GCSE MATHEMATICS

HIGHER TIER

UNIT 4: GEOMETRY AND MEASURES PRACTICE BOOK

Modules 1-10 | 450 original questions | Formula guides, full methods and marking guidance

FREE WEBSITE EDITION

Student name: _____

Target grade: _____ Start date: _____ Completion date: _____

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MODULE 01 / G1-G25

Unit 4 Contents and Study Plan

MODULES 1-10 • FREE WEBSITE EDITION

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Begin with the short fluency check, then move through Standard Higher, Multi-step Higher and Grade 7-9 Challenge. Formula guides and all worked answers are collected at the end.

Module	Topic	Specification	Score
1	Angle Reasoning, Congruence, Constructions and Loci	G1-G6	____ / ____
2	Transformations, Similarity and Scale Factors	G7,G8,G19	____ / ____
3	Circle Theorems and Geometric Proof	G9,G10	____ / ____
4	Mensuration, Circles, Sectors and Composite Areas	G14,G16-G18	____ / ____
5	3D Geometry, Surface Area, Volume and Frustums	G12,G13,G16,G17,G19	____ / ____
6	Pythagoras, Right-angle Trigonometry and 3D Problems	G20,G21	____ / ____
7	Non-right-angled Trigonometry and Triangle Area	G22,G23	____ / ____
8	Bearings, Coordinates and Geometric Modelling	G11,G15	____ / ____
9	Vectors and Geometric Proof	G24,G25	____ / ____
10	Mixed Higher Geometry and Measures Problem Solving	G1-G25 synthesis	____ / ____

Book structure

- 8 purposeful fluency questions per module
- 12 Standard Higher questions per module
- 15 multi-step Higher exam questions per module
- 10 Grade 7-9 challenge questions per module
- Formula and method bank for every module
- All answers, formulas and worked methods at the end

Unit 4 Specification Coverage Map

INDEPENDENTLY AUTHORED PRACTICE MAP

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Every question is independently authored. This map identifies the GCSE Mathematics skills covered by each module. It is not a reproduction or adaptation of an AQA question paper, mark scheme, examiner report or other assessment material.

Module	Specification alignment	Design guidance
M2	G7,G8,G19	Transformations, Similarity and Scale Factors
M3	G9,G10	Circle Theorems and Geometric Proof
M4	G14,G16-G18	Mensuration, Circles, Sectors and Composite Areas
M5	G12,G13,G16,G17,G19	3D Geometry, Surface Area, Volume and Frustums
M6	G20,G21	Pythagoras, Right-angle Trigonometry and 3D Problems
M7	G22,G23	Non-right-angled Trigonometry and Triangle Area
M8	G11,G15	Bearings, Coordinates and Geometric Modelling
M9	G24,G25	Vectors and Geometric Proof
M10	G1-G25 synthesis	Mixed Higher Geometry and Measures Problem Solving

Specification references above identify curriculum coverage only. All questions, diagrams, solutions and marking guidance in this resource are independently authored.

MODULE 01 / G1-G25

How to Use and Check Your Work

STUDENT ROUTINE AND QUALITY STANDARD

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

1 Attempt Work without notes first. Draw and label the geometry, then select a theorem, formula, vector or trigonometric relationship.

2 Mark Use the worked answer. Credit a method only when its mathematical setup is visible.

3 Repair Identify whether the error was modelling, units, algebra, arithmetic or interpretation; then redo from blank.

4 Retest Repeat a similar question after 2-7 days without support.

5 Present Give exact values where appropriate, sensible accuracy, units and a concluding statement.

Higher answer standard

- Define the variables and governing relationship
- State the geometric reason or theorem at every reasoning step
- Keep exact values and unrounded calculator values until the final line
- Distinguish average rates (chords) from instantaneous rates (tangents)
- Check whether a result is physically and mathematically reasonable

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. A regular 7-gon has an exterior angle of 360/7°. Find its interior angle and explain the relationship. [U4-M01-Q01] [2 marks]

2. An isosceles triangle has vertex angle 26°. A line through the vertex is perpendicular to the base. Prove that it bisects the vertex angle. [U4-M01-Q02] [3 marks]

3. Two parallel lines are cut by transversals forming a triangle. Two interior angles are 40° and 63°. Find the third angle and justify every angle fact. [U4-M01-Q03] [3 marks]

4. The interior angle of a regular polygon is 147.273°. Determine its number of sides without measuring. [U4-M01-Q04] [3 marks]

5. Triangles ABC and DEF have AB=10, AC=13, angle A=52°, DE=10, DF=13, angle D=52°. Prove the triangles congruent and state one result that follows. [U4-M01-Q05] [3 marks]

6. Describe accurately the locus of points inside an angle that are equidistant from its sides and at most 3 cm from its vertex. State the construction boundaries. [U4-M01-Q06] [3 marks]

7. **A quadrilateral has one pair of opposite sides equal and parallel. Prove it is a parallelogram using a diagonal and triangle congruence (case 7). [U4-M01-Q07]** **[4 marks]**

8. **The angles of a 8-gon are in the ratio 2:3:4:5:6:6:6:6. Explain how to find every angle and calculate the scale unit. [U4-M01-Q08]** **[4 marks]**

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Explain why SSA data do not guarantee triangle congruence, and describe how a construction can show this ambiguity (example 9). [U4-M01-Q09] [4 marks]

2. A regular 8-gon has an exterior angle of 360/8°. Find its interior angle and explain the relationship. [U4-M01-Q10] [3 marks]

3. An isosceles triangle has vertex angle 44°. A line through the vertex is perpendicular to the base. Prove that it bisects the vertex angle. [U4-M01-Q11] [4 marks]

4. Two parallel lines are cut by transversals forming a triangle. Two interior angles are 49° and 65°. Find the third angle and justify every angle fact. [U4-M01-Q12] [4 marks]

5. The interior angle of a regular polygon is 144°. Determine its number of sides without measuring. [U4-M01-Q13] [4 marks]

6. Triangles ABC and DEF have AB=9, AC=12, angle A=52°, DE=9, DF=12, angle D=52°. Prove the triangles congruent and state one result that follows. [U4-M01-Q14] [4 marks]

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **Describe accurately the locus of points inside an angle that are equidistant from its sides and at most 7 cm from its vertex. State the construction boundaries. [U4-M01-Q15]** **[4 marks]**

8. **A quadrilateral has one pair of opposite sides equal and parallel. Prove it is a parallelogram using a diagonal and triangle congruence (case 16). [U4-M01-Q16]** **[4 marks]**

9. **The angles of a 5-gon are in the ratio 2:3:4:5:6. Explain how to find every angle and calculate the scale unit. [U4-M01-Q17]** **[4 marks]**

10. **Explain why SSA data do not guarantee triangle congruence, and describe how a construction can show this ambiguity (example 18). [U4-M01-Q18]** **[4 marks]**

11. **A regular 9-gon has an exterior angle of 360/9°. Find its interior angle and explain the relationship. [U4-M01-Q19]** **[3 marks]**

12. **An isosceles triangle has vertex angle 62°. A line through the vertex is perpendicular to the base. Prove that it bisects the vertex angle. [U4-M01-Q20]** **[4 marks]**

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. Two parallel lines are cut by transversals forming a triangle. Two interior angles are 58° and 67°. Find the third angle and justify every angle fact. [U4-M01-Q21] [4 marks]

2. The interior angle of a regular polygon is 140°. Determine its number of sides without measuring. [U4-M01-Q22] [4 marks]

3. Triangles ABC and DEF have AB=8, AC=11, angle A=52°, DE=8, DF=11, angle D=52°. Prove the triangles congruent and state one result that follows. [U4-M01-Q23] [4 marks]

4. Describe accurately the locus of points inside an angle that are equidistant from its sides and at most 6 cm from its vertex. State the construction boundaries. [U4-M01-Q24] [4 marks]

5. A quadrilateral has one pair of opposite sides equal and parallel. Prove it is a parallelogram using a diagonal and triangle congruence (case 25). [U4-M01-Q25] [4 marks]

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. The angles of a 6-gon are in the ratio 2:3:4:5:6:6. Explain how to find every angle and calculate the scale unit. [U4-M01-Q26] [4 marks]

7. Explain why SSA data do not guarantee triangle congruence, and describe how a construction can show this ambiguity (example 27). [U4-M01-Q27] [4 marks]

8. A regular 10-gon has an exterior angle of 360/10°. Find its interior angle and explain the relationship. [U4-M01-Q28] [3 marks]

9. An isosceles triangle has vertex angle 80°. A line through the vertex is perpendicular to the base. Prove that it bisects the vertex angle. [U4-M01-Q29] [4 marks]

10. Two parallel lines are cut by transversals forming a triangle. Two interior angles are 67° and 62°. Find the third angle and justify every angle fact. [U4-M01-Q30] [4 marks]

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **The interior angle of a regular polygon is 135°. Determine its number of sides without measuring. [U4-M01-Q31]** **[4 marks]**

12. **Triangles ABC and DEF have AB=7, AC=10, angle A=52°, DE=7, DF=10, angle D=52°. Prove the triangles congruent and state one result that follows. [U4-M01-Q32]** **[4 marks]**

13. **Describe accurately the locus of points inside an angle that are equidistant from its sides and at most 5 cm from its vertex. State the construction boundaries. [U4-M01-Q33]** **[4 marks]**

14. **A quadrilateral has one pair of opposite sides equal and parallel. Prove it is a parallelogram using a diagonal and triangle congruence (case 34). [U4-M01-Q34]** **[4 marks]**

15. **The angles of a 7-gon are in the ratio 2:3:4:5:6:6:6. Explain how to find every angle and calculate the scale unit. [U4-M01-Q35]** **[4 marks]**

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. Explain why SSA data do not guarantee triangle congruence, and describe how a construction can show this ambiguity (example 36). [U4-M01-Q36] [4 marks]

2. A regular 11-gon has an exterior angle of 360/11°. Find its interior angle and explain the relationship. [U4-M01-Q37] [4 marks]

3. An isosceles triangle has vertex angle 98°. A line through the vertex is perpendicular to the base. Prove that it bisects the vertex angle. [U4-M01-Q38] [5 marks]

4. Two parallel lines are cut by transversals forming a triangle. Two interior angles are 76° and 64°. Find the third angle and justify every angle fact. [U4-M01-Q39] [5 marks]

5. The interior angle of a regular polygon is 158.824°. Determine its number of sides without measuring. [U4-M01-Q40] [5 marks]

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. Triangles ABC and DEF have AB=6, AC=9, angle A=52°, DE=6, DF=9, angle D=52°. Prove the triangles congruent and state one result that follows. [U4-M01-Q41] [5 marks]

7. Describe accurately the locus of points inside an angle that are equidistant from its sides and at most 4 cm from its vertex. State the construction boundaries. [U4-M01-Q42] [5 marks]

8. A quadrilateral has one pair of opposite sides equal and parallel. Prove it is a parallelogram using a diagonal and triangle congruence (case 43). [U4-M01-Q43] [4 marks]

9. The angles of a 8-gon are in the ratio 2:3:4:5:6:6:6:6. Explain how to find every angle and calculate the scale unit. [U4-M01-Q44] [4 marks]

10. Explain why SSA data do not guarantee triangle congruence, and describe how a construction can show this ambiguity (example 45). [U4-M01-Q45] [4 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Enlarge P(2,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q01] [2 marks]

2. Two similar shapes have length scale factor 1.5. The smaller area is 49 cm². Find the larger area and explain the exponent used. [U4-M02-Q02] [3 marks]

3. Similar solids have volume ratio 1:64. The smaller volume is 50 cm³. Find the larger volume and the length scale factor. [U4-M02-Q03] [3 marks]

4. Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 4). [U4-M02-Q04] [3 marks]

5. A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 5). [U4-M02-Q05] [4 marks]

6. Similar shapes have corresponding lengths 10 and 13. The smaller area is 40 cm². Find the larger area. [U4-M02-Q06] [4 marks]

7. Prove that a rotation followed by its inverse maps every point back to itself, using centre-to-point vectors (case 7). [U4-M02-Q07] [4 marks]

8. A photograph is enlarged with width scale factor 3/2 and then reduced with scale factor -2/3 about a different centre. State the overall size factor and explain the negative sign (case 8). [U4-M02-Q08] [4 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Enlarge P(5,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q09] [3 marks]

2. Two similar shapes have length scale factor 1.5. The smaller area is 64 cm². Find the larger area and explain the exponent used. [U4-M02-Q10] [3 marks]

3. Similar solids have volume ratio 1:27. The smaller volume is 90 cm³. Find the larger volume and the length scale factor. [U4-M02-Q11] [4 marks]

4. Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 12). [U4-M02-Q12] [3 marks]

5. A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 13). [U4-M02-Q13] [4 marks]

6. Similar shapes have corresponding lengths 6 and 9. The smaller area is 56 cm². Find the larger area. [U4-M02-Q14] [4 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **Prove that a rotation followed by its inverse maps every point back to itself, using centre-to-point vectors (case 15). [U4-M02-Q15]** [4 marks]

8. **A photograph is enlarged with width scale factor 3/2 and then reduced with scale factor -2/3 about a different centre. State the overall size factor and explain the negative sign (case 16). [U4-M02-Q16]** [4 marks]

9. **Enlarge P(3,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q17]** [3 marks]

10. **Two similar shapes have length scale factor 1.5. The smaller area is 81 cm². Find the larger area and explain the exponent used. [U4-M02-Q18]** [3 marks]

11. **Similar solids have volume ratio 1:8. The smaller volume is 130 cm³. Find the larger volume and the length scale factor. [U4-M02-Q19]** [4 marks]

12. **Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 20). [U4-M02-Q20]** [3 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 21). [U4-M02-Q21] [5 marks]

2. Similar shapes have corresponding lengths 8 and 11. The smaller area is 72 cm². Find the larger area. [U4-M02-Q22] [4 marks]

3. Prove that a rotation followed by its inverse maps every point back to itself, using centre-to-point vectors (case 23). [U4-M02-Q23] [4 marks]

4. A photograph is enlarged with width scale factor 3/2 and then reduced with scale factor -2/3 about a different centre. State the overall size factor and explain the negative sign (case 24). [U4-M02-Q24] [4 marks]

5. Enlarge P(6,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q25] [3 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. Two similar shapes have length scale factor 1.5. The smaller area is 100 cm². Find the larger area and explain the exponent used. [U4-M02-Q26] [3 marks]

7. Similar solids have volume ratio 1:64. The smaller volume is 170 cm³. Find the larger volume and the length scale factor. [U4-M02-Q27] [4 marks]

8. Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 28). [U4-M02-Q28] [3 marks]

