant intersection. Find its coordinates and explain why there is only one positive intersection.** [5 marks]

9. **A tangent to y=x^2 at x=a passes through (0,-9). Using tangent gradient 2a, find a.** [5 marks]

10. **A model R=40x/(x^2+4), x>0, represents a rate. Show R<=10 and identify equality.** [5 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. For f(x)=4x-7, find f(3). [1 mark]

2. For g(x)=x^2+2, find g(-4). [1 mark]

3. If f(x)=2x+1, solve f(x)=15. [2 marks]

4. For f(x)=3x-5, find f^-1(x). [2 marks]

5. If f(x)=x+4 and g(x)=2x, find fg(3). [2 marks]

6. Describe y=f(x)+6. [1 mark]

7. Describe y=f(x-2). [1 mark]

8. State why f(x)=x^2 on all real numbers has no inverse function. [2 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. **f(x)=5x-4 and g(x)=x^2. Find gf(-2).** [3 marks]

2. **f(x)=2x+7. Show that f^-1(f(x))=x.** [3 marks]

3. **f(x)=x^2-3, x>=0. Find f^-1(x), including its domain.** [4 marks]

4. **f(x)=3x+2 and g(x)=x-5. Find fg(x) and gf(x).** [3 marks]

5. **Given f(x)=ax+3 and f(4)=23, find a and f^-1(x).** [4 marks]

6. **Solve fg(x)=19 for f(x)=2x+1 and g(x)=x^2-3, x>0.** [4 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **Describe the transformation y=-f(x)+4.** [3 marks]

8. **Describe y=f(3x).** [3 marks]

9. **The point (5,-2) lies on y=f(x). Give the corresponding point on y=f(x-4)+3.** [3 marks]

10. **The point (-3,7) lies on y=f(x). Give the corresponding point on y=2f(-x).** [4 marks]

11. **f(x)=(x-1)/(x+2). State the excluded input and find f(4).** [3 marks]

12. **Given f(x)=1/(x-3), solve f(x)=2.** [3 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **f(x)=2x-3 and g(x)=x^2+4. Solve gf(x)=20.** [4 marks]

2. **f(x)=(3x-2)/(x+4). Find f^-1(x).** [5 marks]

3. **f(x)=x^2-6x+11, restricted to x>=3. Find its inverse.** [5 marks]

4. **f(x)=ax+b and f(f(x))=9x-8. Given a>0, find a and b.** [5 marks]

5. **Functions f and g satisfy f(x)=4x+1 and fg(x)=12x-7. Find g(x).** [4 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **f(x)=x/(x+1). Show that f^-1(x)=x/(1-x), stating restrictions.** [5 marks]

7. **A point (a,b) lies on y=f(x). Find its image on y=-2f(3(x-4))+5.** [5 marks]

8. **The graph y=f(x) has range -2<=y<=7. Find the range of y=3-2f(x).** [4 marks]

9. **Let f(x)=(2x+1)/(x-3) and g(x)=(3x+1)/(x-2). Show that f and g are inverse functions, stating the necessary restrictions.** [5 marks]

10. **For f(x)=x^2+1 and g(x)=3x-2, solve fg(x)=gf(x).** [5 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **The transformation y=f(x-p)+q maps a turning point (-2,5) to (4,-1). Find p and q.** [4 marks]

12. **The graph y=1/x is transformed by a horizontal stretch factor 2, then translated (3,-4). Find an equation.** [5 marks]

13. **A function maps x to (x-2)^2 on domain x>=2. Its inverse maps 25 to what value? Explain.** [3 marks]

14. **h(x)=f(x+1)-2. Given h(4)=7, find a value of f.** [3 marks]

15. **f(x)=kx+6 and f^-1(10)=2. Find k.** [3 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Extended reasoning. Plan the model before calculating.

1. **A linear function f satisfies f(2)=7 and f^-1(11)=4. Find f and hence f^5(0), where f^5 means five repeated applications.** [5 marks]

2. **f(x)=(x+1)/(x-1). Prove that f(f(x))=x and state all necessary restrictions.** [5 marks]

3. **Functions f(x)=x^2 and g(x)=x+2 are given. Explain why fg(x)=gf(x) has only one real solution and find it.** [4 marks]

4. **A graph y=f(x) is invariant under the transformation y=f(-x). What symmetry must it have? Give a non-quadratic example.** [4 marks]

5. **For f(x)=ax+b with a!=0, determine when f is its own inverse.** [5 marks]

MODULE 09 / A7, A13

Functions, Composites and Transformations

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the challenge set. Present a clear mathematical argument.

6. **The graph y=f(x) contains (2,5). It is transformed to y=3f(2x+4)-7. Find the corresponding point and explain each coordinate.** [5 marks]

7. **f(x)=√ (x+4) and g(x)=x^2-4 with domain x>=0. Show f and g are inverses and identify both ranges.** [5 marks]

8. **Solve f^-1(x)=f(x) for f(x)=2x-3.** [4 marks]

9. **A transformation sends every point (x,y) on y=f(x) to (2-x,3y+1). Write the transformed equation.** [5 marks]

10. **A function f has domain 1<=x<=6 and range -4<=f(x)<=2. State domain and range of h(x)=5-2f(x+3).** [5 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Using x_(n+1)=(10+x_n)/3 with x_1=2, find x_2. [1 mark]

2. Continue x_(n+1)=√(6+x_n) from x_1=3 to find x_2. [1 mark]

3. State the fixed-point equation for x_(n+1)=4/(x_n+1). [2 marks]

4. Show that x=2 is a root of x^3-3x-2=0. [1 mark]

5. Find the sign of f(x)=x^2-5 at x=2 and x=3. [2 marks]

6. State what a sign change of a continuous function across an interval proves. [2 marks]

7. Rearrange x^2+x-8=0 into x=√(8-x), taking the positive branch. [2 marks]

8. Why should calculator values be kept unrounded during iteration? [1 mark]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Higher-tier application. Show the governing relationship and exact working.

1. **Use x_(n+1)=√ (10-x_n), x_1=3, to find x_2 and x_3 to 4 d.p.** [3 marks]

2. **Show that x^3+x-5=0 has a root between 1 and 2.** [3 marks]

3. **Locate the positive root of x^2+x-7=0 between consecutive tenths.** [3 marks]

4. **The iteration x_(n+1)=12/(x_n+2) converges to 3. Show that 3 satisfies the related equation.** [3 marks]

5. **Find the exact fixed points of x_(n+1)=6/(x_n+1).** [3 marks]

6. **Starting x_1=2, apply x_(n+1)=(x_n+5/x_n)/2 twice.** [4 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **Use trial values to solve x^3=20 to 1 decimal place.** [3 marks]

8. **For f(x)=x^3-2x-4, show that the root is exactly 2.** [2 marks]

9. **Explain why x_(n+1)=√ (4x_n+5) can only generate non-negative iterates.** [2 marks]

10. **A sequence generated by x_(n+1)=0.4x_n+6 converges. Find its limit.** [3 marks]

11. **Use x_(n+1)=∛ (7x_n+6), x_1=2, to find x_2.** [2 marks]

12. **A root is known to lie in [4.26,4.27]. State the root to 2 decimal places if further testing shows it is below 4.265.** [2 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. **Use x_(n+1)=∛ (2x_n+20), x_1=3, to obtain the root to 3 decimal places.** [5 marks]

2. **Show that x^3-4x-1=0 has a root between 2.1 and 2.2, then give it to 2 d.p.** [5 marks]

3. **The iteration x_(n+1)=√ (15-2x_n), x_1=3, converges. Find the limiting value exactly.** [4 marks]

4. **Rearrange x^3+x=12 into an iteration and use it from x_1=2 to estimate the positive root.** [5 marks]

5. **A rectangle has area 40 cm^2 and length 3 cm more than width. Form an equation and solve for its dimensions.** [4 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Continue the multi-step set. Explain decisions where requested.

6. **The line y=x+1 meets the circle x^2+y^2=25. Find both intersection points exactly.** [5 marks]

7. **A quadratic sequence begins 6,15,28,45. Find its nth term and solve for the term equal to 276.** [5 marks]

8. **Solve simultaneously y=x^2-2x-3 and y=2x+5, giving exact solutions.** [5 marks]

9. **A function f(x)=x^3+x-9. Use a change-of-sign argument to show exactly one root lies between 1 and 2.** [5 marks]

10. **The iterative model P_(n+1)=1.04P_n-120 starts with P_0=5000. Find P_3 and interpret.** [4 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **A formula gives x_(n+1)=2+3/x_n. Starting at 3, determine the limit exactly.** **[4 marks]**

12. **Solve (x-1)/(x+2)=2/x, stating excluded values.** **[5 marks]**

13. **A curve y=x^2+kx+9 touches the x-axis. Find k.** **[4 marks]**

14. **Use bounds to show √ (30) rounds to 5.5 to 1 decimal place.** **[4 marks]**

15. **A geometric sequence has third term 18 and sixth term 486. Find the common ratio and first term.** **[4 marks]**

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. **For x>0, the iteration x_(n+1)=(2x_n+12/x_n^2)/3 is used. Show any positive fixed point solves x^3=12, then compute two iterates from x_1=2.** [5 marks]

2. **The equation x^3-6x+1=0 has three real roots. Use sign changes over four integer test points to justify this.** [5 marks]

3. **An iteration converges to L, where L=5-6/L. Find possible limits and explain why the starting value matters.** [5 marks]

4. **A quadratic x^2-px+12=0 has integer roots differing by 2. Find all positive p.** [5 marks]

5. **Solve x^4-13x^2+36=0.** [4 marks]

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **A tangent from point $(13,0)$ touches $x^2+y^2=25$ at $P=(a,b)$, $b>0$. Find P.** [5 marks]

7. **A sequence satisfies $u_{n+1}=4u_n-3$ and $u_1=2$. Conjecture and prove a formula.** [5 marks]

8. **The functions $f(x)=x^2-4x$ and $g(x)=kx-3$ are tangent for two possible k. Find k.** [5 marks]

9. **A student claims that $x_{n+1}=\sqrt[3]{10-x_n}$, starting at $x_1=2$, is an iteration for $x^3+x-10=0$ and that $x_2=28/13$. Assess both claims.** [5 marks]

10. **A model has fixed point equation $L=\sqrt{aL+6}$. It converges to $L=3$. Find a, then state the other algebraic solution of the squared equation.** [5 marks]

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Index laws	Add powers when multiplying, subtract when dividing, multiply powers for a power of a power.
Identity	An identity is true for all permitted values; prove it by valid algebra.
Parity proof	Represent even integers by 2n and odd integers by 2n+1.
Divisibility	Factor the expression to expose required integer factors.
Algebraic fractions	Factor first, cancel factors only, and state excluded values.
Counterexample	One valid counterexample disproves a universal claim.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Quadratics	Factor by finding two products matching ac and b.
Algebraic fractions	Factor first; cancel factors, never terms; retain original restrictions.
Surds	Collect like surds and rationalise with a conjugate.
Perfect squares	Compare with $(ax \pm b)^2$.
Positivity	Complete the square to expose a non-negative term.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Changing the subject	Undo operations in reverse order while preserving equality.
Variable in a denominator	Multiply through by denominators first and record excluded values.
Variable appears twice	Collect all terms containing the required subject, then factor it out.
Square roots	State sign/domain conditions, square, solve and check for extraneous roots.
Contextual equations	Define the unknown, form the equation, solve, reject invalid roots and interpret.
Quadratic rearrangement	Move all terms to one side, then factorise, complete the square or use the quadratic formula.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Quadratic methods	Factorise where possible; otherwise complete the square or use $x=(-b\pm\sqrt{(b^2-4ac)})/(2a)$.
Discriminant	For $ax^2+bx+c=0$: $D>0$ gives two distinct real roots, $D=0$ one repeated root, and $D<0$ no real roots.
Linear simultaneous equations	Eliminate one variable by matching coefficients, then substitute back and check both equations.
Linear–quadratic systems	Substitute the linear expression into the quadratic relation; each valid root gives a coordinate pair.
Model formation	Define the unknown, translate every condition into an equation, solve, then reject roots that violate the context.
Hidden quadratics	For equations in x^4 and x^2, substitute $u=x^2$ and remember that real solutions require $u\geq 0$.
Tangency	At a tangent there is one repeated intersection, so the resulting quadratic has discriminant zero.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 05 / A22

Linear and Quadratic Inequalities

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Linear inequalities	Solve like an equation, but reverse the inequality whenever multiplying or dividing by a negative number.
Compound inequalities	Apply each operation to all three parts; preserve open and closed endpoints.
Quadratic inequalities	Find the roots, sketch or use a sign chart, then select intervals with the required sign.
Rational inequalities	Put everything over one denominator, mark zeros and excluded values, and test every interval.
Absolute value	$\|u\|<a$ means $-a<u<a$; $\|u\|\geq a$ means $u\leq -a$ or $u\geq a$, for $a\geq 0$.
Parameter conditions	Use the discriminant, turning point, root sum/product and leading coefficient together.
Context	Intersect the algebraic solution with practical restrictions such as positive lengths, integer counts and time domains.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Linear sequences	For common difference d, use dn plus a constant fixed from any term.
Quadratic sequences	A constant second difference 2a gives leading coefficient a in an^2+bn+c.
Geometric sequences	Use $u_n=ar^{(n-1)}$; consecutive terms have constant ratio r.
Recursive sequences	State starting term(s) and the recurrence; substitute carefully one step at a time.
Membership	Set the nth-term formula equal to the value and require n to be a positive integer.
Limiting values	If a recurrence converges, replace consecutive terms by L, solve, then apply domain/sign conditions.
Series totals	For an arithmetic series use $S_n=n(\text{first}+\text{last})/2$.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Gradient	Use $m=(y_2-y_1)/(x_2-x_1)$. Parallel lines share a gradient; perpendicular gradients multiply to -1.
Line equation	Use $y-y_1=m(x-x_1)$, then rearrange into the requested form.
Distance and midpoint	Distance is $\sqrt{[(\Delta x)^2+(\Delta y)^2]}$; midpoint is the mean of corresponding coordinates.
Circle form	$(x-a)^2+(y-b)^2=r^2$ has centre (a,b) and radius r.
Tangents	A tangent is perpendicular to the radius at the point of contact.
Line-circle intersections	Substitute the line into the circle; two roots mean a secant, one repeated root a tangent, and no real roots no intersection.
Coordinate proof	Use exact gradients, lengths or midpoints to justify parallel, perpendicular, equal or bisected relationships.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Quadratic features	Use factorised form for roots and completed-square form for the turning point.
Intersections	Equate the two expressions for y, solve, then substitute back.
Reciprocal graphs	For y=a/(x-h)+k, asymptotes are x=h and y=k.
Exponential graphs	a^x is positive; solve intersections numerically or with logarithms where permitted.
Rates from graphs	Chord gradient is average rate; tangent gradient is instantaneous rate.
Areas under graphs	On time graphs, signed area gives displacement; total area gives distance.
Graph validation	Label intercepts, stationary points, asymptotes and relevant domain/range.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 09 / A7, A13

Functions, Composites and Transformations

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Composite order	fg(x)=f(g(x)): apply the right-hand function first.
Inverse functions	Set y=f(x), rearrange for x, then swap x and y; state restrictions.
One-to-one condition	An inverse function requires each output to come from one permitted input.
Translations	y=f(x-a)+b translates by vector (a,b).
Stretches/reflections	Inside changes x-coordinates inversely; outside changes y-coordinates directly.
Point mapping	Set the new inside expression equal to the old x-coordinate, then transform the y-value.
Checks	Verify f^-1(f(x))=x on the stated domain.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Iteration	Choose a rearrangement x=g(x), substitute repeatedly and retain full precision.
Fixed point	If iterates converge to L, then L=g(L); solve and select a limit consistent with the iteration.
Root bracketing	For a continuous function, opposite signs at endpoints guarantee at least one root.
Accuracy	To justify rounding, test the appropriate half-unit boundaries.
Numerical caution	An iteration may diverge, oscillate or approach different fixed points; do not assume convergence.
Synthesis	Translate context to algebra, solve, reject invalid roots and interpret units.
Verification	Substitute solutions into the original equation, especially after squaring or clearing denominators.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$12a^3b^4$.	**2**	M1 method; A1 answer.
2	$3x^2/y^3$.	**2**	M1 method; A1 answer.
3	2(9)(2)-3(8)=36-24=12.	**2**	M1 method; A1 answer.
4	It is an identity because it is true for every x.	**1**	B1: correct result or statement.
5	2n and 2n+2.	**1**	B1: correct result or statement.
6	y^6/x^4.	**2**	M1 method; A1 answer.
7	f=3(16)/6=8.	**2**	M1 method; A1 answer.
8	n(n+1)(n+2).	**1**	B1: correct result or statement.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Numerator=12-2(-2)(5)=32; denominator=-7; value=-32/7.	**3**	M1 setup; M1 process; A1 conclusion.
2	$-12a^{-2}b^2=-12b^2/a^2$.	**3**	M1 setup; M1 process; A1 conclusion.
3	Expand: $n^2+6n+9-(n^2-6n+9)=12n$.	**3**	M1 setup; M1 process; A1 conclusion.
4	Correct identity: $(a+b)^2=a^2+2ab+b^2$. For a=b=1, left=4 but $a^2+b^2=2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$64x^6y^9/(8x^4y)=8x^2y^8$.	**3**	M1 setup; M1 process; A1 conclusion.
6	4(-1/8)-2(-1/2)(9)=-1/2+9=17/2.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Let them be 2n+1 and 2n+3. Sum=4n+4=4(n+1).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$(n+1)^2-n^2=2n+1$, which is odd.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	(2y+3x)/(xy), with x≠0 and y≠0.	**3**	M1 setup; M1 process; A1 conclusion.
10	Compare with $(ax-3)^2=a^2x^2-6ax+9$. -6a=-12 gives a=2, so k=4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	n and n+1 are consecutive, so one is even; their product is therefore even.	**3**	M1 setup; M1 process; A1 conclusion.
12	r+h=4√2; A=2π(3√2)(4√2)=48π.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Product=(2x+3)(x-1)=$2x^2$+x-3, not the stated expression. Therefore it is not an identity; e.g. x=2 gives actual area 7 but stated expression 20.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	n^2+n=n(n+1) is even. Adding 6, also even, keeps the result even.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Let odd integer=2n+1. Square=$4n^2$+4n+1=4n(n+1)+1. Since n(n+1) is even, write it 2k; result=8k+1.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Numerator=9x^-4y^6×2x^5y^-1=18xy^5. Divide by 6xy^2 to get 3y^3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	n^2+3n=n(n+3). If n is even, product even. If n is odd, n+3 is even. Hence every term is even.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$(n+1)^2$-n^2=2n+1=41, so n=20. Squares are 400 and 441.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Among three consecutive integers one is divisible by 3 and at least one is even, so the product contains factors 2 and 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Expanding gives x^2+2+1/x^2-[x^2-2+1/x^2]=4. All x-dependent terms cancel.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Expand $2ax^2+(2b-3a)x-3b$. Thus $2a=6 \Rightarrow a=3$; $-3b=-6 \Rightarrow b=2$; middle coefficient $4-9=-5$, not 1. Conditions are inconsistent, so no such a,b exist.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Sum$=n+(n+1)+(n+2)=3(n+1)$, always divisible by 3 but not always by 6. For $n=2$ sum$=9$. It is divisible by 6 only when $n+1$ is even.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Treat $b+c$ as one term u: $(a+u)^2-(a-u)^2=4au=4a(b+c)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$p^2+q^2=(p+q)^2-2pq=49-20=29$. $p^3+q^3=(p+q)^3-3pq(p+q)=343-210=133$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
13	$n^4-n^2=n^2(n-1)(n+1)$. Three consecutive factors give a factor 3 and at least one even; additionally n^2 or the two neighbours supply a total factor 4. Hence divisible by 12.	**5**	M1 setup; M2–M4 development; A1 conclusion.
14	First$=(a-b)(a+b)/(a+b)^2=(a-b)/(a+b)$. Dividing by the same fraction gives 1. Restrictions: $a+b\neq0$ and $a-b\neq0$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	Difference$=-8xy$. So $-8xy=24$ gives $xy=-3$. Integer pairs: $(1,-3),(-1,3),(3,-1),(-3,1)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 01 / A1-A4,A6