9. A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 29). [U4-M02-Q29] [5 marks]

10. Similar shapes have corresponding lengths 10 and 13. The smaller area is 88 cm². Find the larger area. [U4-M02-Q30] [4 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Prove that a rotation followed by its inverse maps every point back to itself, using centre-to-point vectors (case 31). [U4-M02-Q31]** **[4 marks]**

12. **A photograph is enlarged with width scale factor 3/2 and then reduced with scale factor -2/3 about a different centre. State the overall size factor and explain the negative sign (case 32). [U4-M02-Q32]** **[4 marks]**

13. **Enlarge P(4,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q33]** **[3 marks]**

14. **Two similar shapes have length scale factor 1.5. The smaller area is 121 cm². Find the larger area and explain the exponent used. [U4-M02-Q34]** **[3 marks]**

15. **Similar solids have volume ratio 1:27. The smaller volume is 210 cm³. Find the larger volume and the length scale factor. [U4-M02-Q35]** **[4 marks]**

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 36). [U4-M02-Q36] [3 marks]

2. A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 37). [U4-M02-Q37] [5 marks]

3. Similar shapes have corresponding lengths 6 and 9. The smaller area is 104 cm². Find the larger area. [U4-M02-Q38] [4 marks]

4. Prove that a rotation followed by its inverse maps every point back to itself, using centre-to-point vectors (case 39). [U4-M02-Q39] [4 marks]

5. A photograph is enlarged with width scale factor 3/2 and then reduced with scale factor -2/3 about a different centre. State the overall size factor and explain the negative sign (case 40). [U4-M02-Q40] [4 marks]

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **Enlarge P(2,-3) about C(1,-2) by scale factor -3. Give P′. [U4-M02-Q41]** **[4 marks]**

7. **Two similar shapes have length scale factor 1.5. The smaller area is 144 cm². Find the larger area and explain the exponent used. [U4-M02-Q42]** **[3 marks]**

8. **Similar solids have volume ratio 1:8. The smaller volume is 250 cm³. Find the larger volume and the length scale factor. [U4-M02-Q43]** **[5 marks]**

9. **Describe the single transformation equivalent to reflection in the x-axis followed by reflection in the y-axis (case 44). [U4-M02-Q44]** **[3 marks]**

10. **A triangle is transformed by rotation 90° clockwise about (2,-1), then translation by vector (3,4). Explain which properties are invariant and whether the combined transformation is a rotation alone (case 45). [U4-M02-Q45]** **[5 marks]**

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Angle AOB at the centre is 48°. Find angle ACB at the circumference subtended by the same arc. [U4-M03-Q01] [2 marks]

2. AB is a diameter and C lies on the circle. Angle BAC=43°. Find angle ABC. [U4-M03-Q02] [3 marks]

3. ABCD is cyclic. Angle ABC=92°. Find angle ADC and give the theorem. [U4-M03-Q03] [3 marks]

4. A tangent at A makes an angle of 38° with chord AB. Find the angle in the alternate segment. [U4-M03-Q04] [3 marks]

5. Tangents PA and PB touch a circle. PA=11 cm. Find PB and prove the equality. [U4-M03-Q05] [4 marks]

6. A chord of length 8 cm is 4 cm from the centre. Find the radius exactly. [U4-M03-Q06] [4 marks]

7. Two angles standing on chord AB are x+0 and 3x-19 degrees in the same segment. Find x. [U4-M03-Q07] [4 marks]

8. Prove that the angle between two tangents from P equals 180° minus angle AOB at the centre (case 8). [U4-M03-Q08] **[5 marks]**

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. A circle-theorem diagram contains a tangent, a cyclic quadrilateral and a diameter. Write a logically ordered strategy to solve an unknown angle (case 9). [U4-M03-Q09] [5 marks]

2. Angle AOB at the centre is 66°. Find angle ACB at the circumference subtended by the same arc. [U4-M03-Q10] [2 marks]

3. AB is a diameter and C lies on the circle. Angle BAC=52°. Find angle ABC. [U4-M03-Q11] [3 marks]

4. ABCD is cyclic. Angle ABC=101°. Find angle ADC and give the theorem. [U4-M03-Q12] [3 marks]

5. A tangent at A makes an angle of 47° with chord AB. Find the angle in the alternate segment. [U4-M03-Q13] [3 marks]

6. Tangents PA and PB touch a circle. PA=12 cm. Find PB and prove the equality. [U4-M03-Q14] [4 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. A chord of length 16 cm is 5 cm from the centre. Find the radius exactly. [U4-M03-Q15] [4 marks]

8. Two angles standing on chord AB are x+3 and 3x-18 degrees in the same segment. Find x. [U4-M03-Q16] [4 marks]

9. Prove that the angle between two tangents from P equals 180° minus angle AOB at the centre (case 17). [U4-M03-Q17] [5 marks]

10. A circle-theorem diagram contains a tangent, a cyclic quadrilateral and a diameter. Write a logically ordered strategy to solve an unknown angle (case 18). [U4-M03-Q18] [5 marks]

11. Angle AOB at the centre is 84°. Find angle ACB at the circumference subtended by the same arc. [U4-M03-Q19] [2 marks]

12. AB is a diameter and C lies on the circle. Angle BAC=61°. Find angle ABC. [U4-M03-Q20] [3 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. ABCD is cyclic. Angle ABC=110°. Find angle ADC and give the theorem. [U4-M03-Q21] [3 marks]

2. A tangent at A makes an angle of 56° with chord AB. Find the angle in the alternate segment. [U4-M03-Q22] [3 marks]

3. Tangents PA and PB touch a circle. PA=13 cm. Find PB and prove the equality. [U4-M03-Q23] [4 marks]

4. A chord of length 14 cm is 6 cm from the centre. Find the radius exactly. [U4-M03-Q24] [4 marks]

5. Two angles standing on chord AB are x+0 and 3x-22 degrees in the same segment. Find x. [U4-M03-Q25] [4 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. Prove that the angle between two tangents from P equals 180° minus angle AOB at the centre (case 26). [U4-M03-Q26] [5 marks]

7. A circle-theorem diagram contains a tangent, a cyclic quadrilateral and a diameter. Write a logically ordered strategy to solve an unknown angle (case 27). [U4-M03-Q27] [5 marks]

8. Angle AOB at the centre is 102°. Find angle ACB at the circumference subtended by the same arc. [U4-M03-Q28] [2 marks]

9. AB is a diameter and C lies on the circle. Angle BAC=70°. Find angle ABC. [U4-M03-Q29] [3 marks]

10. ABCD is cyclic. Angle ABC=119°. Find angle ADC and give the theorem. [U4-M03-Q30] [3 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. A tangent at A makes an angle of 35° with chord AB. Find the angle in the alternate segment. [U4-M03-Q31] [3 marks]

12. Tangents PA and PB touch a circle. PA=14 cm. Find PB and prove the equality. [U4-M03-Q32] [4 marks]

13. A chord of length 12 cm is 3 cm from the centre. Find the radius exactly. [U4-M03-Q33] [4 marks]

14. Two angles standing on chord AB are x+3 and 3x-21 degrees in the same segment. Find x. [U4-M03-Q34] [4 marks]

15. Prove that the angle between two tangents from P equals 180° minus angle AOB at the centre (case 35). [U4-M03-Q35] [5 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. A circle-theorem diagram contains a tangent, a cyclic quadrilateral and a diameter. Write a logically ordered strategy to solve an unknown angle (case 36). [U4-M03-Q36] [5 marks]

2. Angle AOB at the centre is 120°. Find angle ACB at the circumference subtended by the same arc. [U4-M03-Q37] [2 marks]

3. AB is a diameter and C lies on the circle. Angle BAC=24°. Find angle ABC. [U4-M03-Q38] [3 marks]

4. ABCD is cyclic. Angle ABC=128°. Find angle ADC and give the theorem. [U4-M03-Q39] [3 marks]

5. A tangent at A makes an angle of 44° with chord AB. Find the angle in the alternate segment. [U4-M03-Q40] [3 marks]

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. Tangents PA and PB touch a circle. PA=7 cm. Find PB and prove the equality. [U4-M03-Q41] [4 marks]

7. A chord of length 10 cm is 4 cm from the centre. Find the radius exactly. [U4-M03-Q42] [4 marks]

8. Two angles standing on chord AB are x+0 and 3x-20 degrees in the same segment. Find x. [U4-M03-Q43] [4 marks]

9. Prove that the angle between two tangents from P equals 180° minus angle AOB at the centre (case 44). [U4-M03-Q44] [5 marks]

10. A circle-theorem diagram contains a tangent, a cyclic quadrilateral and a diameter. Write a logically ordered strategy to solve an unknown angle (case 45). [U4-M03-Q45] [5 marks]

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q01] [2 marks]

2. A sector has radius 4 cm and angle 45°. Find its area and arc length exactly. [U4-M04-Q02] [3 marks]

3. A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q03] [4 marks]

4. A sector of radius 6 cm has perimeter 12+2.5π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q04] [4 marks]

5. A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q05] [4 marks]

6. Two similar sectors have radii 8 and 11. The smaller area is 105π cm². Find the larger area. [U4-M04-Q06] [4 marks]

7. A regular 11-gon is inscribed in a circle radius 9. Split it into congruent triangles and state an exact area expression. [U4-M04-Q07] [5 marks]

8. **A circular path has outer radius 12 m and inner radius 10 m. Find its exact area and the cost at £11 per m². [U4-M04-Q08]** **[4 marks]**

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q09] [2 marks]

2. A sector has radius 4 cm and angle 30°. Find its area and arc length exactly. [U4-M04-Q10] [3 marks]

3. A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q11] [4 marks]

4. A sector of radius 6 cm has perimeter 12+2π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q12] [4 marks]

5. A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q13] [4 marks]

6. Two similar sectors have radii 8 and 11. The smaller area is 90π cm². Find the larger area. [U4-M04-Q14] [4 marks]

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **A regular 5-gon is inscribed in a circle radius 9. Split it into congruent triangles and state an exact area expression. [U4-M04-Q15]** **[5 marks]**

8. **A circular path has outer radius 12 m and inner radius 10 m. Find its exact area and the cost at £13 per m². [U4-M04-Q16]** **[4 marks]**

9. **Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q17]** **[2 marks]**

10. **A sector has radius 4 cm and angle 150°. Find its area and arc length exactly. [U4-M04-Q18]** **[3 marks]**

11. **A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q19]** **[4 marks]**

12. **A sector of radius 6 cm has perimeter 12+1.5π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q20]** **[4 marks]**

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q21] **[4 marks]**

2. Two similar sectors have radii 8 and 11. The smaller area is 75π cm². Find the larger area. [U4-M04-Q22] **[4 marks]**

3. A regular 6-gon is inscribed in a circle radius 9. Split it into congruent triangles and state an exact area expression. [U4-M04-Q23] **[5 marks]**

4. A circular path has outer radius 12 m and inner radius 10 m. Find its exact area and the cost at £15 per m². [U4-M04-Q24] **[4 marks]**

5. Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q25] **[2 marks]**

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. A sector has radius 4 cm and angle 135°. Find its area and arc length exactly. [U4-M04-Q26] [3 marks]

7. A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q27] [4 marks]

8. A sector of radius 6 cm has perimeter 12+1π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q28] [4 marks]

9. A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q29] [4 marks]

10. Two similar sectors have radii 8 and 11. The smaller area is 60π cm². Find the larger area. [U4-M04-Q30] [4 marks]

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **A regular 7-gon is inscribed in a circle radius 9. Split it into congruent triangles and state an exact area expression. [U4-M04-Q31]** **[5 marks]**

12. **A circular path has outer radius 12 m and inner radius 10 m. Find its exact area and the cost at £11 per m². [U4-M04-Q32]** **[4 marks]**

13. **Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q33]** **[2 marks]**

14. **A sector has radius 4 cm and angle 120°. Find its area and arc length exactly. [U4-M04-Q34]** **[3 marks]**

15. **A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q35]** **[4 marks]**

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. **A sector of radius 6 cm has perimeter 12+5π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q36]** **[4 marks]**

2. **A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q37]** **[4 marks]**

3. **Two similar sectors have radii 8 and 11. The smaller area is 45π cm². Find the larger area. [U4-M04-Q38]** **[4 marks]**

4. **A regular 8-gon is inscribed in a circle radius 9. Split it into congruent triangles and state an exact area expression. [U4-M04-Q39]** **[5 marks]**

5. **A circular path has outer radius 12 m and inner radius 10 m. Find its exact area and the cost at £13 per m². [U4-M04-Q40]** **[4 marks]**

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **Find exactly the area and circumference of a circle of radius 3 cm. [U4-M04-Q41]** **[2 marks]**

7. **A sector has radius 4 cm and angle 105°. Find its area and arc length exactly. [U4-M04-Q42]** **[3 marks]**

8. **A shape consists of a 10 by 9 rectangle with a semicircle of diameter 10 attached. Find its exact area and perimeter. [U4-M04-Q43]** **[4 marks]**

9. **A sector of radius 6 cm has perimeter 12+4.5π cm. Determine its angle by forming an arc-length equation. [U4-M04-Q44]** **[4 marks]**

10. **A circle has area 245π cm². Find its radius and circumference exactly. [U4-M04-Q45]** **[4 marks]**

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Find the volume and total surface area of a cylinder radius 3 cm, height 5 cm. [U4-M05-Q01] [3 marks]

2. Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q02] [3 marks]

3. A cone has radius 5 cm, height 7 cm and slant height √(74). Find volume and total surface area. [U4-M05-Q03] [4 marks]

4. A square-based pyramid has base side 8 cm and perpendicular height 8 cm. Find its volume. [U4-M05-Q04] [3 marks]

5. A frustum is made by removing a similar cone of linear scale factor 1/3 from a cone of volume 168π cm³. Find the frustum volume. [U4-M05-Q05] [5 marks]

6. A solid consists of a hemisphere radius 8 on a cylinder of the same radius and height 10. Find its exact volume. [U4-M05-Q06] [4 marks]

7. Similar containers have surface-area ratio 81:121. Their volume difference is 130 cm³. Form an equation for the smaller volume. [U4-M05-Q07] [5 marks]

8. A sphere of radius 10 is melted into cones radius 5, height 5. Find the exact number of complete cones possible. [U4-M05-Q08] **[5 marks]**

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Find the volume and total surface area of a cylinder radius 3 cm, height 6 cm. [U4-M05-Q09] [3 marks]

2. Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q10] [3 marks]

3. A cone has radius 5 cm, height 8 cm and slant height √(89). Find volume and total surface area. [U4-M05-Q11] [4 marks]

4. A square-based pyramid has base side 8 cm and perpendicular height 9 cm. Find its volume. [U4-M05-Q12] [3 marks]

5. A frustum is made by removing a similar cone of linear scale factor 1/2 from a cone of volume 264π cm³. Find the frustum volume. [U4-M05-Q13] [5 marks]

6. A solid consists of a hemisphere radius 8 on a cylinder of the same radius and height 11. Find its exact volume. [U4-M05-Q14] [4 marks]

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **Similar containers have surface-area ratio 81:121. Their volume difference is 170 cm³. Form an equation for the smaller volume. [U4-M05-Q15]** [5 marks]

8. **A sphere of radius 10 is melted into cones radius 5, height 6. Find the exact number of complete cones possible. [U4-M05-Q16]** [5 marks]

9. **Find the volume and total surface area of a cylinder radius 3 cm, height 7 cm. [U4-M05-Q17]** [3 marks]

10. **Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q18]** [3 marks]

11. **A cone has radius 5 cm, height 9 cm and slant height √(106). Find volume and total surface area. [U4-M05-Q19]** [4 marks]

12. **A square-based pyramid has base side 8 cm and perpendicular height 10 cm. Find its volume. [U4-M05-Q20]** [3 marks]

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A frustum is made by removing a similar cone of linear scale factor 1/4 from a cone of volume 360π cm³. Find the frustum volume. [U4-M05-Q21] [5 marks]

2. A solid consists of a hemisphere radius 8 on a cylinder of the same radius and height 5. Find its exact volume. [U4-M05-Q22] [4 marks]

3. Similar containers have surface-area ratio 81:121. Their volume difference is 210 cm³. Form an equation for the smaller volume. [U4-M05-Q23] [5 marks]

4. A sphere of radius 10 is melted into cones radius 5, height 7. Find the exact number of complete cones possible. [U4-M05-Q24] [5 marks]

5. Find the volume and total surface area of a cylinder radius 3 cm, height 8 cm. [U4-M05-Q25] [3 marks]

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. **Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q26]** **[3 marks]**

7. **A cone has radius 5 cm, height 10 cm and slant height √(125). Find volume and total surface area. [U4-M05-Q27]** **[4 marks]**

8. **A square-based pyramid has base side 8 cm and perpendicular height 11 cm. Find its volume. [U4-M05-Q28]** **[3 marks]**

9. **A frustum is made by removing a similar cone of linear scale factor 1/3 from a cone of volume 456π cm³. Find the frustum volume. [U4-M05-Q29]** **[5 marks]**

10. **A solid consists of a hemisphere radius 8 on a cylinder of the same radius and height 6. Find its exact volume. [U4-M05-Q30]** **[4 marks]**

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **Similar containers have surface-area ratio 81:121. Their volume difference is 250 cm³. Form an equation for the smaller volume. [U4-M05-Q31]** **[5 marks]**

12. **A sphere of radius 10 is melted into cones radius 5, height 8. Find the exact number of complete cones possible. [U4-M05-Q32]** **[5 marks]**

13. **Find the volume and total surface area of a cylinder radius 3 cm, height 9 cm. [U4-M05-Q33]** **[3 marks]**

14. **Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q34]** **[3 marks]**

15. **A cone has radius 5 cm, height 11 cm and slant height √(146). Find volume and total surface area. [U4-M05-Q35]** **[4 marks]**

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. A square-based pyramid has base side 8 cm and perpendicular height 5 cm. Find its volume. [U4-M05-Q36] [3 marks]

2. A frustum is made by removing a similar cone of linear scale factor 1/2 from a cone of volume 552π cm³. Find the frustum volume. [U4-M05-Q37] [5 marks]

3. A solid consists of a hemisphere radius 8 on a cylinder of the same radius and height 7. Find its exact volume. [U4-M05-Q38] [4 marks]

4. Similar containers have surface-area ratio 81:121. Their volume difference is 290 cm³. Form an equation for the smaller volume. [U4-M05-Q39] [5 marks]

5. A sphere of radius 10 is melted into cones radius 5, height 9. Find the exact number of complete cones possible. [U4-M05-Q40] [5 marks]

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **Find the volume and total surface area of a cylinder radius 3 cm, height 10 cm. [U4-M05-Q41]** **[3 marks]**

7. **Find the volume and surface area of a sphere radius 4 cm in exact form. [U4-M05-Q42]** **[3 marks]**

8. **A cone has radius 5 cm, height 5 cm and slant height √(50). Find volume and total surface area. [U4-M05-Q43]** **[4 marks]**

9. **A square-based pyramid has base side 8 cm and perpendicular height 6 cm. Find its volume. [U4-M05-Q44]** **[3 marks]**

10. **A frustum is made by removing a similar cone of linear scale factor 1/4 from a cone of volume 648π cm³. Find the frustum volume. [U4-M05-Q45]** **[5 marks]**

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. A right triangle has legs 5 and 8. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q01] [3 marks]

2. In a right triangle, hypotenuse 15 and one leg 6. Find the other leg. [U4-M06-Q02] [3 marks]

3. A line of length 14 makes angle 34° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q03] [4 marks]

4. A cuboid measures 8 by 11 by 11. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q04] [4 marks]

5. A ladder length 17 m reaches 9 m vertically. Find its angle with the ground. [U4-M06-Q05] [4 marks]

6. Use exact values to simplify $20\sin30°+13\cos60°$. [U4-M06-Q06] [3 marks]

7. A pyramid has apex vertically above the centre of a square base side 22. Its vertical height is 14. Find the angle between a sloping edge and the base. [U4-M06-Q07] [5 marks]

8. Prove algebraically that a triangle with sides 24, 10, 26 is right-angled. [U4-M06-Q08] **[5 marks]**

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. A right triangle has legs 6 and 16. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q09] [3 marks]

2. In a right triangle, hypotenuse 14 and one leg 7. Find the other leg. [U4-M06-Q10] [3 marks]

3. A line of length 13 makes angle 58° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q11] [4 marks]

4. A cuboid measures 9 by 10 by 12. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q12] [4 marks]

5. A ladder length 16 m reaches 10 m vertically. Find its angle with the ground. [U4-M06-Q13] [4 marks]

6. Use exact values to simplify $22\sin30°+12\cos60°$. [U4-M06-Q14] [3 marks]

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **A pyramid has apex vertically above the centre of a square base side 10. Its vertical height is 13. Find the angle between a sloping edge and the base. [U4-M06-Q15]** **[5 marks]**

8. **Prove algebraically that a triangle with sides 35, 12, 37 is right-angled. [U4-M06-Q16]** **[5 marks]**

9. **A right triangle has legs 7 and 15. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q17]** **[3 marks]**

10. **In a right triangle, hypotenuse 22 and one leg 8. Find the other leg. [U4-M06-Q18]** **[3 marks]**

11. **A line of length 12 makes angle 46° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q19]** **[4 marks]**

12. **A cuboid measures 10 by 9 by 13. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q20]** **[4 marks]**

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A ladder length 15 m reaches 11 m vertically. Find its angle with the ground. [U4-M06-Q21] [4 marks]

2. Use exact values to simplify $10\sin30°+11\cos60°$. [U4-M06-Q22] [3 marks]

3. A pyramid has apex vertically above the centre of a square base side 12. Its vertical height is 12. Find the angle between a sloping edge and the base. [U4-M06-Q23] [5 marks]

4. Prove algebraically that a triangle with sides 48, 14, 50 is right-angled. [U4-M06-Q24] [5 marks]

5. A right triangle has legs 8 and 14. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q25] [3 marks]

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. **In a right triangle, hypotenuse 21 and one leg 9. Find the other leg. [U4-M06-Q26]** **[3 marks]**

7. **A line of length 20 makes angle 34° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q27]** **[4 marks]**

8. **A cuboid measures 11 by 8 by 14. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q28]** **[4 marks]**

9. **A ladder length 14 m reaches 5 m vertically. Find its angle with the ground. [U4-M06-Q29]** **[4 marks]**

10. **Use exact values to simplify $12\sin30°+10\cos60°$. [U4-M06-Q30]** **[3 marks]**

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **A pyramid has apex vertically above the centre of a square base side 14. Its vertical height is 11. Find the angle between a sloping edge and the base. [U4-M06-Q31]** **[5 marks]**

12. **Prove algebraically that a triangle with sides 63, 16, 65 is right-angled. [U4-M06-Q32]** **[5 marks]**

13. **A right triangle has legs 9 and 13. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q33]** **[3 marks]**

14. **In a right triangle, hypotenuse 20 and one leg 10. Find the other leg. [U4-M06-Q34]** **[3 marks]**

15. **A line of length 19 makes angle 58° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q35]** **[4 marks]**

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. A cuboid measures 5 by 16 by 8. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q36] [4 marks]

2. A ladder length 13 m reaches 6 m vertically. Find its angle with the ground. [U4-M06-Q37] [4 marks]

3. Use exact values to simplify 14sin30°+9cos60°. [U4-M06-Q38] [3 marks]

4. A pyramid has apex vertically above the centre of a square base side 16. Its vertical height is 10. Find the angle between a sloping edge and the base. [U4-M06-Q39] [5 marks]

5. Prove algebraically that a triangle with sides 80, 18, 82 is right-angled. [U4-M06-Q40] [5 marks]

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **A right triangle has legs 10 and 12. Find the hypotenuse exactly and to 3 s.f. [U4-M06-Q41]** **[3 marks]**

7. **In a right triangle, hypotenuse 19 and one leg 11. Find the other leg. [U4-M06-Q42]** **[3 marks]**

8. **A line of length 18 makes angle 46° with the horizontal. Find its vertical and horizontal components. [U4-M06-Q43]** **[4 marks]**

9. **A cuboid measures 6 by 15 by 9. Find its space diagonal exactly and to 3 s.f. [U4-M06-Q44]** **[4 marks]**

10. **A ladder length 21 m reaches 7 m vertically. Find its angle with the ground. [U4-M06-Q45]** **[4 marks]**

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. In triangle ABC, a=5, A=28°, B=53°. Find b using the sine rule. [U4-M07-Q01] **[4 marks]**

2. Two sides of a triangle are 6 and 9, with included angle 31°. Find the third side. [U4-M07-Q02] **[4 marks]**

3. A triangle has sides 7, 10, 15. Find the angle opposite side 15. [U4-M07-Q03] **[4 marks]**

4. Find the area of a triangle with sides 8, 11 and included angle 37°. [U4-M07-Q04] **[3 marks]**

5. A triangle has area 32.4, sides 9 and 12 enclosing an acute angle. Find that angle. [U4-M07-Q05] **[4 marks]**

6. Explain the ambiguous sine-rule case for a=10, b=13, A=43°, and determine whether a second angle is possible. [U4-M07-Q06] **[5 marks]**

7. A quadrilateral is split by a diagonal 14. On one side the adjacent sides are 11,16 with included angle 46°. Describe a complete method for total area. [U4-M07-Q07] **[5 marks]**

8. Prove the cosine rule reduces to Pythagoras when the included angle is 90° (case 8). [U4-M07-Q08] **[4 marks]**

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. In triangle ABC, a=6, A=52°, B=77°. Find b using the sine rule. [U4-M07-Q09] **[4 marks]**

2. Two sides of a triangle are 7 and 8, with included angle 55°. Find the third side. [U4-M07-Q10] **[4 marks]**

3. A triangle has sides 8, 9, 15. Find the angle opposite side 15. [U4-M07-Q11] **[4 marks]**

4. Find the area of a triangle with sides 9, 10 and included angle 61°. [U4-M07-Q12] **[3 marks]**

5. A triangle has area 33, sides 10 and 11 enclosing an acute angle. Find that angle. [U4-M07-Q13] **[4 marks]**

6. Explain the ambiguous sine-rule case for a=11, b=12, A=31°, and determine whether a second angle is possible. [U4-M07-Q14] **[5 marks]**

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. A quadrilateral is split by a diagonal 13. On one side the adjacent sides are 5,15 with included angle 34°. Describe a complete method for total area. [U4-M07-Q15] [5 marks]

8. Prove the cosine rule reduces to Pythagoras when the included angle is 90° (case 16). [U4-M07-Q16] [4 marks]

9. In triangle ABC, a=7, A=40°, B=65°. Find b using the sine rule. [U4-M07-Q17] [4 marks]

10. Two sides of a triangle are 8 and 16, with included angle 43°. Find the third side. [U4-M07-Q18] [4 marks]

11. A triangle has sides 9, 8, 15. Find the angle opposite side 15. [U4-M07-Q19] [4 marks]

12. Find the area of a triangle with sides 10, 9 and included angle 49°. [U4-M07-Q20] [3 marks]

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A triangle has area 33, sides 11 and 10 enclosing an acute angle. Find that angle. [U4-M07-Q21] [4 marks]

2. Explain the ambiguous sine-rule case for a=5, b=11, A=55°, and determine whether a second angle is possible. [U4-M07-Q22] [5 marks]

3. A quadrilateral is split by a diagonal 12. On one side the adjacent sides are 6,14 with included angle 58°. Describe a complete method for total area. [U4-M07-Q23] [5 marks]

4. Prove the cosine rule reduces to Pythagoras when the included angle is 90° (case 24). [U4-M07-Q24] [4 marks]

5. In triangle ABC, a=8, A=28°, B=53°. Find b using the sine rule. [U4-M07-Q25] [4 marks]

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. **Two sides of a triangle are 9 and 15, with included angle 31°. Find the third side. [U4-M07-Q26]** **[4 marks]**

7. **A triangle has sides 10, 16, 24. Find the angle opposite side 24. [U4-M07-Q27]** **[4 marks]**

8. **Find the area of a triangle with sides 11, 8 and included angle 37°. [U4-M07-Q28]** **[3 marks]**

9. **A triangle has area 13.5, sides 5 and 9 enclosing an acute angle. Find that angle. [U4-M07-Q29]** **[4 marks]**

10. **Explain the ambiguous sine-rule case for a=6, b=10, A=43°, and determine whether a second angle is possible. [U4-M07-Q30]** **[5 marks]**

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **A quadrilateral is split by a diagonal 11. On one side the adjacent sides are 7,13 with included angle 46°. Describe a complete method for total area. [U4-M07-Q31]** **[5 marks]**

12. **Prove the cosine rule reduces to Pythagoras when the included angle is 90° (case 32). [U4-M07-Q32]** **[4 marks]**

13. **In triangle ABC, a=9, A=52°, B=77°. Find b using the sine rule. [U4-M07-Q33]** **[4 marks]**

14. **Two sides of a triangle are 10 and 14, with included angle 55°. Find the third side. [U4-M07-Q34]** **[4 marks]**

15. **A triangle has sides 11, 15, 24. Find the angle opposite side 24. [U4-M07-Q35]** **[4 marks]**

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. Find the area of a triangle with sides 5, 16 and included angle 61°. [U4-M07-Q36] [3 marks]