Algebraic Language, Substitution and Proof

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Every integer is $2k$ or $2k+1$. Squares are $4k^2$ (remainder 0) or $4k^2+4k+1$ (remainder 1), never 2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Contrapositive: if n is odd, $n=2k+1$ and $n^2=4k^2+4k+1$ is odd. Therefore an even square cannot come from an odd n.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	It must be $(x\pm6)^2=x^2\pm12x+36$, so $k=12$ or -12.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Use $2n+1, 2n+3$. Sum$=8n^2+16n+10=8(n^2+2n+1)+2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Square: $x^2+2+1/x^2=25$, so first$=23$. Cube identity gives $x^3+1/x^3=(x+1/x)^3-3(x+1/x)=125-15=110$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Assume $\sqrt{3}=a/b$ in lowest terms. Then $a^2=3b^2$, so 3 divides a; write $a=3k$. Then $b^2=3k^2$, so 3 divides b, contradicting lowest terms.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Expression$=(n+1)^2+4$. Test squares 4,9,16,25: $(n+1)^2=0,5,12,21$. Only 0 is square, giving $n=-1$ and value 4.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Let $A=n^2+n$, which is even. Difference$=(A+1)^2-A^2=2A+1$, odd.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Identity $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$. First factor is zero.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	$(x-y)(x+y)=45$. The positive factor pairs (1,45), (3,15) and (5,9) give the absolute pairs (23,22), (9,6) and (7,2). Since only the squares matter, x and y may each have either sign. Hence the 12 ordered pairs are $(\pm23,\pm22)$, $(\pm9,\pm6)$ and $(\pm7,\pm2)$, where the signs are chosen independently.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$(x-5)(x+5)$.	**2**	M1 method; A1 answer.
2	$2x^2+7x-15$.	**2**	M1 method; A1 answer.
3	$3x(2x+3)$.	**2**	M1 method; A1 answer.
4	$x-3$, $x\neq-3$.	**2**	M1 method; A1 answer.
5	$x^3+6x^2+12x+8$.	**3**	M1 setup; M1 process; A1 conclusion.
6	$(2x-1)(x-3)$.	**3**	M1 setup; M1 process; A1 conclusion.
7	$4\sqrt{3}+5\sqrt{3}=9\sqrt{3}$.	**2**	M1 method; A1 answer.
8	$4(\sqrt{5}+1)/4=\sqrt{5}+1$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$(3x+2)(2x-1)$.	**3**	M1 setup; M1 process; A1 conclusion.
2	First, $(x+3)(x-4)=x^2-x-12$. Then $(2x-1)(x^2-x-12)=2x^3-3x^2-23x+12$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$(x-6)(x+2)/[(x-3)(x+3)]$; no cancellation. $x\neq\pm3$.	**3**	M1 setup; M1 process; A1 conclusion.
4	$[2(x-1)+3x]/[x(x-1)]=(5x-2)/[x(x-1)]$, $x\neq0,1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$(x-4)(x+4)/[(x+3)(x+4)]=(x-4)/(x+3)$, $x\neq-4,-3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$5(2-\sqrt{3})/(4-3)=10-5\sqrt{3}$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	$4x\sqrt{3}$.	**3**	M1 setup; M1 process; A1 conclusion.
8	$(3x+1)(4x-5)$.	**3**	M1 setup; M1 process; A1 conclusion.
9	$(3/x)(x^2/6)=x/2$, $x\neq0$.	**3**	M1 setup; M1 process; A1 conclusion.
10	$[(x-1)-(x+1)]/[(x+1)(x-1)]=-2/(x^2-1)$, $x\neq\pm1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$x(x^2-4)=x(x-2)(x+2)$.	**3**	M1 setup; M1 process; A1 conclusion.
12	Expansion gives $x^3-x^2+x+x^2-x+1=x^3+1$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$x \neq 3$; fraction $= x+3$, so $x+3=8$ and $x=5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	First $=(x-2)(x+2)/[(x+2)(x-1)]=(x-2)/(x-1)$. Multiply by $(x-1)/(x+2)$: $(x-2)/(x+2)$, with original restrictions $x \neq -2,1$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Expand both cubes; b^3 and a^2b terms cancel, leaving $2a^3+6ab^2=2a(a^2+3b^2)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	$(x\pm7)^2=x^2\pm14x+49$, so $k=\pm14$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$(2\sqrt{3}+3\sqrt{3})/\sqrt{3}=5$.	**3**	M1 setup; M1 process; A1 conclusion.
6	$3(\sqrt{7}-\sqrt{2})/(7-2)=3(\sqrt{7}-\sqrt{2})/5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$x(x+1)=3(x-2)$; $x^2-2x+6=0$, discriminant $=-20$, so no real solutions.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Difference of squares: $(x^2-4)(x^2+4)=(x-2)(x+2)(x^2+4)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Common denominator: $[2x+(x-3)]/[(x-3)(x+3)]=3(x-1)/(x^2-9)$, $x\neq\pm3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$x(4x^2-12x+9)=x(2x-3)^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$(14+6\sqrt{5})+(14-6\sqrt{5})=28$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Square: $x^2+2+1/x^2=16$, so answer 14.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	$[(a^2-b^2)/(ab)]\div[(a^2+b^2)/(ab)]=(a^2-b^2)/(a^2+b^2)$, $a,b\neq0$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
14	Common denominator x^2-1: numerator $2x^2+2$. So $2x^2+2=3x^2-3$; $x^2=5$; $x=\pm\sqrt{5}$, with $x\neq\pm1$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	Complete square: $(x+2)^2+4>0$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 02 / A4,A6

Expansion, Factorisation and Algebraic Fractions

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Restriction $x \neq 1$. Simplifies to $x+1=0$, so $x=-1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Expand or use Sophie Germain identity: $x^4+4=x^4+4x^2+4-4x^2=(x^2+2)^2-(2x)^2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	First two combine to $2x/(x^2-1)$, so expression=0, $x \neq \pm1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Require $x \geq 0$. Square: $x^2-x-6=0 \Rightarrow x=3$ or -2; only $x=3$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	$1/(2-\sqrt{3})=2+\sqrt{3}$, so cube$=(2+\sqrt{3})^3=26+15\sqrt{3}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$a^2+b^2=25-6=19$; $a^4+b^4=19^2-2(9)=343$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	$(x^3-y^3)(x^3+y^3)=(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Difference$=x^2b^2+y^2a^2-2abxy=(bx-ay)^2 \geq 0$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	Numerator zero gives $x=2,3$; denominator excludes $x=\pm2$, so only $x=3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$(x+3)^2+4$, so minimum 4 at $x=-3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ________________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	x=(y+7)/3.	**2**	M1 method; A1 answer.
2	r=C/(2π).	**2**	M1 method; A1 answer.
3	10x-15=4x+9; 6x=24; x=4.	**2**	M1 method; A1 answer.
4	x+3=20; x=17.	**2**	M1 method; A1 answer.
5	2A=bh; h=2A/b.	**2**	M1 method; A1 answer.
6	3=5x; x=3/5.	**2**	M1 method; A1 answer.
7	v=u-at.	**2**	M1 method; A1 answer.
8	2x/3=12; 2x=36; x=18.	**2**	M1 method; A1 answer.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$4y=3x-5$; $3x=4y+5$; $x=(4y+5)/3$.	**3**	M1 setup; M1 process; A1 conclusion.
2	$r^2=V/(\pi h)$; for radius $r\geq 0$, $r=\sqrt{[V/(\pi h)]}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$y(x-2)=5$; $yx=5+2y$; $x=(5+2y)/y=2+5/y$, $y\neq 0$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Multiply by 15: $5(2x-1)-3(x+4)=30$; $7x-17=30$; $x=47/7$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$2b=P-2a$; $b=P/2-a$.	**3**	M1 setup; M1 process; A1 conclusion.
6	$x\neq -1$. $4=x(x+1)$; $x^2+x-4=0$; $x=(-1\pm\sqrt{17})/2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	$2at=v^2-u^2$; $t=(v^2-u^2)/(2a)$, $a\neq0$.	**3**	M1 setup; M1 process; A1 conclusion.
8	$x=(y-b)/a$.	**2**	M1 method; A1 answer.
9	Multiply by 4: $3x-6+2x+2=20$; $5x=24$; $x=24/5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$mc^2=E-K$; $m=(E-K)/c^2$, $c\neq0$.	**3**	M1 setup; M1 process; A1 conclusion.
11	$2x+3=25$; $x=11$. Check: $\sqrt{25}=5$.	**3**	M1 setup; M1 process; A1 conclusion.
12	$p(q+r)=q-r$; $pq+pr=q-r$; $q(p-1)=-r(1+p)$; $q=r(1+p)/(1-p)$, $p\neq1$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$y(x-4)=2x+3$; $x(y-2)=4y+3$; $x=(4y+3)/(y-2)$, $y\neq 2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	$1/v=1/f-1/u=(u-f)/(fu)$; $v=fu/(u-f)$, $u\neq f$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	$x\neq\pm1$. Multiply by x^2-1: $2(x+1)+3(x-1)=x^2-1$; $x^2-5x=0$; $x=0$ or 5.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Let width w, length $2w+5$. $2[(2w+5)+w]=62$; $6w+10=62$; $w=26/3$ cm, length$=67/3$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Let $n-1,n,n+1$. Product$=n^3-n$ and sum$=3n$. Equation $n^3-n=3n+12$, so $n^3-4n-12=0$. $n=3$ is a root; integers are 2,3,4.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$x^2+(x+7)^2=(x+8)^2$; $x^2-2x-15=0$; $(x-5)(x+3)=0$. Length requires $x=5$; sides 5,12,13.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Square: $y^2=(x-a)/b$; $by^2=x-a$; $x=a+by^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$x\neq 3,-1$. $(x+2)(x+1)=2x(x-3)$; $x^2+3x+2=2x^2-6x$; $x^2-9x-2=0$; $x=(9\pm\sqrt{89})/2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Rate=(27.70-18.20)/5=£1.90/km. Fixed charge=18.20-7(1.90)=£4.90. C=1.9d+4.9; C(20)=£42.90.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	$x^2+2yx-A=0$. Quadratic formula: $x=-y\pm\sqrt{y^2+A}$. Non-negative required root is $x=-y+\sqrt{y^2+A}$, when valid.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Let width x. $x(x+4)=96$; $x^2+4x-96=0=(x+12)(x-8)$; positive x=8, length=12.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$1.2x-15=0.9x$; $0.3x=15$; $x=50$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	$S-2\pi r^2=2\pi rh$; $h=(S-2\pi r^2)/(2\pi r)=S/(2\pi r)-r$, $r\neq0$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	Set $\sqrt{x+1}=11-3x$, requiring right side≥0. Square: $x+1=121-66x+9x^2$; $9x^2-67x+120=0=(9x-40)(x-3)$. Candidates 40/9,3; 40/9 makes right side negative, so x=3.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	Let one be x, other 17-x. $x(17-x)=60$; $x^2-17x+60=0=(x-5)(x-12)$; numbers 5 and 12.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 03 / A5,A17,A21