2. A triangle has area 14.4, sides 6 and 8 enclosing an acute angle. Find that angle. [U4-M07-Q37] [4 marks]

3. Explain the ambiguous sine-rule case for a=7, b=9, A=31°, and determine whether a second angle is possible. [U4-M07-Q38] [5 marks]

4. A quadrilateral is split by a diagonal 10. On one side the adjacent sides are 8,12 with included angle 34°. Describe a complete method for total area. [U4-M07-Q39] [5 marks]

5. Prove the cosine rule reduces to Pythagoras when the included angle is 90° (case 40). [U4-M07-Q40] [4 marks]

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **In triangle ABC, a=10, A=40°, B=65°. Find b using the sine rule. [U4-M07-Q41]** **[4 marks]**

7. **Two sides of a triangle are 11 and 13, with included angle 43°. Find the third side. [U4-M07-Q42]** **[4 marks]**

8. **A triangle has sides 5, 14, 17. Find the angle opposite side 17. [U4-M07-Q43]** **[4 marks]**

9. **Find the area of a triangle with sides 6, 15 and included angle 49°. [U4-M07-Q44]** **[3 marks]**

10. **A triangle has area 33.6, sides 7 and 16 enclosing an acute angle. Find that angle. [U4-M07-Q45]** **[4 marks]**

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Find the reverse bearing of 020°. [U4-M08-Q01] [2 marks]

2. A ship travels 6 km on bearing 027°, then 9 km on bearing 107°. Find its displacement using a triangle method. [U4-M08-Q02] [5 marks]

3. Points A(-1,3) and B(7,-2) are given. Find midpoint, gradient and distance AB. [U4-M08-Q03] [4 marks]

4. Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q04] [5 marks]

5. A bearing diagram gives AB=13, AC=16 and angle BAC=78°. Find BC. [U4-M08-Q05] [4 marks]

6. A point P is equidistant from A(-10,2) and B(10,2). Write its locus equation and explain. [U4-M08-Q06] [3 marks]

7. A line through (11,-11) has gradient 9. Find its equation and both axis intercepts. [U4-M08-Q07] [4 marks]

8. Explain how three-figure bearings, sine/cosine rules and scale drawings can independently check a location problem (case 8). [U4-M08-Q08] **[4 marks]**

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Find the reverse bearing of 076°. [U4-M08-Q09] [2 marks]

2. A ship travels 6 km on bearing 083°, then 9 km on bearing 163°. Find its displacement using a triangle method. [U4-M08-Q10] [5 marks]

3. Points A(0,1) and B(8,-4) are given. Find midpoint, gradient and distance AB. [U4-M08-Q11] [4 marks]

4. Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q12] [5 marks]

5. A bearing diagram gives AB=13, AC=16 and angle BAC=34°. Find BC. [U4-M08-Q13] [4 marks]

6. A point P is equidistant from A(-10,2) and B(10,2). Write its locus equation and explain. [U4-M08-Q14] [3 marks]

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. A line through (11,-11) has gradient 9. Find its equation and both axis intercepts. [U4-M08-Q15] [4 marks]

8. Explain how three-figure bearings, sine/cosine rules and scale drawings can independently check a location problem (case 16). [U4-M08-Q16] [4 marks]

9. Find the reverse bearing of 132°. [U4-M08-Q17] [2 marks]

10. A ship travels 6 km on bearing 139°, then 9 km on bearing 219°. Find its displacement using a triangle method. [U4-M08-Q18] [5 marks]

11. Points A(1,4) and B(9,-1) are given. Find midpoint, gradient and distance AB. [U4-M08-Q19] [4 marks]

12. Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q20] [5 marks]

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. A bearing diagram gives AB=13, AC=16 and angle BAC=50°. Find BC. [U4-M08-Q21] [4 marks]

2. A point P is equidistant from A(-10,2) and B(10,2). Write its locus equation and explain. [U4-M08-Q22] [3 marks]

3. A line through (11,-11) has gradient 9. Find its equation and both axis intercepts. [U4-M08-Q23] [4 marks]

4. Explain how three-figure bearings, sine/cosine rules and scale drawings can independently check a location problem (case 24). [U4-M08-Q24] [4 marks]

5. Find the reverse bearing of 048°. [U4-M08-Q25] [2 marks]

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. **A ship travels 6 km on bearing 055°, then 9 km on bearing 135°. Find its displacement using a triangle method. [U4-M08-Q26]** **[5 marks]**

7. **Points A(2,2) and B(10,-3) are given. Find midpoint, gradient and distance AB. [U4-M08-Q27]** **[4 marks]**

8. **Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q28]** **[5 marks]**

9. **A bearing diagram gives AB=13, AC=16 and angle BAC=106°. Find BC. [U4-M08-Q29]** **[4 marks]**

10. **A point P is equidistant from A(-10,2) and B(10,2). Write its locus equation and explain. [U4-M08-Q30]** **[3 marks]**

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **A line through (11,-11) has gradient 9. Find its equation and both axis intercepts. [U4-M08-Q31]** **[4 marks]**

12. **Explain how three-figure bearings, sine/cosine rules and scale drawings can independently check a location problem (case 32). [U4-M08-Q32]** **[4 marks]**

13. **Find the reverse bearing of 104°. [U4-M08-Q33]** **[2 marks]**

14. **A ship travels 6 km on bearing 111°, then 9 km on bearing 191°. Find its displacement using a triangle method. [U4-M08-Q34]** **[5 marks]**

15. **Points A(3,5) and B(11,0) are given. Find midpoint, gradient and distance AB. [U4-M08-Q35]** **[4 marks]**

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q36] [5 marks]

2. A bearing diagram gives AB=13, AC=16 and angle BAC=62°. Find BC. [U4-M08-Q37] [4 marks]

3. A point P is equidistant from A(-10,2) and B(10,2). Write its locus equation and explain. [U4-M08-Q38] [3 marks]

4. A line through (11,-11) has gradient 9. Find its equation and both axis intercepts. [U4-M08-Q39] [4 marks]

5. Explain how three-figure bearings, sine/cosine rules and scale drawings can independently check a location problem (case 40). [U4-M08-Q40] [4 marks]

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **Find the reverse bearing of 020°. [U4-M08-Q41]** **[2 marks]**

7. **A ship travels 6 km on bearing 027°, then 9 km on bearing 107°. Find its displacement using a triangle method. [U4-M08-Q42]** **[5 marks]**

8. **Points A(-3,3) and B(5,-2) are given. Find midpoint, gradient and distance AB. [U4-M08-Q43]** **[4 marks]**

9. **Find the equation of the perpendicular bisector of A(0,0) and B(16,12). [U4-M08-Q44]** **[5 marks]**

10. **A bearing diagram gives AB=13, AC=16 and angle BAC=78°. Find BC. [U4-M08-Q45]** **[4 marks]**

MODULE 09 / G24,G25

Vectors and Geometric Proof

FLUENCY CHECK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

A short prerequisite check. Work accurately, then move promptly to Higher demand.

1. Given a=(2,3) and b=(-3,2), find 2a-b. [U4-M09-Q01] [2 marks]

2. OA=a and OB=b. Point P divides AB in ratio 3:4. Find OP. [U4-M09-Q02] [4 marks]

3. In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q03] [3 marks]

4. ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 4). [U4-M09-Q04] [4 marks]

5. P lies on OA with OP=6/13a. Q lies on OB with OQ=6/13b. Prove PQ is parallel to AB. [U4-M09-Q05] [4 marks]

6. Given OP=a+7b and OQ=3a+21b, prove O,P,Q are collinear. [U4-M09-Q06] [4 marks]

7. In triangle ABC, P divides AB in ratio 2:1 and Q divides AC in ratio 2:1. Prove PQ∥BC using vectors (case 7). [U4-M09-Q07] [5 marks]

8. A vector proof gives XY=kAB. Explain what can be concluded for k>0, k<0 and |k|=1 (case 8). [U4-M09-Q08] **[4 marks]**

MODULE 09 / G24,G25

Vectors and Geometric Proof

STANDARD HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Higher-tier application. Show the governing relationship and exact working.

1. Given a=(4,6) and b=(-6,4), find 2a-b. [U4-M09-Q09] [2 marks]

2. OA=a and OB=b. Point P divides AB in ratio 5:7. Find OP. [U4-M09-Q10] [4 marks]

3. In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q11] [3 marks]

4. ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 12). [U4-M09-Q12] [4 marks]

5. P lies on OA with OP=2/7a. Q lies on OB with OQ=2/7b. Prove PQ is parallel to AB. [U4-M09-Q13] [4 marks]

6. Given OP=a+3b and OQ=6a+18b, prove O,P,Q are collinear. [U4-M09-Q14] [4 marks]

MODULE 09 / G24,G25

Vectors and Geometric Proof

STANDARD HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the Higher set. Interpret each result in context.

7. **In triangle ABC, P divides AB in ratio 2:1 and Q divides AC in ratio 2:1. Prove PQ∥BC using vectors (case 15). [U4-M09-Q15]** **[5 marks]**

8. **A vector proof gives XY=kAB. Explain what can be concluded for k>0, k<0 and |k|=1 (case 16). [U4-M09-Q16]** **[4 marks]**

9. **Given a=(6,4) and b=(-4,6), find 2a-b. [U4-M09-Q17]** **[2 marks]**

10. **OA=a and OB=b. Point P divides AB in ratio 7:5. Find OP. [U4-M09-Q18]** **[4 marks]**

11. **In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q19]** **[3 marks]**

12. **ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 20). [U4-M09-Q20]** **[4 marks]**

MODULE 09 / G24,G25

Vectors and Geometric Proof

MULTI-STEP HIGHER • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Original multi-step exam-style questions. Show every connected stage.

1. P lies on OA with OP=4/7a. Q lies on OB with OQ=4/7b. Prove PQ is parallel to AB. [U4-M09-Q21] [4 marks]

2. Given OP=a+5b and OQ=4a+20b, prove O,P,Q are collinear. [U4-M09-Q22] [4 marks]

3. In triangle ABC, P divides AB in ratio 2:1 and Q divides AC in ratio 2:1. Prove PQ∥BC using vectors (case 23). [U4-M09-Q23] [5 marks]

4. A vector proof gives XY=kAB. Explain what can be concluded for k>0, k<0 and |k|=1 (case 24). [U4-M09-Q24] [4 marks]

5. Given a=(2,7) and b=(-7,2), find 2a-b. [U4-M09-Q25] [2 marks]

MODULE 09 / G24,G25

Vectors and Geometric Proof

MULTI-STEP HIGHER • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Continue the multi-step set. Explain decisions where requested.

6. OA=a and OB=b. Point P divides AB in ratio 3:3. Find OP. [U4-M09-Q26] [4 marks]

7. In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q27] [3 marks]

8. ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 28). [U4-M09-Q28] [4 marks]

9. P lies on OA with OP=6/12a. Q lies on OB with OQ=6/12b. Prove PQ is parallel to AB. [U4-M09-Q29] [4 marks]

10. Given OP=a+7b and OQ=7a+49b, prove O,P,Q are collinear. [U4-M09-Q30] [4 marks]

MODULE 09 / G24,G25

Vectors and Geometric Proof

MULTI-STEP HIGHER • PART C

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the multi-step set. Check units, accuracy and reasonableness.

11. **In triangle ABC, P divides AB in ratio 2:1 and Q divides AC in ratio 2:1. Prove PQ∥BC using vectors (case 31). [U4-M09-Q31]** **[5 marks]**

12. **A vector proof gives XY=kAB. Explain what can be concluded for k>0, k<0 and |k|=1 (case 32). [U4-M09-Q32]** **[4 marks]**

13. **Given a=(4,5) and b=(-5,4), find 2a-b. [U4-M09-Q33]** **[2 marks]**

14. **OA=a and OB=b. Point P divides AB in ratio 5:6. Find OP. [U4-M09-Q34]** **[4 marks]**

15. **In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q35]** **[3 marks]**

MODULE 09 / G24,G25

Vectors and Geometric Proof

GRADE 7-9 CHALLENGE • PART A

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Extended reasoning. Plan the model before calculating.

1. ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 36). [U4-M09-Q36] [4 marks]

2. P lies on OA with OP=2/6a. Q lies on OB with OQ=2/6b. Prove PQ is parallel to AB. [U4-M09-Q37] [4 marks]

3. Given OP=a+3b and OQ=5a+15b, prove O,P,Q are collinear. [U4-M09-Q38] [4 marks]

4. In triangle ABC, P divides AB in ratio 2:1 and Q divides AC in ratio 2:1. Prove PQ∥BC using vectors (case 39). [U4-M09-Q39] [5 marks]

5. A vector proof gives XY=kAB. Explain what can be concluded for k>0, k<0 and |k|=1 (case 40). [U4-M09-Q40] [4 marks]

MODULE 09 / G24,G25

Vectors and Geometric Proof

GRADE 7-9 CHALLENGE • PART B

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Complete the challenge set. Present a clear mathematical argument.

6. **Given a=(6,3) and b=(-3,6), find 2a-b. [U4-M09-Q41]** **[2 marks]**

7. **OA=a and OB=b. Point P divides AB in ratio 7:4. Find OP. [U4-M09-Q42]** **[4 marks]**

8. **In triangle OAB, M is midpoint AB. Prove OM=½(a+b). [U4-M09-Q43]** **[3 marks]**

9. **ABCD is a parallelogram with AB=a and AD=b. Express AC and prove diagonals bisect each other (variant 44). [U4-M09-Q44]** **[4 marks]**

10. **P lies on OA with OP=4/11a. Q lies on OB with OQ=4/11b. Prove PQ is parallel to AB. [U4-M09-Q45]** **[4 marks]**

FLUENCY AND RECALL • U4-M10-Q01 • 4 marks

1. A sector has radius 6 cm and angle 80°. Find its arc length and area in terms of π.

FLUENCY AND RECALL • U4-M10-Q02 • 4 marks

2. Two sides of a triangle are 7 cm and 10 cm with included angle 50°. Find the third side to 3 significant figures.

FLUENCY AND RECALL • U4-M10-Q03 • 4 marks

3. In triangle ABC, angle A=35°, side a=8 cm and side b=10 cm. Find the acute possible value of angle B to 1 decimal place.

FLUENCY AND RECALL • U4-M10-Q04 • 4 marks

4. Two solids are mathematically similar. Their corresponding length scale factor, small to large, is 1.4. The smaller volume is 250 cm³. Find the larger volume and the area scale factor.

FLUENCY AND RECALL • U4-M10-Q05 • 4 marks

5. OA=a and OB=b. Point P divides AB internally in the ratio AP:PB=2:3. Express OP in terms of a and b.

FLUENCY AND RECALL • U4-M10-Q06 • 5 marks

6. Find the equation of the perpendicular bisector of the segment joining (2,5) and (8,1).

FLUENCY AND RECALL • U4-M10-Q07 • 4 marks

7. A closed cylinder has radius 3 cm and height 8 cm. Find its total surface area in terms of π and as a decimal to 3 significant figures.

FLUENCY AND RECALL • U4-M10-Q08 • 4 marks

8. Point B is 4 km east and 7 km north of A. Find the bearing of B from A and distance AB.

STANDARD HIGHER • U4-M10-Q09 • 4 marks

9. A regular polygon has 8 sides. Find each exterior and interior angle. A diagonal and two adjacent sides form an isosceles triangle: state its base angles.

STANDARD HIGHER • U4-M10-Q10 • 4 marks

10. A sector has radius 7 cm and angle 85°. Find its arc length and area in terms of π.

STANDARD HIGHER • U4-M10-Q11 • 4 marks

11. Two sides of a triangle are 8 cm and 11 cm with included angle 53°. Find the third side to 3 significant figures.

STANDARD HIGHER • U4-M10-Q12 • 4 marks

12. In triangle ABC, angle A=37°, side a=9 cm and side b=11 cm. Find the acute possible value of angle B to 1 decimal place.

STANDARD HIGHER • U4-M10-Q13 • 4 marks

13. Two solids are mathematically similar. Their corresponding length scale factor, small to large, is 1.5. The smaller volume is 300 cm³. Find the larger volume and the area scale factor.

STANDARD HIGHER • U4-M10-Q14 • 4 marks

14. OA=a and OB=b. Point P divides AB internally in the ratio AP:PB=3:3. Express OP in terms of a and b.

STANDARD HIGHER • U4-M10-Q15 • 5 marks

15. Find the equation of the perpendicular bisector of the segment joining (3,6) and (9,0).

STANDARD HIGHER • U4-M10-Q16 • 4 marks

16. A closed cylinder has radius 4 cm and height 9 cm. Find its total surface area in terms of π and as a decimal to 3 significant figures.

STANDARD HIGHER • U4-M10-Q17 • 4 marks

17. Point B is 5 km east and 8 km north of A. Find the bearing of B from A and distance AB.

STANDARD HIGHER • U4-M10-Q18 • 4 marks

18. A regular polygon has 9 sides. Find each exterior and interior angle. A diagonal and two adjacent sides form an isosceles triangle: state its base angles.

STANDARD HIGHER • U4-M10-Q19 • 4 marks

19. A sector has radius 8 cm and angle 90°. Find its arc length and area in terms of π.

STANDARD HIGHER • U4-M10-Q20 • 4 marks

20. Two sides of a triangle are 9 cm and 12 cm with included angle 56°. Find the third side to 3 significant figures.

MULTI-STEP REASONING • U4-M10-Q21 • 4 marks

21. In triangle ABC, angle A=39°, side a=10 cm and side b=12 cm. Find the acute possible value of angle B to 1 decimal place.

MULTI-STEP REASONING • U4-M10-Q22 • 4 marks

22. Two solids are mathematically similar. Their corresponding length scale factor, small to large, is 1.6. The smaller volume is 350 cm³. Find the larger volume and the area scale factor.

MULTI-STEP REASONING • U4-M10-Q23 • 4 marks

23. OA=a and OB=b. Point P divides AB internally in the ratio AP:PB=4:3. Express OP in terms of a and b.

MULTI-STEP REASONING • U4-M10-Q24 • 5 marks

24. Find the equation of the perpendicular bisector of the segment joining (4,7) and (10,-1).

MULTI-STEP REASONING • U4-M10-Q25 • 4 marks

25. A closed cylinder has radius 5 cm and height 10 cm. Find its total surface area in terms of π and as a decimal to 3 significant figures.

MULTI-STEP REASONING • U4-M10-Q26 • 4 marks

26. Point B is 6 km east and 9 km north of A. Find the bearing of B from A and distance AB.

MULTI-STEP REASONING • U4-M10-Q27 • 4 marks

27. A regular polygon has 10 sides. Find each exterior and interior angle. A diagonal and two adjacent sides form an isosceles triangle: state its base angles.

MULTI-STEP REASONING • U4-M10-Q28 • 4 marks

28. A sector has radius 9 cm and angle 95°. Find its arc length and area in terms of π.

MULTI-STEP REASONING • U4-M10-Q29 • 4 marks

29. Two sides of a triangle are 10 cm and 13 cm with included angle 59°. Find the third side to 3 significant figures.

MULTI-STEP REASONING • U4-M10-Q30 • 4 marks

30. In triangle ABC, angle A=41°, side a=11 cm and side b=13 cm. Find the acute possible value of angle B to 1 decimal place.

MULTI-STEP REASONING • U4-M10-Q31 • 4 marks

31. Two solids are mathematically similar. Their corresponding length scale factor, small to large, is 1.7. The smaller volume is 400 cm³. Find the larger volume and the area scale factor.

MULTI-STEP REASONING • U4-M10-Q32 • 4 marks

32. OA=a and OB=b. Point P divides AB internally in the ratio AP:PB=5:3. Express OP in terms of a and b.

MULTI-STEP REASONING • U4-M10-Q33 • 5 marks

33. Find the equation of the perpendicular bisector of the segment joining (5,8) and (11,-2).

MULTI-STEP REASONING • U4-M10-Q34 • 4 marks

34. A closed cylinder has radius 6 cm and height 11 cm. Find its total surface area in terms of π and as a decimal to 3 significant figures.

MULTI-STEP REASONING • U4-M10-Q35 • 4 marks

35. Point B is 7 km east and 10 km north of A. Find the bearing of B from A and distance AB.

GRADE 7–9 CHALLENGE • U4-M10-Q36 • 4 marks

36. A regular polygon has 11 sides. Find each exterior and interior angle. A diagonal and two adjacent sides form an isosceles triangle: state its base angles.

GRADE 7–9 CHALLENGE • U4-M10-Q37 • 4 marks

37. A sector has radius 10 cm and angle 100°. Find its arc length and area in terms of π.

GRADE 7–9 CHALLENGE • U4-M10-Q38 • 4 marks

38. Two sides of a triangle are 11 cm and 14 cm with included angle 62°. Find the third side to 3 significant figures.

GRADE 7–9 CHALLENGE • U4-M10-Q39 • 4 marks

39. In triangle ABC, angle A=43°, side a=12 cm and side b=14 cm. Find the acute possible value of angle B to 1 decimal place.

GRADE 7–9 CHALLENGE • U4-M10-Q40 • 4 marks

40. Two solids are mathematically similar. Their corresponding length scale factor, small to large, is 1.8. The smaller volume is 450 cm³. Find the larger volume and the area scale factor.

GRADE 7–9 CHALLENGE • U4-M10-Q41 • 4 marks

41. OA=a and OB=b. Point P divides AB internally in the ratio AP:PB=6:3. Express OP in terms of a and b.

GRADE 7–9 CHALLENGE • U4-M10-Q42 • 5 marks

42. Find the equation of the perpendicular bisector of the segment joining (6,9) and (12,-3).

GRADE 7–9 CHALLENGE • U4-M10-Q43 • 4 marks

43. A closed cylinder has radius 7 cm and height 12 cm. Find its total surface area in terms of π and as a decimal to 3 significant figures.

GRADE 7–9 CHALLENGE • U4-M10-Q44 • 4 marks

44. Point B is 8 km east and 11 km north of A. Find the bearing of B from A and distance AB.

GRADE 7–9 CHALLENGE • U4-M10-Q45 • 4 marks

45. A regular polygon has 12 sides. Find each exterior and interior angle. A diagonal and two adjacent sides form an isosceles triangle: state its base angles.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Polygon sum	(n-2)×180°.
Regular exterior angle	360°/n.
Congruence	SSS, SAS, ASA and RHS only.
Parallel lines	Use corresponding, alternate and co-interior facts with reasons.
Loci	Translate each condition to a boundary, then intersect regions.
Proof	State equal facts, criterion, then the corresponding conclusion.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Enlargement	$P'=C+k(P-C)$; negative k places image on the opposite ray.
Similarity	Length k, area k^2, volume k^3.
Transformation description	State type and all defining data.
Reflections	Two intersecting reflections produce a rotation through twice the angle.
Invariance	Isometries preserve length, angle and area.
Combined transformations	Track representative points in the stated order.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Centre theorem	Angle at centre = 2× angle at circumference.
Semicircle	Angle subtended by a diameter is 90°.
Same segment	Angles standing on the same chord are equal.
Cyclic quadrilateral	Opposite angles sum to 180°.
Tangent facts	Radius ⊥ tangent; tangents from one external point are equal.
Alternate segment	Tangent-chord angle equals angle in opposite segment.
Chord	Perpendicular from centre bisects a chord.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Circle	$C=2\pi r$; $A=\pi r^2$.
Sector	$\text{Area}=\theta/360\times\pi r^2$.
Arc	$\text{Length}=\theta/360\times 2\pi r$.
Composite area	Split/add/subtract and exclude internal edges.
Similarity	Areas scale with k^2.
Exact form	Retain π and surds unless a decimal is requested.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Prism	V=cross-sectional area×length.
Cylinder	$V=\pi r^2h$; $SA=2\pi r^2+2\pi rh$.
Pyramid/cone	V=⅓×base area×height.
Sphere	$V=4\pi r^3/3$; $SA=4\pi r^2$.
Cone area	Curved area=πrl.
Frustum	large similar solid minus small solid.
Similarity	Volumes scale with k^3.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Pythagoras	$a^2+b^2=c^2$.
SOHCAHTOA	sin=O/H, cos=A/H, tan=O/A.
3D	Identify a right triangle in a plane, often in two stages.
Exact values	Know trig values at 0°,30°,45°,60°,90°.
Elevation/depression	Use horizontal parallel lines and alternate angles.
Accuracy	Degree mode; round only the final answer.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Sine rule	a/sinA=b/sinB=c/sinC.
Cosine rule	$a^2=b^2+c^2-2bc\ \cos A$.
Triangle area	½ab sinC.
Ambiguous case	sin gives acute and obtuse candidates; test angle sum.
Choice	Use cosine rule for SAS/SSS; sine rule for known opposite pairs.
Accuracy	Keep full values until the end.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Bearings	Clockwise from north, three figures.
Reverse	Add/subtract 180°.
Coordinates	Gradient=Δy/Δx; midpoint averages coordinates.
Distance	$\sqrt{((\Delta x)^2+(\Delta y)^2)}$.
Perpendicular	Gradients multiply to -1.
Bearing triangles	Convert bearings to interior angles before applying trig.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

MODULE 09 / G24,G25

Vectors and Geometric Proof

FORMULAS AND METHOD BANK

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use these methods before checking the answer tables. Write the setup first, substitute values, calculate, then give a final answer with units.

When to use it	Formula / method
Position vectors	AB=OB-OA.
Section formula	For AP:PB=m:n, OP=(n a+m b)/(m+n).
Collinearity	Show one vector is a scalar multiple of another.
Parallelism	XY=kAB.
Midpoint	Average the endpoint position vectors.
Proof	Define base vectors, find two routes, compare.

Marking reminder: a correct setup can earn method marks even when a later arithmetic error occurs.

Circle measures

Arc length=$(\theta/360)\times 2\pi r$; sector area=$(\theta/360)\times \pi r^2$.

Pythagoras & trigonometry

$a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse; $\cos\theta$=adjacent/hypotenuse; $\tan\theta$=opposite/adjacent.

Non-right triangles

$a/\sin A=b/\sin B=c/\sin C$; $c^2=a^2+b^2-2ab\cos C$; area=$\frac{1}{2}ab\sin C$.

Similarity

Length factor k; area factor k^2; volume factor k^3.

Coordinate geometry

Gradient=$(y_2-y_1)/(x_2-x_1)$; midpoint=$((x_1+x_2)/2,(y_1+y_2)/2)$; perpendicular gradients multiply to -1.

Vectors

AB=OB−OA; divide a segment by adding the required fraction of its displacement vector.

Mensuration

Cylinder volume=πr^2h; cylinder TSA=$2\pi r^2+2\pi rh$; prism volume=cross-sectional area×length.

Bearings

Measure clockwise from north and write with three figures. Use a scale diagram or trigonometry.