Formulae, Rearrangement, Equations and Modelling

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$ycx+yd=ax+b$; $x(yc-a)=b-yd$; $x=(b-yd)/(yc-a)$, provided $yc\neq a$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	$x\neq\pm1$. $2x/(x^2-1)=3/4$; $8x=3x^2-3$; $3x^2-8x-3=0$; $x=3$ or $-1/3$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	$x^2(x+3)=112$; $x^3+3x^2-112=0$. $x=4$ is a root and gives positive dimensions; base 4, height 7.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	$yx=x^2+1$; $x^2-yx+1=0$; $x=[y\pm\sqrt{(y^2-4)}]/2$, requiring $y^2\geq4$ for real x.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Consecutive factors of 72 are 8 and 9. Sum $p=17$ and product $q=72$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Let width w, length 2w. Area$=2w^2$; perimeter$=6w$. For positive w, $2w^2=6w$, so $w=3$ and length$=6$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Require $x\geq0$. Isolate $\sqrt{(x+6)}=6-\sqrt{x}$ and square: $x+6=36-12\sqrt{x}+x$, so $\sqrt{x}=5/2$ and $x=25/4$. Check gives $3.5+2.5=6$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$r^3=3V/(4\pi)$; $r=\sqrt[3]{[3V/(4\pi)]}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$120/x+180/(x+20)=5$. Multiply: $120(x+20)+180x=5x(x+20)$; $x^2-40x-480=0$; $x=20\pm4\sqrt{55}$. Positive $x=20+4\sqrt{55}\approx49.7$ km/h.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Discriminant$=144-4k^2=0$, so $k^2=36$ and $k=\pm6$. Both give genuine quadratics and one repeated root.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$4x=24$, so $x=6$.	**2**	M1 method; A1 answer.
2	$(x-3)(x-4)=0$, so $x=3$ or $x=4$.	**2**	M1 method; A1 answer.
3	$x^2=9$, so $x=\pm 3$.	**2**	M1 method; A1 answer.
4	Adding gives $2x=14$, so $x=7$ and $y=4$.	**2**	M1 method; A1 answer.
5	$(x+1)(x+5)=0$, so $x=-1$ or $x=-5$.	**2**	M1 method; A1 answer.
6	$8x-4=3x+11$; $5x=15$; $x=3$.	**2**	M1 method; A1 answer.
7	$x^2-8x+3=(x-4)^2-13$.	**2**	M1 method; A1 answer.
8	$b^2-4ac=(-5)^2-4(2)(1)=17$.	**2**	M1 method; A1 answer.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$x\neq2$. Multiply through: $5=(x+1)(x-2)=x^2-x-2$, so $x^2-x-7=0$. Hence $x=(1\pm\sqrt{29})/2$; neither is 2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$(3x+1)(x-4)=0$, so $x=-1/3$ or $x=4$.	**3**	M1 setup; M1 process; A1 conclusion.
3	Quadratic formula gives $x=[-7\pm\sqrt{(49+24)}]/4=(-7\pm\sqrt{73})/4$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Double the second equation: $10x-2y=30$. Add to get $13x=46$, so $x=46/13$. Then $y=5x-15=35/13$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$x^2-4=2x+3$, so $x^2-2x-7=0$. Thus $x=1\pm2\sqrt{2}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Let $u=x^2$. Then $u^2-13u+36=(u-4)(u-9)=0$. Hence $x^2=4$ or 9, so $x=\pm2,\pm3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	$x^2-6x+9=5x+1$, so $x^2-11x+8=0$. Thus $x=(11\pm\sqrt{89})/2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Equal roots require $k^2-64=0$, so $k=\pm8$.	**3**	M1 setup; M1 process; A1 conclusion.
9	Let width $x>0$. $x(x+7)=60$, so $x^2+7x-60=(x+12)(x-5)=0$. Thus width 5 cm and length 12 cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$x^2-2x-3=3x+1$ gives $x^2-5x-4=0$, so $x=(5\pm\sqrt{41})/2$. For each, $y=3x+1=(17\pm3\sqrt{41})/2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Discriminant $36-4m<0$, so $m>9$.	**3**	M1 setup; M1 process; A1 conclusion.
12	$x\neq-1,2$. Multiply by $(x+1)(x-2)$: $2(x-2)-(x+1)=(x+1)(x-2)$. Thus $x-5=x^2-x-2$, so $x^2-2x+3=0$. Its discriminant is -8, so there are no real solutions.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Let integers be $n, n+1$. $n^2+(n+1)^2=313$ gives $2n^2+2n-312=0$, so $n^2+n-156=0=(n-12)(n+13)$. Positive $n=12$; integers are 12 and 13.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$x(x+1)=12$, so $x^2+x-12=(x+4)(x-3)=0$. Thus $(x,y)=(3,4)$ or $(-4,-3)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Intersections satisfy $x^2-mx-4=0$. Tangency requires discriminant $m^2+16=0$, which has no real m. Therefore no real line of this form is tangent to $y=x^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$x^2-mx+4=0$ has equal roots, so $m^2-16=0$ and $m=\pm4$. Repeated root $x=m/2$. Points are $(2,4)$ for $m=4$ and $(-2,4)$ for $m=-4$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Multiply by x: $x^2-7x+12=0=(x-3)(x-4)$. Hence $x=3$ or 4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$(x+1)^2+(x+8)^2=(2x+1)^2$ gives $2x^2-14x-64=0$, so $x^2-7x-32=0$. Thus $x=(7\pm\sqrt{177})/2$; lengths require $x=(7+\sqrt{177})/2$. Sides are $(9+\sqrt{177})/2$, $(23+\sqrt{177})/2$ and $8+\sqrt{177}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	$y=7-2x$. Substitute: $x^2+(7-2x)^2=13$, giving $5x^2-28x+36=0$. Hence $x=(14\pm4)/5$, so $x=2$ or $18/5$. Then $y=3$ or $-1/5$. Solutions: $(2,3),(18/5,-1/5)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	If $p=0$, the equation is linear, $4x=0$, with one solution. If $p\neq0$, equal roots require $16-4p^2=0$, so $p=\pm2$. Therefore $p\in\{-2,0,2\}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Roots have sum 7/2. Let roots be u and v with u-v=3. Solving u+v=7/2 and u-v=3 gives u=13/4,v=1/4. Product=c/2=13/16, so c=13/8.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Require $x \geq -2$. Squaring gives $3x+4=x^2+4x+4$, so $x^2+x=0$ and x=0 or -1. Both check: 2=2 and 1=1.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Axis is x=-a/2=3, so a=-6. Using (1,0): 1-6+b=0, so b=5. Equation $y=x^2-6x+5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Let $x \neq 0$. x-1/x=15/4, so $4x^2-15x-4=0=(4x+1)(x-4)$. Thus x=4 or -1/4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	Since $(x+y)^2=x^2+2xy+y^2$, 25=19+xy, so xy=6. Thus x,y are roots of $t^2-5t+6=0$, giving ordered pairs (2,3) and (3,2).	**5**	M1 setup; M2–M4 development; A1 conclusion.
14	Let side x>0. $(x+3)^2-x^2=81$, so 6x+9=81 and x=12 cm.	**3**	M1 setup; M1 process; A1 conclusion.
15	Equate: $x^2-4x+1=2-x$, so $x^2-3x-1=0$ and $x=(3\pm\sqrt{13})/2$. Since y=2-x, $y=(1\mp\sqrt{13})/2$. These give two intersection points.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 04 / A17-A19,A21

Linear, Quadratic and Simultaneous Equations

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Discriminant equals the square of the root difference because a=1. Thus $(k+1)^2-4k=25$, so $(k-1)^2=25$. Hence k=6 or -4.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Let $u=x^2\geq0$. Then $u^2-5u+4=(u-1)(u-4)=0$. Both u values are non-negative, giving $x=\pm1,\pm2$. Every real x determines $u=x^2$, so none are lost.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	$(\alpha+\beta)^2=34+2(3)=40$, so $\alpha+\beta=\pm2\sqrt{10}$. Since $p=-(\alpha+\beta)$ and $q=\alpha\beta$, $(p,q)=(2\sqrt{10},3)$ or $(-2\sqrt{10},3)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Equate: $x^2-4x+7=2x+c$, so $x^2-6x+(7-c)=0$. Equal roots require $36-4(7-c)=0$, giving c=-2. Repeated root x=3 and y=4, so contact is (3,4).	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	$(x+y)^2=49$, so $x+y=\pm7$. For sum 7, roots of $t^2-7t+12$ are 3,4. For sum -7, they are -3,-4. Ordered solutions are (3,4), (4,3),(-3,-4),(-4,-3).	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Let roots r,s. rs=r+s, so rs-r-s=0 and (r-1)(s-1)=1. Positive integers greater than 1 give r-1=s-1=1, hence r=s=2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Distinct real roots: $4k^2-4(k+2)>0$, so $(k-2)(k+1)>0$. Positive sum 2k requires k>0; positive product k+2 requires k>-2. Combining gives k>2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Domain $-4\leq x\leq9$. Squaring gives $13+2\sqrt{(x+4)(9-x)}=25$, so $(x+4)(9-x)=36$. Thus $-x^2+5x+36=36$, so $x(x-5)=0$. Both x=0 and x=5 check.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	Intersections solve $x^2-6x+k=0$. Difference between roots is $\sqrt{D}/a=\sqrt{36-4k}=2$. Thus 36-4k=4, giving k=8.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Let numerator x, denominator x+2. (x+3)/(x+5)=4/5 gives 5x+15=4x+20, so x=5. Original fraction is 5/7.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 05 / A22