Polygons

Exterior angle sum=360°; interior sum=$(n-2)\times 180°$; regular exterior=$360°/n$.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Exterior=51.4°. Interior=180°-exterior=129° because the two angles form a straight line.	**2**	M1 method; A1 answer.
2	The two right triangles share the perpendicular, have equal hypotenuses (the equal sides) and each has a right angle. They are congruent by RHS, so corresponding angles at the vertex are equal; each is 13°.	**3**	M1 setup; M1 process; A1 conclusion.
3	Use corresponding/alternate angles to transfer the given angles into the triangle. Triangle angles total 180°, so the third angle is 180-40-63=77°.	**3**	M1 setup; M1 process; A1 conclusion.
4	Exterior=180-147.273=32.7273°. Since exterior angles total 360°, n=360/(32.7273)=11.	**3**	M1 setup; M1 process; A1 conclusion.
5	Two corresponding sides and the included angle are equal, so the triangles are congruent by SAS. Therefore corresponding third sides and corresponding angles are equal.	**3**	M1 setup; M1 process; A1 conclusion.
6	The equidistant boundary is the internal angle bisector. “At most 3 cm” is the inside of a circle centre the vertex, radius 3 cm. Required locus is their intersection on the angle bisector.	**3**	M1 setup; M1 process; A1 conclusion.
7	Draw the diagonal joining an endpoint of one given side to the opposite vertex. The two triangles have the given equal sides, the shared diagonal and equal alternate angles from the parallel sides; SAS congruence gives the other opposite sides/angles equal, so the quadrilateral is a parallelogram.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Interior sum=(8-2)×180=1080°. Ratio total=38. One unit=1080/38=28.4°. Multiply this by each ratio part.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	With two sides and a non-included angle, the third vertex may lie at two intersections of an arc and a ray. This can form two different triangles satisfying the same data, so SSA is not a congruence criterion.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Exterior=45°. Interior=180°-exterior=135° because the two angles form a straight line.	**3**	M1 setup; M1 process; A1 conclusion.
3	The two right triangles share the perpendicular, have equal hypotenuses (the equal sides) and each has a right angle. They are congruent by RHS, so corresponding angles at the vertex are equal; each is 22°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Use corresponding/alternate angles to transfer the given angles into the triangle. Triangle angles total 180°, so the third angle is 180-49-65=66°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Exterior=180-144=36°. Since exterior angles total 360°, n=360/(36)=10.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Two corresponding sides and the included angle are equal, so the triangles are congruent by SAS. Therefore corresponding third sides and corresponding angles are equal.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	The equidistant boundary is the internal angle bisector. “At most 7 cm” is the inside of a circle centre the vertex, radius 7 cm. Required locus is their intersection on the angle bisector.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Draw the diagonal joining an endpoint of one given side to the opposite vertex. The two triangles have the given equal sides, the shared diagonal and equal alternate angles from the parallel sides; SAS congruence gives the other opposite sides/angles equal, so the quadrilateral is a parallelogram.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Interior sum=(5-2)×180=540°. Ratio total=20. One unit=540/20=27°. Multiply this by each ratio part.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	With two sides and a non-included angle, the third vertex may lie at two intersections of an arc and a ray. This can form two different triangles satisfying the same data, so SSA is not a congruence criterion.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Exterior=40°. Interior=180°-exterior=140° because the two angles form a straight line.	**3**	M1 setup; M1 process; A1 conclusion.
12	The two right triangles share the perpendicular, have equal hypotenuses (the equal sides) and each has a right angle. They are congruent by RHS, so corresponding angles at the vertex are equal; each is 31°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Use corresponding/alternate angles to transfer the given angles into the triangle. Triangle angles total 180°, so the third angle is 180-58-67=55°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Exterior=180-140=40°. Since exterior angles total 360°, n=360/(40)=9.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Two corresponding sides and the included angle are equal, so the triangles are congruent by SAS. Therefore corresponding third sides and corresponding angles are equal.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	The equidistant boundary is the internal angle bisector. “At most 6 cm” is the inside of a circle centre the vertex, radius 6 cm. Required locus is their intersection on the angle bisector.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Draw the diagonal joining an endpoint of one given side to the opposite vertex. The two triangles have the given equal sides, the shared diagonal and equal alternate angles from the parallel sides; SAS congruence gives the other opposite sides/angles equal, so the quadrilateral is a parallelogram.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Interior sum=(6-2)×180=720°. Ratio total=26. One unit=720/26=27.7°. Multiply this by each ratio part.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	With two sides and a non-included angle, the third vertex may lie at two intersections of an arc and a ray. This can form two different triangles satisfying the same data, so SSA is not a congruence criterion.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Exterior=36°. Interior=180°-exterior=144° because the two angles form a straight line.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	The two right triangles share the perpendicular, have equal hypotenuses (the equal sides) and each has a right angle. They are congruent by RHS, so corresponding angles at the vertex are equal; each is 40°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Use corresponding/alternate angles to transfer the given angles into the triangle. Triangle angles total 180°, so the third angle is 180-67-62=51°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Exterior=180-135=45°. Since exterior angles total 360°, n=360/(45)=8.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Two corresponding sides and the included angle are equal, so the triangles are congruent by SAS. Therefore corresponding third sides and corresponding angles are equal.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	The equidistant boundary is the internal angle bisector. “At most 5 cm” is the inside of a circle centre the vertex, radius 5 cm. Required locus is their intersection on the angle bisector.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	Draw the diagonal joining an endpoint of one given side to the opposite vertex. The two triangles have the given equal sides, the shared diagonal and equal alternate angles from the parallel sides; SAS congruence gives the other opposite sides/angles equal, so the quadrilateral is a parallelogram.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	Interior sum=(7-2)×180=900°. Ratio total=32. One unit=900/32=28.1°. Multiply this by each ratio part.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 01 / G1-G6

Angle Reasoning, Congruence, Constructions and Loci

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	With two sides and a non-included angle, the third vertex may lie at two intersections of an arc and a ray. This can form two different triangles satisfying the same data, so SSA is not a congruence criterion.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Exterior=32.7°. Interior=180°-exterior=147° because the two angles form a straight line.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	The two right triangles share the perpendicular, have equal hypotenuses (the equal sides) and each has a right angle. They are congruent by RHS, so corresponding angles at the vertex are equal; each is 49°.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Use corresponding/alternate angles to transfer the given angles into the triangle. Triangle angles total 180°, so the third angle is 180-76-64=40°.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Exterior=180-158.824=21.1765°. Since exterior angles total 360°, n=360/(21.1765)=17.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Two corresponding sides and the included angle are equal, so the triangles are congruent by SAS. Therefore corresponding third sides and corresponding angles are equal.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	The equidistant boundary is the internal angle bisector. “At most 4 cm” is the inside of a circle centre the vertex, radius 4 cm. Required locus is their intersection on the angle bisector.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Draw the diagonal joining an endpoint of one given side to the opposite vertex. The two triangles have the given equal sides, the shared diagonal and equal alternate angles from the parallel sides; SAS congruence gives the other opposite sides/angles equal, so the quadrilateral is a parallelogram.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Interior sum=(8-2)×180=1080°. Ratio total=38. One unit=1080/38=28.4°. Multiply this by each ratio part.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	With two sides and a non-included angle, the third vertex	**4**	M1 setup; M1 development; A1

	may lie at two intersections of an arc and a ray. This can form two different triangles satisfying the same data, so SSA is not a congruence criterion.		accuracy; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	CP=(1,-1); multiply by -3 and add C: P′=(-2,1).	**2**	M1 method; A1 answer.
2	Area scale factor=1.5^2=2.25. Larger area=49×2.25=110.25 cm^2.	**3**	M1 setup; M1 process; A1 conclusion.
3	Length scale factor=$\sqrt[3]{(64)}$=4. Larger volume=50×64=3200 cm^3.	**3**	M1 setup; M1 process; A1 conclusion.
4	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.
5	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Length factor=(13)/10. Area factor=$[(13)/10]^2$. Larger area=40×$[(13)/10]^2$=67.6 cm^2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	A rotation multiplies each centre-to-point vector by a rotation matrix/turn of angle θ. The inverse turns by -θ, so the net turn is 0° and the vector is unchanged. Hence every image returns to its original point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Magnitude of overall scale factor=(3/2)(2/3)=1, so size is unchanged. The negative factor reverses each centre-to-point direction through the enlargement centre; because centres differ, the complete mapping must be determined point by point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	CP=(4,-1); multiply by -3 and add C: P′=(-11,1).	**3**	M1 setup; M1 process; A1 conclusion.
2	Area scale factor=1.5^2=2.25. Larger area=64×2.25=144 cm^2.	**3**	M1 setup; M1 process; A1 conclusion.
3	Length scale factor=$\sqrt[3]{(27)}$=3. Larger volume=90×27=2430 cm^3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.
5	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Length factor=(9)/6. Area factor=$[(9)/6]^2$. Larger area=56×$[(9)/6]^2$=126 cm^2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	A rotation multiplies each centre-to-point vector by a rotation matrix/turn of angle θ. The inverse turns by -θ, so the net turn is 0° and the vector is unchanged. Hence every image returns to its original point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Magnitude of overall scale factor=(3/2)(2/3)=1, so size is unchanged. The negative factor reverses each centre-to-point direction through the enlargement centre; because centres differ, the complete mapping must be determined point by point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	CP=(2,-1); multiply by -3 and add C: P′=(-5,1).	**3**	M1 setup; M1 process; A1 conclusion.
10	Area scale factor=1.5^2=2.25. Larger area=81×2.25=182.25 cm^2.	**3**	M1 setup; M1 process; A1 conclusion.
11	Length scale factor=$\sqrt[3]{(8)}$=2. Larger volume=130×8=1040 cm^3.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Length factor=(11)/8. Area factor=$[(11)/8]^2$. Larger area=$72\times[(11)/8]^2=136$ cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	A rotation multiplies each centre-to-point vector by a rotation matrix/turn of angle θ. The inverse turns by -θ, so the net turn is 0° and the vector is unchanged. Hence every image returns to its original point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Magnitude of overall scale factor=(3/2)(2/3)=1, so size is unchanged. The negative factor reverses each centre-to-point direction through the enlargement centre; because centres differ, the complete mapping must be determined point by point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	CP=(5,-1); multiply by -3 and add C: P′=(-14,1).	**3**	M1 setup; M1 process; A1 conclusion.
6	Area scale factor=$1.5^2=2.25$. Larger area=$100\times2.25=225$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
7	Length scale factor=$\sqrt[3]{(64)}=4$. Larger volume=$170\times64=10880$ cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Length factor=(13)/10. Area factor=[(13)/10]². Larger area=88×[(13)/10]²=149 cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	A rotation multiplies each centre-to-point vector by a rotation matrix/turn of angle θ. The inverse turns by -θ, so the net turn is 0° and the vector is unchanged. Hence every image returns to its original point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Magnitude of overall scale factor=(3/2)(2/3)=1, so size is unchanged. The negative factor reverses each centre-to-point direction through the enlargement centre; because centres differ, the complete mapping must be determined point by point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	CP=(3,-1); multiply by -3 and add C: P′=(-8,1).	**3**	M1 setup; M1 process; A1 conclusion.
14	Area scale factor=1.5²=2.25. Larger area=121×2.25=272.25 cm².	**3**	M1 setup; M1 process; A1 conclusion.
15	Length scale factor=$\sqrt[3]{(27)}$=3. Larger volume=210×27=5670 cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 02 / G7,G8,G19

Transformations, Similarity and Scale Factors

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.
2	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Length factor=(9)/6. Area factor=$[(9)/6]^2$. Larger area=$104\times[(9)/6]^2=234$ cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	A rotation multiplies each centre-to-point vector by a rotation matrix/turn of angle θ. The inverse turns by -θ, so the net turn is 0° and the vector is unchanged. Hence every image returns to its original point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Magnitude of overall scale factor=(3/2)(2/3)=1, so size is unchanged. The negative factor reverses each centre-to-point direction through the enlargement centre; because centres differ, the complete mapping must be determined point by point.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	CP=(1,-1); multiply by -3 and add C: P′=(-2,1).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Area scale factor=$1.5^2=2.25$. Larger area=$144\times2.25=324$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
8	Length scale factor=$\sqrt[3]{(8)}=2$. Larger volume=$250\times8=2000$ cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	A rotation of 180° about the origin. Two reflections in intersecting lines give a rotation through twice the angle between the lines.	**3**	M1 setup; M1 process; A1 conclusion.
10	Lengths, angles, area and orientation are preserved. A rotation followed by a translation is an orientation-	**5**	M1 setup; M2–M4 development; A1

	preserving isometry and, unless the net effect is a translation, can be represented as a rotation about a different centre; locate it by perpendicular bisectors of point-image segments.		conclusion.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Angle at centre is twice angle at circumference, so ACB=24°.	**2**	M1 method; A1 answer.
2	Angle ACB=90° (angle in a semicircle). ABC=180-90-43=47°.	**3**	M1 setup; M1 process; A1 conclusion.
3	Opposite angles in a cyclic quadrilateral sum to 180°, so ADC=88°.	**3**	M1 setup; M1 process; A1 conclusion.
4	By the alternate segment theorem, the angle subtended by chord AB at the opposite circumference is 38°.	**3**	M1 setup; M1 process; A1 conclusion.
5	PB=11 cm. OA=OB (radii), OP is common and both radii are perpendicular to tangents, so triangles OAP and OBP are congruent by RHS; hence PA=PB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	The perpendicular from the centre bisects the chord. $r^2=4^2+4^2=32$; $r=\sqrt{32}$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Angles in the same segment are equal: x+0=3x-19; 2x=19; x=9.5.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	OA and OB are perpendicular to the tangents, so angles OAP and OBP are 90°. Angles in quadrilateral OAPB total 360°, hence APB+ AOB+180°=360° and APB=180°-AOB.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Mark the 90° angle in the semicircle and the radius-tangent 90°. Use alternate segment or same-segment angles to transfer a chord angle, then use opposite cyclic angles sum 180° and triangle angle sums. Give a theorem at every step.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Angle at centre is twice angle at circumference, so ACB=33°.	**2**	M1 method; A1 answer.
3	Angle ACB=90° (angle in a semicircle). ABC=180-90-52=38°.	**3**	M1 setup; M1 process; A1 conclusion.
4	Opposite angles in a cyclic quadrilateral sum to 180°, so ADC=79°.	**3**	M1 setup; M1 process; A1 conclusion.
5	By the alternate segment theorem, the angle subtended by chord AB at the opposite circumference is 47°.	**3**	M1 setup; M1 process; A1 conclusion.
6	PB=12 cm. OA=OB (radii), OP is common and both radii are perpendicular to tangents, so triangles OAP and OBP are congruent by RHS; hence PA=PB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	The perpendicular from the centre bisects the chord. $r^2=8^2+5^2=89$; $r=\sqrt{89}$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Angles in the same segment are equal: $x+3=3x-18$; $2x=21$; $x=10.5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	OA and OB are perpendicular to the tangents, so angles OAP and OBP are 90°. Angles in quadrilateral OAPB total 360°, hence APB+ AOB+180°=360° and APB=180°-AOB.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Mark the 90° angle in the semicircle and the radius-tangent 90°. Use alternate segment or same-segment angles to transfer a chord angle, then use opposite cyclic angles sum 180° and triangle angle sums. Give a theorem at every step.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Angle at centre is twice angle at circumference, so ACB=42°.	**2**	M1 method; A1 answer.
12	Angle ACB=90° (angle in a semicircle). ABC=180-90-61=29°.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Opposite angles in a cyclic quadrilateral sum to 180°, so ADC=70°.	**3**	M1 setup; M1 process; A1 conclusion.
2	By the alternate segment theorem, the angle subtended by chord AB at the opposite circumference is 56°.	**3**	M1 setup; M1 process; A1 conclusion.
3	PB=13 cm. OA=OB (radii), OP is common and both radii are perpendicular to tangents, so triangles OAP and OBP are congruent by RHS; hence PA=PB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	The perpendicular from the centre bisects the chord. $r^2=7^2+6^2=85$; $r=\sqrt{85}$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Angles in the same segment are equal: x+0=3x-22; 2x=22; x=11.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	OA and OB are perpendicular to the tangents, so angles OAP and OBP are 90°. Angles in quadrilateral OAPB total 360°, hence APB+ AOB+180°=360° and APB=180°-AOB.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Mark the 90° angle in the semicircle and the radius-tangent 90°. Use alternate segment or same-segment angles to transfer a chord angle, then use opposite cyclic angles sum 180° and triangle angle sums. Give a theorem at every step.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Angle at centre is twice angle at circumference, so ACB=51°.	**2**	M1 method; A1 answer.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Angle ACB=90° (angle in a semicircle). ABC=180-90-70=20°.	**3**	M1 setup; M1 process; A1 conclusion.
10	Opposite angles in a cyclic quadrilateral sum to 180°, so ADC=61°.	**3**	M1 setup; M1 process; A1 conclusion.
11	By the alternate segment theorem, the angle subtended by chord AB at the opposite circumference is 35°.	**3**	M1 setup; M1 process; A1 conclusion.
12	PB=14 cm. OA=OB (radii), OP is common and both radii are perpendicular to tangents, so triangles OAP and OBP are congruent by RHS; hence PA=PB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	The perpendicular from the centre bisects the chord. $r^2=6^2+3^2=45$; $r=\sqrt{45}$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	Angles in the same segment are equal: x+3=3x-21; 2x=24; x=12.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	OA and OB are perpendicular to the tangents, so angles OAP and OBP are 90°. Angles in quadrilateral OAPB total 360°, hence APB+ AOB+180°=360° and APB=180°-AOB.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 03 / G9,G10

Circle Theorems and Geometric Proof

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Mark the 90° angle in the semicircle and the radius-tangent 90°. Use alternate segment or same-segment angles to transfer a chord angle, then use opposite cyclic angles sum 180° and triangle angle sums. Give a theorem at every step.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Angle at centre is twice angle at circumference, so ACB=60°.	**2**	M1 method; A1 answer.
3	Angle ACB=90° (angle in a semicircle). ABC=180-90-24=66°.	**3**	M1 setup; M1 process; A1 conclusion.
4	Opposite angles in a cyclic quadrilateral sum to 180°, so ADC=52°.	**3**	M1 setup; M1 process; A1 conclusion.
5	By the alternate segment theorem, the angle subtended by chord AB at the opposite circumference is 44°.	**3**	M1 setup; M1 process; A1 conclusion.
6	PB=7 cm. OA=OB (radii), OP is common and both radii are perpendicular to tangents, so triangles OAP and OBP are congruent by RHS; hence PA=PB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	The perpendicular from the centre bisects the chord. $r^2=5^2+4^2=41$; $r=\sqrt{41}$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Angles in the same segment are equal: x+0=3x-20; 2x=20; x=10.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	OA and OB are perpendicular to the tangents, so angles OAP and OBP are 90°. Angles in quadrilateral OAPB total 360°, hence APB+ AOB+180°=360° and APB=180°-AOB.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Mark the 90° angle in the semicircle and the radius-tangent 90°. Use alternate segment or same-segment angles to transfer a chord angle, then use opposite cyclic angles sum 180° and triangle angle sums. Give a theorem at every step.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$A=9\pi$ cm²; $C=6\pi$ cm.	**2**	M1 method; A1 answer.
2	Area$=45/360\times\pi\times4^2=2\pi$ cm². Arc$=45/360\times2\pi\times4=1\pi$ cm.	**3**	M1 setup; M1 process; A1 conclusion.
3	Area$=90+\frac{1}{2}\pi(5)^2=90+12.5\pi$. Perimeter$=28+\pi r=28+5\pi$, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Arc length=perimeter-2r$=2.5\pi$. Set $(\theta/360)\times2\pi\times6=2.5\pi$, giving $\theta=75°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Radius$=\sqrt{(245)}=7\sqrt{5}$ cm. Circumference$=2\pi r=14\sqrt{5}\pi$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Area factor$=[(11)/8]^2$. Larger area$=105\pi\times[(11)/8]^2=199\pi$ cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Central angle$=360/11°$. Total area$=\frac{1}{2}\times11\times9^2\times\sin(360/11°)$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Area$=\pi[(12)^2-10^2]=44\pi$ m². Cost$=£11\times44\pi=£484\pi$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$A=9\pi$ cm²; $C=6\pi$ cm.	**2**	M1 method; A1 answer.
2	Area=$30/360\times\pi\times4^2=1.33333\pi$ cm². Arc=$30/360\times2\pi\times4=0.666667\pi$ cm.	**3**	M1 setup; M1 process; A1 conclusion.
3	Area=$90+\frac{1}{2}\pi(5)^2=90+12.5\pi$. Perimeter=$28+\pi r=28+5\pi$, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Arc length=perimeter-2r=2π. Set $(\theta/360)\times2\pi\times6=2\pi$, giving $\theta=60°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Radius=$\sqrt{(245)}=7\sqrt{5}$ cm. Circumference=$2\pi r=14\sqrt{5}\pi$ cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	Area factor=$[(11)/8]^2$. Larger area=$90\pi\times[(11)/8]^2=170\pi$ cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Central angle=360/5°. Total area=½×5×9²×sin(360/5°).	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Area=π[(12)²-10²]=44π m². Cost=£13×44π=£572π.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	A=9π cm²; C=6π cm.	**2**	M1 method; A1 answer.
10	Area=150/360×π×4²=6.66667π cm². Arc=150/360×2π×4=3.33333π cm.	**3**	M1 setup; M1 process; A1 conclusion.
11	Area=90+½π(5)²=90+12.5π. Perimeter=28+πr=28+5π, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Arc length=perimeter-2r=1.5π. Set (θ/360)×2π×6=1.5π, giving θ=45°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Radius=√(245)=7√5 cm. Circumference=2πr=14√5π cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Area factor=[(11)/8]². Larger area=75π×[(11)/8]²=142π cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Central angle=360/6°. Total area=½×6×9²×sin(360/6°).	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Area=π[(12)²-10²]=44π m². Cost=£15×44π=£660π.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	A=9π cm²; C=6π cm.	**2**	M1 method; A1 answer.
6	Area=135/360×π×4²=6π cm². Arc=135/360×2π×4=3π cm.	**3**	M1 setup; M1 process; A1 conclusion.
7	Area=90+½π(5)²=90+12.5π. Perimeter=28+πr=28+5π, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Arc length=perimeter-2r=1π. Set (θ/360)×2π×6=1π, giving θ=30°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Radius=√(245)=7√5 cm. Circumference=2πr=14√5π cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Area factor=[(11)/8]². Larger area=60π×[(11)/8]²=113π cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Central angle=360/7°. Total area=½×7×9²×sin(360/7°).	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	Area=π[(12)²-10²]=44π m². Cost=£11×44π=£484π.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	A=9π cm²; C=6π cm.	**2**	M1 method; A1 answer.
14	Area=120/360×π×4²=5.33333π cm². Arc=120/360×2π×4=2.66667π cm.	**3**	M1 setup; M1 process; A1 conclusion.
15	Area=90+½π(5)²=90+12.5π. Perimeter=28+πr=28+5π, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 04 / G14,G16-G18

Mensuration, Circles, Sectors and Composite Areas

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Arc length=perimeter-2r=5π. Set (θ/360)×2π×6=5π, giving θ=150°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	Radius=√(245)=7√5 cm. Circumference=2πr=14√5π cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Area factor=[(11)/8]². Larger area=45π×[(11)/8]²=85.1π cm².	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Central angle=360/8°. Total area=½×8×9²×sin(360/8°).	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Area=π[(12)²-10²]=44π m². Cost=£13×44π=£572π.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	A=9π cm²; C=6π cm.	**2**	M1 method; A1 answer.
7	Area=105/360×π×4²=4.66667π cm². Arc=105/360×2π×4=2.33333π cm.	**3**	M1 setup; M1 process; A1 conclusion.
8	Area=90+½π(5)²=90+12.5π. Perimeter=28+πr=28+5π, excluding the joined diameter.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Arc length=perimeter-2r=4.5π. Set (θ/360)×2π×6=4.5π, giving θ=135°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	Radius=√(245)=7√5 cm. Circumference=2πr=14√5π cm.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$V=\pi r^2h=45\pi$ cm³. $SA=2\pi r^2+2\pi rh=48\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
2	$V=4/3\pi r^3=(256/3)\pi$ cm³. $SA=4\pi r^2=64\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
3	$V=\frac{1}{3}\pi r^2h=(175/3)\pi$. Total area $=\pi r^2+\pi rl=25\pi+5\pi\sqrt{74}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$V=\frac{1}{3}\times$base area$\times$height$=\frac{1}{3}\times64\times8=512/3$ cm³.	**3**	M1 setup; M1 process; A1 conclusion.
5	Removed volume factor=1/27. Frustum volume=(168)π[1-1/27]=162π cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Volume $=\pi r^2h+\frac{2}{3}\pi r^3=640\pi+(1024/3)\pi=(2944/3)\pi$ cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Length ratio=9:11; volume ratio=729:1331. If smaller volume V, V[((r+2)/r)^3-1]=130, so V=157 cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Ratio of volumes $=[(4/3)\pi10^3]/[(1/3)\pi(5)^2(5)]=32$. Take the whole-number part if only complete cones are allowed.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$V=\pi r^2h=54\pi$ cm³. $SA=2\pi r^2+2\pi rh=54\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
2	$V=4/3\pi r^3=(256/3)\pi$ cm³. $SA=4\pi r^2=64\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
3	$V=\frac{1}{3}\pi r^2h=(200/3)\pi$. Total area $=\pi r^2+\pi rl=25\pi+5\pi\sqrt{89}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$V=\frac{1}{3}\times$base area$\times$height$=\frac{1}{3}\times64\times9=576/3$ cm³.	**3**	M1 setup; M1 process; A1 conclusion.
5	Removed volume factor=1/8. Frustum volume$=(264)\pi[1-1/8]=231\pi$ cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	Volume$=\pi r^2h+\frac{2}{3}\pi r^3=704\pi+(1024/3)\pi=(3136/3)\pi$ cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Length ratio=9:11; volume ratio=729:1331. If smaller volume V, V[((r+2)/r)^3-1]=170, so V=206 cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Ratio of volumes=[(4/3)π10³]/[(1/3)π(5)²(6)]=26.7. Take the whole-number part if only complete cones are allowed.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	V=πr²h=63π cm³. SA=2πr²+2πrh=60π cm².	**3**	M1 setup; M1 process; A1 conclusion.
10	V=4/3πr³=(256/3)π cm³. SA=4πr²=64π cm².	**3**	M1 setup; M1 process; A1 conclusion.
11	V=⅓πr²h=(225/3)π. Total area=πr²+πrl=25π+5π√106.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	V=⅓×base area×height=⅓×64×10=640/3 cm³.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Removed volume factor=1/64. Frustum volume=(360)π[1-1/64]=354π cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Volume=πr²h+⅔πr³=320π+(1024/3)π=(1984/3)π cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Length ratio=9:11; volume ratio=729:1331. If smaller volume V, V[((r+2)/r)^3-1]=210, so V=254 cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Ratio of volumes=[(4/3)π10³]/[(1/3)π(5)²(7)]=22.9. Take the whole-number part if only complete cones are allowed.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	V=πr²h=72π cm³. SA=2πr²+2πrh=66π cm².	**3**	M1 setup; M1 process; A1 conclusion.
6	V=4/3πr³=(256/3)π cm³. SA=4πr²=64π cm².	**3**	M1 setup; M1 process; A1 conclusion.
7	V=⅓πr²h=(250/3)π. Total area=πr²+πrl=25π+5π√125.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	V=⅓×base area×height=⅓×64×11=704/3 cm³.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Removed volume factor=1/27. Frustum volume=(456)π[1-1/27]=439π cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Volume=πr²h+⅔πr³=384π+(1024/3)π=(2176/3)π cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Length ratio=9:11; volume ratio=729:1331. If smaller volume V, V[((r+2)/r)^3-1]=250, so V=303 cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	Ratio of volumes=[(4/3)π10³]/[(1/3)π(5)²(8)]=20. Take the whole-number part if only complete cones are allowed.	**5**	M1 setup; M2–M4 development; A1 conclusion.
13	V=πr²h=81π cm³. SA=2πr²+2πrh=72π cm².	**3**	M1 setup; M1 process; A1 conclusion.
14	V=4/3πr³=(256/3)π cm³. SA=4πr²=64π cm².	**3**	M1 setup; M1 process; A1 conclusion.
15	V=⅓πr²h=(275/3)π. Total area=πr²+πrl=25π+5π√146.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 05 / G12,G13,G16,G17,G19

3D Geometry, Surface Area, Volume and Frustums

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$V=\frac{1}{3}\times$base area$\times$height$=\frac{1}{3}\times64\times5=320/3$ cm³.	**3**	M1 setup; M1 process; A1 conclusion.
2	Removed volume factor=1/8. Frustum volume=$(552)\pi[1-1/8]=483\pi$ cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Volume$=\pi r^2h+\frac{2}{3}\pi r^3=448\pi+(1024/3)\pi=(2368/3)\pi$ cm³.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Length ratio=9:11; volume ratio=729:1331. If smaller volume V, V[((r+2)/r)^3-1]=290, so V=351 cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Ratio of volumes$=[(4/3)\pi10^3]/[(1/3)\pi(5)^2(9)]=17.8$. Take the whole-number part if only complete cones are allowed.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$V=\pi r^2h=90\pi$ cm³. $SA=2\pi r^2+2\pi rh=78\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
7	$V=4/3\pi r^3=(256/3)\pi$ cm³. $SA=4\pi r^2=64\pi$ cm².	**3**	M1 setup; M1 process; A1 conclusion.
8	$V=\frac{1}{3}\pi r^2h=(125/3)\pi$. Total area$=\pi r^2+\pi rl=25\pi+5\pi\sqrt{50}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$V=\frac{1}{3}\times$base area$\times$height$=\frac{1}{3}\times64\times6=384/3$ cm³.	**3**	M1 setup; M1 process; A1 conclusion.
10	Removed volume factor=1/64. Frustum volume$=(648)\pi[1-1/64]=638\pi$ cm³.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$c=\sqrt{(5^2+8^2)}=\sqrt{89}=9.43$.	**3**	M1 setup; M1 process; A1 conclusion.
2	Length$=\sqrt{((15)^2-6^2)}=\sqrt{189}=13.7$.	**3**	M1 setup; M1 process; A1 conclusion.
3	Horizontal$=(14)\cos34°=11.6$; vertical$=(14)\sin34°=7.83$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$d=\sqrt{(8^2+11^2+11^2)}=\sqrt{306}=17.5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$\sin\theta=9/17$; $\theta=\sin^{-1}(9/17)=32.0°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$\sin30°=\cos60°=1/2$, so value$=10+13/2=16.5$.	**3**	M1 setup; M1 process; A1 conclusion.
7	Centre-to-vertex horizontal distance$=11\sqrt{2}$. $\tan\theta=14/(11\sqrt{2})$, so $\theta=42.0°$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$(24)^2+(10)^2=576+100=676=(26)^2$. By converse Pythagoras, it is right-angled.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$c=\sqrt{(6^2+16^2)}=\sqrt{292}=17.1$.	**3**	M1 setup; M1 process; A1 conclusion.
2	Length$=\sqrt{((14)^2-7^2)}=\sqrt{147}=12.1$.	**3**	M1 setup; M1 process; A1 conclusion.
3	Horizontal$=(13)\cos 58°=6.89$; vertical$=(13)\sin 58°=11$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	$d=\sqrt{(9^2+10^2+12^2)}=\sqrt{325}=18$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	$\sin\theta=10/16$; $\theta=\sin^{-1}(10/16)=38.7°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$\sin 30°=\cos 60°=1/2$, so value$=11+12/2=17$.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Centre-to-vertex horizontal distance=$5\sqrt{2}$. $\tan\theta=13/(5\sqrt{2})$, so $\theta=61.5°$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$(35)^2+(12)^2=1225+144=1369=(37)^2$. By converse Pythagoras, it is right-angled.	**5**	M1 setup; M2–M4 development; A1 conclusion.
9	$c=\sqrt{(7^2+15^2)}=\sqrt{274}=16.6$.	**3**	M1 setup; M1 process; A1 conclusion.
10	Length$=\sqrt{((22)^2-8^2)}=\sqrt{420}=20.5$.	**3**	M1 setup; M1 process; A1 conclusion.
11	Horizontal$=(12)\cos46°=8.34$; vertical$=(12)\sin46°=8.63$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	$d=\sqrt{(10^2+9^2+13^2)}=\sqrt{350}=18.7$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$\sin\theta=11/15$; $\theta=\sin^{-1}(11/15)=47.2°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$\sin30°=\cos60°=1/2$, so value=5+11/2=10.5.	**3**	M1 setup; M1 process; A1 conclusion.
3	Centre-to-vertex horizontal distance=$6\sqrt{2}$. $\tan\theta=12/(6\sqrt{2})$, so $\theta=54.7°$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	$(48)^2+(14)^2=2304+196=2500=(50)^2$. By converse Pythagoras, it is right-angled.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	$c=\sqrt{(8^2+14^2)}=\sqrt{260}=16.1$.	**3**	M1 setup; M1 process; A1 conclusion.
6	Length=$\sqrt{((21)^2-9^2)}=\sqrt{360}=19$.	**3**	M1 setup; M1 process; A1 conclusion.
7	Horizontal=$(20)\cos34°=16.6$; vertical=$(20)\sin34°=11.2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	$d=\sqrt{(11^2+8^2+14^2)}=\sqrt{381}=19.5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	$\sin\theta=5/14$; $\theta=\sin^{-1}(5/14)=20.9°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$\sin30°=\cos60°=1/2$, so value=6+10/2=11.	**3**	M1 setup; M1 process; A1 conclusion.
11	Centre-to-vertex horizontal distance=$7\sqrt{2}$. $\tan\theta=11/(7\sqrt{2})$, so $\theta=48.0°$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	$(63)^2+(16)^2=3969+256=4225=(65)^2$. By converse Pythagoras, it is right-angled.	**5**	M1 setup; M2–M4 development; A1 conclusion.
13	$c=\sqrt{(9^2+13^2)}=\sqrt{250}=15.8$.	**3**	M1 setup; M1 process; A1 conclusion.
14	Length=$\sqrt{((20)^2-10^2)}=\sqrt{300}=17.3$.	**3**	M1 setup; M1 process; A1 conclusion.
15	Horizontal=$(19)\cos58°=10.1$; vertical=$(19)\sin58°=16.1$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 06 / G20,G21