Linear and Quadratic Inequalities

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$3x<15$, so $x<5$.	**2**	M1 method; A1 answer.
2	Divide by -2 and reverse the sign: $x\geq-4$.	**2**	M1 method; A1 answer.
3	$-2\leq2x<6$, so $-1\leq x<3$.	**2**	M1 method; A1 answer.
4	-1, 0, 1, 2, 3.	**2**	M1 method; A1 answer.
5	$(x-3)(x+3)>0$, so $x<-3$ or $x>3$.	**2**	M1 method; A1 answer.
6	$x(x-5)\leq0$, so $0\leq x\leq5$.	**2**	M1 method; A1 answer.
7	$-3x>-6$; divide by -3 and reverse: $x<2$.	**2**	M1 method; A1 answer.
8	$-4\leq x<2$.	**1**	B1: correct result or statement.

MODULE 05 / A22

Linear and Quadratic Inequalities

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$7-6x \geq 4x+7$, so $-10x \geq 0$ and $x \leq 0$.	**3**	M1 setup; M1 process; A1 conclusion.
2	Multiply by 6: $2(2x-1)>3(x+4)$, so $4x-2>3x+12$ and $x>14$.	**3**	M1 setup; M1 process; A1 conclusion.
3	From $-3<2-5x$: $-5<-5x$, so $x<1$. From $2-5x \leq 12$: $-5x \leq 10$, so $x \geq -2$. Hence $-2 \leq x<1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$(x+4)(x-3)<0$, so $-4<x<3$.	**3**	M1 setup; M1 process; A1 conclusion.
5	$(2x-1)(x-3) \geq 0$, so $x \leq 1/2$ or $x \geq 3$.	**3**	M1 setup; M1 process; A1 conclusion.
6	$(x+1)(x+3) \leq 0$, so $-3 \leq x \leq -1$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 05 / A22

Linear and Quadratic Inequalities

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	$(x-4)(x+2)<0$, so $-2<x<4$. Integers: -1,0,1,2,3.	**3**	M1 setup; M1 process; A1 conclusion.
8	Critical values are -5 and 1. Product is positive outside them: $x<-5$ or $x>1$.	**3**	M1 setup; M1 process; A1 conclusion.
9	$(3x-1)(x+1)\le 0$, so $-1\le x\le 1/3$.	**3**	M1 setup; M1 process; A1 conclusion.
10	$12+0.08t\le 20$, so $0.08t\le 8$ and $t\le 100$. The maximum whole number is 100 texts.	**3**	M1 setup; M1 process; A1 conclusion.
11	Roots are $x=-1\pm 2\sqrt{2}$. Since the parabola opens upward, $x<-1-2\sqrt{2}$ or $x>-1+2\sqrt{2}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$(x-1)^2\le 4$, so $-2\le x-1\le 2$ and $-1\le x\le 3$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 05 / A22

Linear and Quadratic Inequalities

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Critical values are -2 (zero) and 1 (undefined). A sign chart gives $x<-2$ or $x>1$; endpoints are excluded.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Critical values are -4 and 3/2. The quotient is non-positive for $-4<x\leq 3/2$; $x=-4$ is excluded and $x=3/2$ included.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$x^2-6x+1<0$. Roots are $3\pm 2\sqrt{2}$, so $3-2\sqrt{2}<x<3+2\sqrt{2}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Complete square: $(x-2)^2+k-4$. Its minimum is $k-4$, which must be >0. Hence $k>4$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	For $x=2$, $y=3,4,5$; for $x=3$, $y=3,4$; for $x=4$, $y=3$. Thus (2,3),(2,4),(2,5),(3,3),(3,4),(4,3).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$x(x+5)\leq 84$, so $x^2+5x-84\leq 0=(x+12)(x-7)\leq 0$. Thus $-12\leq x\leq 7$; with $x>0$, $0<x\leq 7$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$x^2-4x+4\geq 3x+4$, so $x^2-7x\geq 0$ and $x(x-7)\geq 0$. Hence $x\leq 0$ or $x\geq 7$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Factor $x(x-2)(x+2)>0$. Sign intervals give $-2<x<0$ or $x>2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 05 / A22

Linear and Quadratic Inequalities

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Critical values are -3,1,3. Sign testing gives $[-3,1)\cup[3,\infty)$. The zeros ±3 are included; $x=1$ is excluded.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Product $k>0$. Sum$=1-k$ must be negative, so $k>1$. Discriminant $(k-1)^2-4k=k^2-6k+1>0$, giving $k<3-2\sqrt{2}$ or $k>3+2\sqrt{2}$. Combined: $k>3+2\sqrt{2}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	$-7<2x-5<7$, so $-2<2x<12$ and $-1<x<6$.	**3**	M1 setup; M1 process; A1 conclusion.
12	$3x+1\le-8$ or $3x+1\ge8$. Hence $x\le-3$ or $x\ge7/3$.	**3**	M1 setup; M1 process; A1 conclusion.
13	$x^2-2x-8<x+2$ gives $x^2-3x-10<0=(x-5)(x+2)<0$. Therefore $-2<x<5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	$-5t^2+20t+1\ge16$ gives $t^2-4t+3\le0=(t-1)(t-3)\le0$. Thus $1\le t\le3$ seconds.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	$x\ne2$. Bring to one fraction: $[1-(x-2)]/(x-2)=(3-x)/(x-2)<0$. Critical values 2 and 3; sign testing gives $x<2$ or $x>3$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 05 / A22

Linear and Quadratic Inequalities

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Expression $=(x-k)^2-9\le 0$ gives $k-3\le x\le k+3$, whose length is always 6. Therefore there is no such k.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Critical values -2,-1,1,2; ±1 excluded and ±2 zeros excluded because strict. Sign testing gives $(-\infty,-2)\cup(-1,1)\cup(2,\infty)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Leading coefficient is positive, so require discriminant <0: $(k-2)^2-4k=k^2-8k+4<0$. Roots are $4\pm2\sqrt{3}$, hence $4-2\sqrt{3}<k<4+2\sqrt{3}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Both sides are non-negative, so square: $(x-2)^2\le(2x+1)^2$. This gives $3x^2+8x-3\ge0=(3x-1)(x+3)\ge0$. Hence $x\le-3$ or $x\ge1/3$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	The roots must be 3 and 4 and the parabola opens upward. Thus $x^2-px+12=(x-3)(x-4)=x^2-7x+12$, so $p=7$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Move left: $[(x-1)(x-4)-(x+3)(x+2)]/[(x+2)(x-4)]\ge0$. Numerator $=-10x-2=-2(5x+1)$. Critical values -2,-1/5,4. Sign testing gives $(-\infty,-2)\cup[-1/5,4)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	First inequality: $(n-3)(n-8)<0$, so $3<n<8$. Second gives $n\le8$. Integer solutions are 4,5,6,7.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	If $a>0$, require discriminant ≤0: $16-4a\le0$, so $a\ge4$. If $a=0$, $-4x+1$ is not always non-negative; if $a<0$ the quadratic tends to $-\infty$. Hence $a\ge4$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	Let $u=x^2$. $(u-1)(u-4)\le0$ gives $1\le x^2\le4$. Hence $-2\le x\le-1$ or $1\le x\le2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	$R-C=-x^2+36x-60>0$, equivalent to $x^2-36x+60<0$. Roots are $18\pm2\sqrt{66}$, approximately 1.75 and 34.25. Therefore integer x values are $2\le x\le34$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Common difference 5, so nth term = 5n+2.	**2**	M1 method; A1 answer.
2	3(20)-8=52.	**1**	B1: correct result or statement.
3	4n+1=97 gives n=24, a positive integer, so yes: it is the 24th term.	**2**	M1 method; A1 answer.
4	The multiplier is 2, so the next terms are 80 and 160.	**1**	B1: correct result or statement.
5	u_2=3(2)-1=5; u_3=3(5)-1=14.	**2**	M1 method; A1 answer.
6	First differences: 5,7,9; second differences: 2,2.	**2**	M1 method; A1 answer.
7	Each term is the sum of the previous two, so the next term is 21.	**1**	B1: correct result or statement.
8	27/81=1/3, so the common ratio is 1/3.	**1**	B1: correct result or statement.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Second difference is 4, so start with $2n^2$. Subtracting gives sequence 2,3,4,5,6 with term n+1. Hence $u_n=2n^2+n+1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$15^2-4(15)+7=225-60+7=172$.	**2**	M1 method; A1 answer.
3	Solve $n^2+n=250$, so $n^2+n-250=0$. The discriminant is $1+1000=1001$, which is not a perfect square, so n is not an integer. Therefore 250 is not a term.	**3**	M1 setup; M1 process; A1 conclusion.
4	$u_n=6\times3^{(n-1)}$.	**2**	M1 method; A1 answer.
5	Ratio $r=10/20=1/2$. Since $ar^2=20$, $a=20/(1/4)=80$.	**3**	M1 setup; M1 process; A1 conclusion.
6	$u_2=11$, $u_3=25$, $u_4=53$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Each term is double the preceding term plus 2: $u_1=1$, $u_{n+1}=2u_n+2$.	**3**	M1 setup; M1 process; A1 conclusion.
8	Second differences are 2, so start with n^2. Difference from n^2 is 7,11,15,19,23 $=4n+3$. Thus $u_n=n^2+4n+3$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$2n^2-3n>100$. Checking $n=8$ gives 104, while $n=7$ gives 77. The first is the 8th term, 104.	**3**	M1 setup; M1 process; A1 conclusion.
10	5,-10,20,-40.	**2**	M1 method; A1 answer.
11	$u_3=10, u_4=17, u_5=27, u_6=44$.	**3**	M1 setup; M1 process; A1 conclusion.
12	For $n=1$: $a+b=5$. For $n=2$: $4a+2b=14$, so $2a+b=7$. Hence $a=2, b=3$; term is $2n^2+3n$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	25+5k+6=46, so 5k=15 and k=3. The tenth term is 100+30+6=136.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	n^2-9n+14=14 gives n(n-9)=0. Valid positive position is n=9; n=0 is not a term position.	**3**	M1 setup; M1 process; A1 conclusion.
3	For consecutive geometric terms, x^2=12×27=324. Positive ratio gives x=18 and r=18/12=3/2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	ar=12 and ar^4=324, so r^3=27 and r=3 (the only real cube root). Then a=4. Thus first term 4, ratio 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	u_2=10k-2; u_3=k(10k-2)-2=22. Thus $10k^2$-2k-24=0, or $5k^2$-k-12=0=(5k-?); k=[1±√241]/10.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Second difference 6 gives leading term $3n^2$. Let $u_n=3n^2+bn+c$. Difference u_2-u_1=9+b=7, so b=-2. Using u_1=2 gives 3-2+c=2, c=1. Hence $3n^2$-2n+1.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	n^2+4=7n-2 gives n^2-7n+6=0=(n-1)(n-6). Hence n=1 or 6.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Arithmetic sequence: last term=18+24(4)=114. Sum=25(18+114)/2=1650.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	First differences: 2p+1 and 5p-3. Second difference=3p-4=10, so p=14/3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Let terms a, ar, ar^2. Since ar=20, $a+ar^2=a+400/a=50$. Thus $a^2-50a+400=0$, giving a=10 or 40. Terms are 10,20,40 or 40,20,10.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	At a limit, $L=\sqrt{(6L+4)}$. Thus $L^2-6L-4=0$, so $L=3\pm\sqrt{13}$. The positive value is $3+\sqrt{13}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	The kth odd number is 2k-1. Sum = n/2[first+last] = $n/2[1+(2n-1)]=n^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	Complete square: $(n-2.5)^2-13/4$. Nearest positive integers are n=2 and n=3; both give -3. Minimum term is -3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	Need $1.06^n>1.5$. Using logs, $n>\log(1.5)/\log(1.06)\approx6.96$. First whole n is 7; $P_7\approx1202.9$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	Fourth term=a^3. $(1+\sqrt{2})^2=3+2\sqrt{2}$; multiply by $1+\sqrt{2}$ to get $7+5\sqrt{2}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / A23-A25

Sequences: Quadratic, Geometric and Recursive

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	The nth term is $n^2+n=n(n+1)$. For $n>1$ both factors exceed 1, so every term after the first is composite.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Possible squares are 1,4,9. Completing square or solving: value 1 gives $n^2-7n+16=0$, no integer roots; value 4 gives $n^2-7n+13=0$, none; value 9 gives $n^2-7n+8=0$, none. Therefore there are no such positions.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Product $a\cdot ar\cdot ar^2=(ar)^3=216$, so middle term $ar=6$. Sum gives $a+6+36/a=21$, so $a^2-15a+36=0$. Thus $a=3$ or 12, giving triples (3,6,12) and (12,6,3).	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	$u_2=2^2-2=2$. Therefore $u_n=2$ for every n by induction.	**3**	M1 setup; M1 process; A1 conclusion.
5	$n^3-n=n(n-1)(n+1)$, a product of three consecutive integers. One is divisible by 3 and at least one is even, so the product is divisible by 6.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$9+3b+c=2$ and $64+8b+c=42$. Subtract:$55+5b=40$, so $b=-3$; then $c=2$. Term$=n^2-3n+2=(n-1)(n-2)$, so zeros occur at $n=1,2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	If $u_n=\sqrt{2}$, then $u_{n+1}=2/\sqrt{2}=\sqrt{2}$. Since $u_1=\sqrt{2}$, induction gives $u_n=\sqrt{2}$ for every n.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Work backwards: $u_4=47-29=18$; $u_3=29-18=11$; $u_2=18-11=7$; $u_1=11-7=4$. First terms are 4,7.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Binomial expansions have opposite signs on odd powers of $\sqrt{5}$, so those terms cancel. Even powers become integer powers of 5; the remaining doubled sum has integer coefficients. Hence u_n is an integer.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	$L=1+1/L$, so $L^2-L-1=0$. Thus $L=(1\pm\sqrt{5})/2$. Since all terms and the limit are positive, reject $(1-\sqrt{5})/2$; $L=(1+\sqrt{5})/2$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Gradient=(15-3)/(8-2)=12/6=2.	**2**	M1 method; A1 answer.
2	Gradient -3; y-intercept 7.	**1**	B1: correct result or statement.
3	((−4+6)/2,(5−1)/2)=(1,2).	**2**	M1 method; A1 answer.
4	Length=$\sqrt{[(4-1)^2+(6-2)^2]}=\sqrt{25}=5$.	**2**	M1 method; A1 answer.
5	y=4x-5.	**1**	B1: correct result or statement.
6	Centre (0,0), radius 7.	**1**	B1: correct result or statement.
7	Perpendicular gradients multiply to -1, so gradient=-1/2.	**1**	B1: correct result or statement.
8	$3^2+4^2=9+16=25$, so yes.	**1**	B1: correct result or statement.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Use y+2=5(x-3), so y=5x-17.	**3**	M1 setup; M1 process; A1 conclusion.
2	Gradient=(-5-7)/(4+2)=-2. Using (-2,7): y-7=-2(x+2), so y=-2x+3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Perpendicular gradient=-1/3. y-2=-(1/3)(x-6), hence y=-x/3+4.	**3**	M1 setup; M1 process; A1 conclusion.
4	Rearrange: y=3x+9/2, so m=3.	**2**	M1 method; A1 answer.
5	2x+5=-x-1 gives x=-2; y=1. Intersection (-2,1).	**3**	M1 setup; M1 process; A1 conclusion.
6	Midpoint (5,3); segment gradient=4/6=2/3, so perpendicular gradient=-3/2. Equation y-3=-(3/2)(x-5).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Complete squares: $(x-4)^2-16+(y+3)^2-9-11=0$, so $(x-4)^2+(y+3)^2=36$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Centre (-2,5), radius $\sqrt{13}$.	**2**	M1 method; A1 answer.
9	Radius gradient=4/3, so tangent gradient=-3/4. $y-4=-(3/4)(x-3)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Gradient AB=(10-2)/4=2; gradient BC=(14-10)/2=2. Equal gradients with common B show collinearity.	**3**	M1 setup; M1 process; A1 conclusion.
11	Distance=$\sqrt{[8^2+(-6)^2]}=\sqrt{100}=10$.	**2**	M1 method; A1 answer.
12	6+3b=12, so b=2. Line 3x+2y=12 gives y=-3x/2+6; gradient=-3/2.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Given line has y=(2/3)x-4, gradient 2/3; perpendicular gradient=-3/2. y+1=-(3/2)(x-4). Thus 3x+2y=10.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	One third of the way from A to B: A+(1/3)(12,-6)=(-2,5)+(4,-2)=(2,3).	**3**	M1 setup; M1 process; A1 conclusion.
3	Gradient QR=12/6=2. Hence gradient QP=(5+1)/(k-2)=2; 6=2(k-2), so k=5.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Centre is midpoint (2,4). Diameter length=$\sqrt{(8^2+6^2)}$=10, so radius 5. Equation $(x-2)^2+(y-4)^2=25$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Substitute: $x^2+(x+1)^2=25$, so $2x^2+2x-24=0$; $x^2+x-12=0$; x=3 or -4. Then y=4 or -3. Points (3,4),(-4,-3).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Then y=±4. At (3,4): y-4=-(3/4)(x-3). At (3,-4): y+4=(3/4)(x-3).	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Gradient AB=2/6=1/3; gradient BC=6/(-2)=-3. Product=-1, so AB⊥BC and the right angle is at B.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Vectors from first point are (6,2) and (3,7). Area=1/2|6·7-2·3|=18 square units.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Radius²=(-5)²+12²=169. Equation (x-4)²+(y+2)²=169.	**3**	M1 setup; M1 process; A1 conclusion.
10	Radius gradient=8, so tangent gradient=-1/8. y-8=-(1/8)(x-1). Set y=0: -8=-(x-1)/8, so x-1=64 and A=(65,0).	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Perpendicular through P has gradient -1/2: y+1=-(x-5)/2. Intersect with y=2x+3: 2x+4=-(x-5)/2; 4x+8=-x+5, x=-3/5, y=9/5. Distance from P is √[(28/5)²+(14/5)²]=14√5/5.	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	3x-2=-x+10 gives x=3,y=7, so P=(3,7). Through origin and P, gradient=7/3; equation y=7x/3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	At x=6, y²=100-36=64, so y=±8. Chord length=8-(-8)=16.	**3**	M1 setup; M1 process; A1 conclusion.
14	Substitute: (1+m²)x²+10mx+25=25, so x[(1+m²)x+10m]=0. Since (0,5) is always an intersection, tangency requires the other root also x=0, hence m=0.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	D=A+C-B=(0,4). Diagonal BD through (8,5),(0,4) has gradient 1/8; y-4=x/8, so y=x/8+4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / A8-A10,A16

Straight Lines, Coordinates and Circles

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Substitute to get $(1+k^2)x^2+12kx+16=0$. Tangency requires discriminant $144k^2-64(1+k^2)=0$, so $80k^2=64$, $k^2=4/5$ and $k=\pm2/\sqrt{5}=\pm2\sqrt{5}/5$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Circle is $(x-3)^2+(y+2)^2=25$, centre $(3,-2)$. Radius to $(6,2)$ has gradient $4/3$; tangent gradient$=-3/4$. Hence $y-2=-(3/4)(x-6)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Midpoint $AC=((-2+6)/2,(1+9)/2)=(2,5)$. Midpoint $BD=((4+0)/2,(3+7)/2)=(2,5)$. Equal midpoints prove the diagonals bisect each other.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Let line $y-7=m(x-4)$, or $mx-y+(7-4m)=0$. Distance from the origin to the line equals $\sqrt{65}$: $(7-4m)^2/(m^2+1)=65$. This gives $49-56m+16m^2=65m^2+65$, so $49m^2+56m+16=0=(7m+4)^2$. Thus $m=-4/7$ and the tangent is $y-7=-(4/7)(x-4)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Equate squared distances: $(x+2)^2+(y-3)^2=(x-6)^2+(y+1)^2$. Simplifying gives $16x-8y-24=0$, so $y=2x-3$. This is the perpendicular bisector of AB.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Subtract equations: $x^2-(x-6)^2=0$ gives $12x-36=0$, $x=3$. Then $y^2=25-9=16$, so points are $(3,4)$ and $(3,-4)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	The triangle is right-angled at the origin, so its hypotenuse from $(6,0)$ to $(0,8)$ is a diameter. Centre $(3,4)$, radius 5; equation $(x-3)^2+(y-4)^2=25$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	The sloping lines meet at $(2,5)$. Their intersections with $y=k$ have $x=(k-1)/2$ and $x=7-k$, so horizontal base length$=\|7-k-(k-1)/2\|=3\|5-k\|/2$. Height$=\|k-5\|$. Area$=3(k-5)^2/4=27/2$, so $(k-5)^2=18$ and $k=5\pm3\sqrt{2}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	Let tangent point (a,b). It satisfies $a^2+b^2=36$ and radius OP is perpendicular to TP, so $(a,b)\cdot(a-10,b)=0$. Hence $36-10a=0$, $a=18/5$. Then $b^2=36-324/25=576/25$, so $b=\pm24/5$. Points $(18/5,\pm24/5)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Equidistance from A and B gives the perpendicular bisector $x=2$. Since P lies on $y=x^2$, $y=4$. Thus $P=(2,4)$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	At x=0, y=6, so the intercept is (0,6).	**1**	B1: correct result or statement.
2	x=-3 or x=3.	**1**	B1: correct result or statement.
3	x=0 and y=0.	**2**	M1 method; A1 answer.
4	3^0=1, so (0,1).	**1**	B1: correct result or statement.
5	y=x^3+2.	**2**	M1 method; A1 answer.
6	(14-2)/(5-1)=3.	**2**	M1 method; A1 answer.
7	Area=12x7=84 m.	**2**	M1 method; A1 answer.
8	(4,-7).	**1**	B1: correct result or statement.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	y=(x-1)(x-5), so roots 1,5. Also y=(x-3)^2-4, so turning point (3,-4).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	y=a(x+2)(x-7). At x=0, -14a=-28, so a=2; y=2(x+2)(x-7).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	x(x^2-4)=x(x-2)(x+2)=0; intercepts at x=-2,0,2.	**3**	M1 setup; M1 process; A1 conclusion.
4	x^2+1=4x-2, so x^2-4x+3=0; x=1 or 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Translation by vector (3,2); asymptotes become x=3 and y=2.	**3**	M1 setup; M1 process; A1 conclusion.
6	x=log(11)/log(2)=3.46 (3 s.f.).	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	[4.1^2-3.9^2]/0.2=(16.81-15.21)/0.2=8.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	(-4-11)/(8-2)=-15/6=-2.5.	**3**	M1 setup; M1 process; A1 conclusion.
9	Area of trapezium=(6+18)x8/2=96 m.	**3**	M1 setup; M1 process; A1 conclusion.
10	(x+3)(x+4)=x^2+7x+12, so p=7.	**3**	M1 setup; M1 process; A1 conclusion.
11	Since 2(x+1)^2>=0, y>=-5.	**3**	M1 setup; M1 process; A1 conclusion.
12	k=xy=-24. Then y=-24/(-3)=8.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	x^2-4x+7=2x+3 gives x^2-6x+4=0, so x=3± √ (5). Then y=9± 2√ (5).	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	y=(x-3)^2-7=x^2-6x+2, so b=-6 and c=2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Divide by (x-2): x^2-3x-4=(x-4)(x+1). Hence (x-2)(x-4)(x+1).	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	For y=0, 12/(x-1)=3, so x=5: (5,0). At x=0, y=-12-3=-15: (0,-15). Asymptotes x=1,y=-3.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	For F=3^x+x-20: F(2)=-9 and F(3)=10, so a root lies between. Testing gives F(2.5)≈-1.91 and F(2.6)≈-0.002; the solution is about 2.6001, hence x=2.6 to 1 d.p.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	v=0 at t=6. Signed area: triangle 0--6 is 54; triangle 6--8 is -6, displacement 48 m. Total distance=54+6=60 m.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	From 1 to3: (2-6)/2=-2. From3 to7: (10-2)/4=2. The curve decreases on average then increases; a minimum is suggested between/at the intervals.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	From 3 to 3.1, gradient=(3.1^2-3^2)/0.1=6.1. From 3 to 3.01, gradient=(3.01^2-3^2)/0.01=6.01. These values approach 6, so the tangent gradient is approximately 6.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Other side=12-x, so A=x(12-x)=-x^2+12x=-(x-6)^2+36. Maximum 36 cm^2 at x=6.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	h=-5(t-2)^2+22, so maximum 22 m at t=2. Ground: 5t^2-20t-2=0, t=2+√(110)/5≈4.10 s (positive root).	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	First area=(10+25)x12/2=210. Remaining=180=25T, so T=7.2 s.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	y=a(x+1)(x-2)(x-5). At x=0, y=10a=20, so a=2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	8/x=x+2 gives x^2+2x-8=0, so x=2 or -4. Points (2,4) and (-4,-2).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	[500(1.08^7)-500(1.08^2)]/5≈54.7 people per year.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	At x=3, y=10, so contact (3,10). Normal gradient=-1/7; y-10=-(x-3)/7.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 08 / A11, A12, A14, A15, A18

Graphs and Graphical Interpretation

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Axis is average of roots: (2+k)/2=5, so k=8. y=a(x-2)(x-8); at 0,16a=6, so a=3/8. Expanding: b=-15/4,c=6.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Intersection gives x^2-(4+m)x+7=0. Tangency means discriminant 0: (4+m)^2-28=0, so m=-4± 2√(7).	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	[1/b-1/a]/(b-a)=[(a-b)/(ab)]/(b-a)=-1/(ab).	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	y=x(x+2)(x-5)=x^3-3x^2-10x, so p=-3,q=-10. y(-2)=0,y(3)=-30; gradient=-30/5=-6.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Signed trapezia give 5.5+1.5-0.5-0.5+1.5+5.5+11.5=24.5 m. Using absolute areas gives total distance 5.5+1.5+0.5+0.5+1.5+5.5+11.5=26.5 m.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Translate 1 left, stretch vertically by factor 3, reflect in the x-axis, then translate 4 up. Horizontal asymptote y=4.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	2[x-(3+√5)][x-(3-√5)]=2[(x-3)^2-5]=2x^2-12x+8.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	x^2=12/x gives x^3=12, so x=∛(12), y=12^(2/3). For x>0, x^3 is strictly increasing, so x^3=12 has one positive solution.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	Tangent at (a,a^2): y-a^2=2a(x-a), so y=2ax-a^2. At x=0, -a^2=-9, hence a=± 3.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	R<=10 is equivalent to 40x<=10x^2+40, or 0<=10(x-2)^2, always true. Equality at x=2.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 09 / A7, A13

Functions, Composites and Transformations

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: ______________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	f(3)=12-7=5.	**1**	B1: correct result or statement.
2	16+2=18.	**1**	B1: correct result or statement.
3	2x+1=15, so x=7.	**2**	M1 method; A1 answer.
4	f^-1(x)=(x+5)/3.	**2**	M1 method; A1 answer.
5	g(3)=6, then f(6)=10.	**2**	M1 method; A1 answer.
6	Translation 6 units upwards.	**1**	B1: correct result or statement.
7	Translation 2 units to the right.	**1**	B1: correct result or statement.
8	It is not one-to-one: positive and negative inputs can give the same output.	**2**	M1 method; A1 answer.

MODULE 09 / A7, A13

Functions, Composites and Transformations

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	f(-2)=-14; g(-14)=196.	**3**	M1 setup; M1 process; A1 conclusion.
2	f^-1(x)=(x-7)/2, so f^-1(2x+7)=(2x+7-7)/2=x.	**3**	M1 setup; M1 process; A1 conclusion.
3	y=x^2-3 gives x=√ (y+3). Thus f^-1(x)=√ (x+3), domain x>=-3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	fg=3(x-5)+2=3x-13; gf=(3x+2)-5=3x-3.	**3**	M1 setup; M1 process; A1 conclusion.
5	4a+3=23, a=5; f^-1(x)=(x-3)/5.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	fg=2(x^2-3)+1=2x^2-5. Set 19: x^2=12, so x=2√ 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / A7, A13

Functions, Composites and Transformations

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Reflect y=f(x) in the x-axis, then translate 4 units up.	**3**	M1 setup; M1 process; A1 conclusion.
8	Horizontal stretch with scale factor 1/3 (compression towards the y-axis).	**3**	M1 setup; M1 process; A1 conclusion.
9	Translate by (4,3), giving (9,1).	**3**	M1 setup; M1 process; A1 conclusion.
10	Reflection in y-axis sends x to 3; vertical stretch factor 2 gives y=14. Point (3,14).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	x!=-2. f(4)=3/6=1/2.	**3**	M1 setup; M1 process; A1 conclusion.
12	1/(x-3)=2, so x-3=1/2 and x=7/2.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 09 / A7, A13

Functions, Composites and Transformations

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	(2x-3)^2+4=20, so (2x-3)^2=16; 2x-3=± 4; x=7/2 or -1/2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Let y=(3x-2)/(x+4). yx+4y=3x-2, x(y-3)=-2-4y, so x=(4y+2)/(3-y). Thus f^-1(x)=(4x+2)/(3-x).	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	f=(x-3)^2+2. Since x>=3, x=3+√ (y-2). Thus f^-1(x)=3+√ (x-2), x>=2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	f(f(x))=a(ax+b)+b=a^2x+b(a+1). Thus a^2=9 gives a=3; 4b=-8, so b=-2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	4g(x)+1=12x-7, so g(x)=3x-2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	y=x/(x+1): yx+y=x, so x(y-1)=-y and x=y/(1-y). Original x!=-1; inverse input x!=1.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Inside 3(x-4)=a gives x=4+a/3. Output=-2b+5. Image (4+a/3,5-2b).	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	When f=7 output=-11; when f=-2 output=7. Range -11<=y<=7.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / A7, A13

Functions, Composites and Transformations

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Rearrange y=(2x+1)/(x-3): yx-3y=2x+1, so x(y-2)=3y+1 and x=(3y+1)/(y-2)=g(y). Hence g=f^-1. The domain of f excludes 3 and the domain of g excludes 2.	5	M1 setup; M2–M4 development; A1 conclusion.
10	fg=(3x-2)^2+1=9x^2-12x+5; gf=3(x^2+1)-2=3x^2+1. Hence 6x^2-12x+4=0; 3x^2-6x+2=0; x=1± √ 3/3.	5	M1 setup; M2–M4 development; A1 conclusion.
11	Translation vector is (6,-6), so p=6 and q=-6.	4	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Horizontal stretch 2 gives y=1/(x/2)=2/x. Translation gives y=2/(x-3)-4.	5	M1 setup; M2–M4 development; A1 conclusion.
13	Inverse is 2+√ (x), so 25 maps to 7. The positive root is selected by x>=2.	3	M1 setup; M1 process; A1 conclusion.
14	h(4)=f(5)-2=7, so f(5)=9.	3	M1 setup; M1 process; A1 conclusion.
15	f^-1(10)=2 means f(2)=10. Thus 2k+6=10, k=2.	3	M1 setup; M1 process; A1 conclusion.

MODULE 09 / A7, A13

Functions, Composites and Transformations

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	f^-1(11)=4 means f(4)=11. Gradient=(11-7)/2=2; f=2x+3. Iterating from 0: 3,9,21,45,93, so f^5(0)=93.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Substitute: [(x+1)/(x-1)+1]/[(x+1)/(x-1)-1]=[2x/(x-1)]/[2/(x-1)]=x. Need x!=1; also f(x)!=1, which holds for no finite permitted x.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	(x+2)^2=x^2+2 gives 4x+4=2, so x=-1/2; the reduced equation is linear, so only one real solution.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	It must be symmetric about the y-axis (an even function). One non-quadratic example is f(x)=x^4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	f(f(x))=a^2x+b(a+1)=x. Hence a^2=1 and b(a+1)=0. Either a=-1 with any b, or a=1 with b=0 (identity).	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Set 2x+4=2, so x=-1. Output=3(5)-7=8. Corresponding point (-1,8).	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	f(g(x))=√ (x^2)=x for x>=0; g(f(x))=(√ (x+4))^2-4=x for x>=-4. Range f: y>=0; range g: y>=-4.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	f^-1=(x+3)/2. Set 2x-3=(x+3)/2: 4x-6=x+3, so x=3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	New X=2-x, so old x=2-X; new y=3f(2-X)+1. Hence y=3f(2-x)+1.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Need 1<=x+3<=6, so -2<=x<=3. Output extremes: f=2 gives1; f=-4 gives13. Range 1<=h<=13.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	x_2=(10+2)/3=4.	**1**	B1: correct result or statement.
2	x_2=√ 9=3.	**1**	B1: correct result or statement.
3	x=4/(x+1).	**2**	M1 method; A1 answer.
4	8-6-2=0.	**1**	B1: correct result or statement.
5	f(2)=-1; f(3)=4.	**2**	M1 method; A1 answer.
6	There is at least one root in the interval.	**2**	M1 method; A1 answer.
7	x^2=8-x, so x=√ (8-x).	**2**	M1 method; A1 answer.
8	Premature rounding accumulates error and may alter the final rounded answer.	**1**	B1: correct result or statement.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$x_2=\sqrt{7}=2.6458$; $x_3=\sqrt{(10-2.645751...)}=2.7119$.	**3**	M1 setup; M1 process; A1 conclusion.
2	$f(1)=-3$ and $f(2)=5$; the continuous function changes sign.	**3**	M1 setup; M1 process; A1 conclusion.
3	$f(2.1)=-0.49$ and $f(2.2)=0.04$, so $2.1<x<2.2$.	**3**	M1 setup; M1 process; A1 conclusion.
4	$3=12/(3+2)$ is false, so the claim is inconsistent: 3 is not a fixed point. The fixed points solve $x^2+2x-12=0$.	**3**	M1 setup; M1 process; A1 conclusion.
5	$x=6/(x+1)$, so $x^2+x-6=0$; $x=2$ or -3.	**3**	M1 setup; M1 process; A1 conclusion.
6	$x_2=(2+2.5)/2=2.25$; $x_3=(2.25+5/2.25)/2\approx2.23611$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	$2.7^3=19.683<20$. The rounding boundary is 2.75, and $2.75^3=20.796875>20$. Therefore the root is below 2.75 and above 2.7, so $x=2.7$ to 1 decimal place.	**3**	M1 setup; M1 process; A1 conclusion.
8	$f(2)=8-4-4=0$.	**2**	M1 method; A1 answer.
9	The principal square root is always non-negative whenever its input is defined.	**2**	M1 method; A1 answer.
10	At limit $L=0.4L+6$, so $0.6L=6$ and $L=10$.	**3**	M1 setup; M1 process; A1 conclusion.
11	x_2=∛ 20≈2.714.	**2**	M1 method; A1 answer.
12	4.26.	**2**	M1 method; A1 answer.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Iterates: 3.000, 2.96250, 2.95964, 2.95942, 2.95940; root≈2.959.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	$f(2.1)=9.261-8.4-1=-0.139$; $f(2.2)=10.648-8.8-1=0.848$. Refinement gives $x\approx2.1149$, so 2.11.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	$L^2=15-2L$, so $L^2+2L-15=0$; $L=3$ or -5. Iterates are non-negative and start at 3, so limit 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Use $x_{n+1}=\sqrt[3]{12-x_n}$. Iterates 2.1544,2.1425,2.1434,... so root≈2.143.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Let width x: $x(x+3)=40$, so $x^2+3x-40=0=(x+8)(x-5)$. Positive $x=5$; dimensions 5 cm by 8 cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$x^2+(x+1)^2=25$ gives $2x^2+2x-24=0$; $x=3$ or -4. Points (3,4),(-4,-3).	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Second difference4, so term$=2n^2+3n+1$. Set 276: $2n^2+3n-275=0=(2n+25)(n-11)$; $n=11$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$x^2-2x-3=2x+5$ gives $x^2-4x-8=0$, so $x=2\pm2\sqrt{3}$; $y=9\pm4\sqrt{3}$.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	f(1)=-7,f(2)=1, so a root lies there. Also f is strictly increasing because for b>a, f(b)-f(a)=(b-a)(a^2+ab+b^2+1)>0; hence only one.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	P_1=5080; P_2=5163.2; P_3=5249.728. After three periods the model predicts about 5250 units.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	L=2+3/L, so L^2-2L-3=0; L=3 or -1. Starting at 3 remains 3, so limit 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	x(x-1)=2(x+2); x^2-3x-4=0; x=4 or -1. Excluded x=0,-2; both roots valid.	**5**	M1 setup; M2–M4 development; A1 conclusion.
13	Discriminant k^2-36=0, so k=± 6.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	Rounding interval is [5.45,5.55). Squares: 5.45^2=29.7025<30<30.8025=5.55^2, so √ 30 is in the interval.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	ar^2=18, ar^5=486; r^3=27, so r=3 and a=18/9=2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 10 / A20; selected A1-A25 synthesis

Iteration, Numerical Methods and Mixed Higher Algebra

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: ______________________________ Date: _____________ Score: ______ / ______

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	At a fixed point x=(2x+12/x^2)/3, so 3x=2x+12/x^2 and x^3=12. x_2=7/3; x_3=[14/3+12/(49/9)]/3=1010/441≈2.290.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	f(-3)=-8,f(-2)=5 gives a root (-3,-2); f(0)=1,f(1)=-4 gives (0,1); f(2)=-3,f(3)=10 gives (2,3). Three disjoint intervals give three roots.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	L^2-5L+6=0, so L=2 or3. Different fixed points may be attracting/repelling for the chosen rearrangement; the initial value determines which basin or whether convergence occurs.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Positive factor pairs of12 are 1 and12, 2 and6, 3 and4; none differ by2. Negative roots give negative p. Therefore there is no positive p satisfying the condition.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Let u=x^2. u^2-13u+36=(u-4)(u-9)=0. Thus x=± 2 or ± 3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Radius OP is perpendicular to AP, so (a,b) dot (a-13,b)=0: a^2-13a+b^2=0. Since a^2+b^2=25, 25-13a=0, a=25/13. Then b=√ (25-a^2)=60/13.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Terms suggest u_n=4^(n-1)+1. Base n=1 gives2. If true, u_(n+1)=4(4^(n-1)+1)-3=4^n+1, so true by induction.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Intersection x^2-(4+k)x+3=0. Tangency: (4+k)^2-12=0, so k=-4± 2√ 3.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	The rearrangement is valid because x^3=10-x. But x_2=∛ (8)=2, not 28/13. Since x=2 is already the exact root, every later iterate remains 2.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	9=3a+6, so a=1. Squaring gives L^2-L-6=0, roots3,-2. The square-root iteration cannot converge to negative -2.	**5**	M1 setup; M2–M4 development; A1 conclusion.