Pythagoras, Right-angle Trigonometry and 3D Problems

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$d=\sqrt{(5^2+16^2+8^2)}=\sqrt{345}=18.6$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$\sin\theta=6/13$; $\theta=\sin^{-1}(6/13)=27.5°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$\sin30°=\cos60°=1/2$, so value=7+9/2=11.5.	**3**	M1 setup; M1 process; A1 conclusion.
4	Centre-to-vertex horizontal distance=$8\sqrt{2}$. $\tan\theta=10/(8\sqrt{2})$, so $\theta=41.5°$.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	$(80)^2+(18)^2=6400+324=6724=(82)^2$. By converse Pythagoras, it is right-angled.	**5**	M1 setup; M2–M4 development; A1 conclusion.
6	$c=\sqrt{(10^2+12^2)}=\sqrt{244}=15.6$.	**3**	M1 setup; M1 process; A1 conclusion.
7	Length=$\sqrt{((19)^2-11^2)}=\sqrt{240}=15.5$.	**3**	M1 setup; M1 process; A1 conclusion.
8	Horizontal=$(18)\cos46°=12.5$; vertical=$(18)\sin46°=12.9$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$d=\sqrt{(6^2+15^2+9^2)}=\sqrt{342}=18.5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$\sin\theta=7/21$; $\theta=\sin^{-1}(7/21)=19.5°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	$b/\sin(53°)=5/\sin(28°)$; $b=5\sin(53°)/\sin(28°)=8.51$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$c^2=6^2+9^2-2(6)(9)\cos31°$; $c=4.94$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$\cos C=[7^2+10^2-(15)^2]/[2(7)(10)]$; $C=122.9°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Area$=\frac{1}{2}ab\sin C=\frac{1}{2}(8)(11)\sin37°=26.5$.	**3**	M1 setup; M1 process; A1 conclusion.
5	$\sin C=2\text{Area}/(ab)=0.6$. Acute $C=\sin^{-1}(0.6)=36.9°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$\sin B=b\sin A/a=0.887$. If this is ≤1, B may be $\sin^{-1}$(value) or $180°-\sin^{-1}$(value); retain only cases with $A+B<180°$. This test determines whether there are 0, 1 or 2 triangles.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Use cosine rule to find the diagonal if not already known, then find each triangle area using $\frac{1}{2}ab\sin C$ (or sine rule to obtain missing angles/sides). Add the two triangle areas, keeping unrounded values.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$c^2=a^2+b^2-2ab\cos90°$. Since $\cos90°=0$, $c^2=a^2+b^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	b/sin(77°)=6/sin(52°); b=6sin(77°)/sin(52°)=7.42.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	$c^2=7^2+8^2-2(7)(8)\cos 55°$; c=6.98.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	$\cos C=[8^2+9^2-(15)^2]/[2(8)(9)]$; C=123.7°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Area=½ab sin C=½(9)(10)sin61°=39.4.	**3**	M1 setup; M1 process; A1 conclusion.
5	sin C=2Area/(ab)=0.6. Acute $C=\sin^{-1}(0.6)=36.9°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	sin B=b sin A/a=0.562. If this is ≤1, B may be $\sin^{-1}$(value) or 180°-$\sin^{-1}$(value); retain only cases with A+B<180°. This test determines whether there are 0, 1 or 2 triangles.	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Use cosine rule to find the diagonal if not already known, then find each triangle area using ½ab sin C (or sine rule to obtain missing angles/sides). Add the two triangle areas, keeping unrounded values.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	$c^2=a^2+b^2-2ab\cos90°$. Since $\cos90°=0$, $c^2=a^2+b^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	$b/\sin(65°)=7/\sin(40°)$; $b=7\sin(65°)/\sin(40°)=9.87$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	$c^2=8^2+16^2-2(8)(16)\cos43°$; $c=11.5$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	$\cos C=[9^2+8^2-(15)^2]/[2(9)(8)]$; $C=123.7°$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Area=½ab sin C=½(10)(9)sin49°=34.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	sin C=2Area/(ab)=0.6. Acute C=$\sin^{-1}(0.6)$=36.9°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	sin B=b sin A/a=1.8. If this is ≤1, B may be $\sin^{-1}$(value) or 180°-$\sin^{-1}$(value); retain only cases with A+B<180°. This test determines whether there are 0, 1 or 2 triangles.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Use cosine rule to find the diagonal if not already known, then find each triangle area using ½ab sin C (or sine rule to obtain missing angles/sides). Add the two triangle areas, keeping unrounded values.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	$c^2=a^2+b^2-2ab\cos90°$. Since cos90°=0, $c^2=a^2+b^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	b/sin(53°)=8/sin(28°); b=8sin(53°)/sin(28°)=13.6.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	$c^2=9^2+15^2-2(9)(15)\cos31°$; c=8.64.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	cos C=[$10^2+16^2-(24)^2$]/[2(10)(16)]; C=133.4°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Area=½ab sin C=½(11)(8)sin37°=26.5.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	sin C=2Area/(ab)=0.6. Acute C=$\sin^{-1}(0.6)$=36.9°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	sin B=b sin A/a=1.14. If this is ≤1, B may be $\sin^{-1}$(value) or 180°-$\sin^{-1}$(value); retain only cases with A+B<180°. This test determines whether there are 0, 1 or 2 triangles.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Use cosine rule to find the diagonal if not already known, then find each triangle area using ½ab sin C (or sine rule to obtain missing angles/sides). Add the two triangle areas, keeping unrounded values.	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	$c^2=a^2+b^2-2ab\cos 90°$. Since cos90°=0, $c^2=a^2+b^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	b/sin(77°)=9/sin(52°); b=9sin(77°)/sin(52°)=11.1.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
14	$c^2=10^2+14^2-2(10)(14)\cos 55°$; c=11.6.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	cos C=$[11^2+15^2-(24)^2]/[2(11)(15)]$; C=134.2°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 07 / G22,G23

Non-right-angled Trigonometry and Triangle Area

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Area=½ab sin C=½(5)(16)sin61°=35.	**3**	M1 setup; M1 process; A1 conclusion.
2	sin C=2Area/(ab)=0.6. Acute C=$\sin^{-1}(0.6)$=36.9°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	sin B=b sin A/a=0.662. If this is ≤1, B may be $\sin^{-1}$(value) or 180°-$\sin^{-1}$(value); retain only cases with A+B<180°. This test determines whether there are 0, 1 or 2 triangles.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	Use cosine rule to find the diagonal if not already known, then find each triangle area using ½ab sin C (or sine rule to obtain missing angles/sides). Add the two triangle areas, keeping unrounded values.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	$c^2=a^2+b^2-2ab\cos90°$. Since cos90°=0, $c^2=a^2+b^2$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	b/sin(65°)=10/sin(40°); b=10sin(65°)/sin(40°)=14.1.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	$c^2=11^2+13^2-2(11)(13)\cos43°$; c=8.99.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	cos C=$[5^2+14^2-(17)^2]/[2(5)(14)]$; C=119.1°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Area=½ab sin C=½(6)(15)sin49°=34.	**3**	M1 setup; M1 process; A1 conclusion.
10	sin C=2Area/(ab)=0.6. Acute C=$\sin^{-1}(0.6)$=36.9°.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	200°.	**2**	M1 method; A1 answer.
2	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=√(6²+9²-2(6)(9)cos100°)=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Midpoint=(3,0.5); gradient=-5/8; distance=√(8²+5²)=√89.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Cosine rule: BC²=(13)²+(16)²-2(13)(16)cos(78°), so BC=18.4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	AB is horizontal with midpoint (0,2), so its perpendicular bisector is x=0. Every point on x=0 is equidistant from A and B.	**3**	M1 setup; M1 process; A1 conclusion.
7	y+11=(9)(x-11), so y=(9)x-110. y-intercept=-(110); set y=0 to get x=12.2222.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Convert bearings to interior angles, solve algebraically with sine/cosine rules, then draw an accurate scale diagram. Agreement within drawing tolerance supports the result; the calculation remains the precise answer.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	256°.	**2**	M1 method; A1 answer.
2	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=√(6²+9²-2(6)(9)cos100°)=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
3	Midpoint=(4,-1.5); gradient=-5/8; distance=√(8²+5²)=√89.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	Cosine rule: BC²=(13)²+(16)²-2(13)(16)cos(34°), so BC=8.95.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	AB is horizontal with midpoint (0,2), so its perpendicular bisector is x=0. Every point on x=0 is equidistant from A and B.	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	y+11=(9)(x-11), so y=(9)x-110. y-intercept=-(110); set y=0 to get x=12.2222.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Convert bearings to interior angles, solve algebraically with sine/cosine rules, then draw an accurate scale diagram. Agreement within drawing tolerance supports the result; the calculation remains the precise answer.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	312°.	**2**	M1 method; A1 answer.
10	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=√(6²+9²-2(6)(9)cos100°)=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
11	Midpoint=(5,1.5); gradient=-5/8; distance=√(8²+5²)=√89.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Cosine rule: $BC^2=(13)^2+(16)^2-2(13)(16)\cos(50°)$, so BC=12.6.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	AB is horizontal with midpoint (0,2), so its perpendicular bisector is x=0. Every point on x=0 is equidistant from A and B.	**3**	M1 setup; M1 process; A1 conclusion.
3	y+11=(9)(x-11), so y=(9)x-110. y-intercept=-(110); set y=0 to get x=12.2222.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Convert bearings to interior angles, solve algebraically with sine/cosine rules, then draw an accurate scale diagram. Agreement within drawing tolerance supports the result; the calculation remains the precise answer.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	228°.	**2**	M1 method; A1 answer.
6	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=$\sqrt{(6^2+9^2-2(6)(9)\cos100°)}$=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
7	Midpoint=(6,-0.5); gradient=-5/8; distance=$\sqrt{(8^2+5^2)}=\sqrt{89}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	Cosine rule: $BC^2=(13)^2+(16)^2-2(13)(16)\cos(106°)$, so BC=23.2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	AB is horizontal with midpoint (0,2), so its perpendicular bisector is x=0. Every point on x=0 is equidistant from A and B.	**3**	M1 setup; M1 process; A1 conclusion.
11	y+11=(9)(x-11), so y=(9)x-110. y-intercept=-(110); set y=0 to get x=12.2222.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
12	Convert bearings to interior angles, solve algebraically with sine/cosine rules, then draw an accurate scale diagram. Agreement within drawing tolerance supports the result; the calculation remains the precise answer.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	284°.	**2**	M1 method; A1 answer.
14	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=$\sqrt{(6^2+9^2-2(6)(9)\cos100°)}$=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
15	Midpoint=(7,2.5); gradient=-5/8; distance=$\sqrt{(8^2+5^2)}=\sqrt{89}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 08 / G11,G15

Bearings, Coordinates and Geometric Modelling

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.
2	Cosine rule: $BC^2=(13)^2+(16)^2-2(13)(16)\cos(62°)$, so BC=15.2.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	AB is horizontal with midpoint (0,2), so its perpendicular bisector is x=0. Every point on x=0 is equidistant from A and B.	**3**	M1 setup; M1 process; A1 conclusion.
4	y+11=(9)(x-11), so y=(9)x-110. y-intercept=-(110); set y=0 to get x=12.2222.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	Convert bearings to interior angles, solve algebraically with sine/cosine rules, then draw an accurate scale diagram. Agreement within drawing tolerance supports the result; the calculation remains the precise answer.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	200°.	**2**	M1 method; A1 answer.
7	Included turn angle is 100° (reverse first direction then compare). By cosine rule, displacement=$\sqrt{(6^2+9^2-2(6)(9)\cos100°)}$=11.7 km.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	Midpoint=(1,0.5); gradient=-5/8; distance=$\sqrt{(8^2+5^2)}=\sqrt{89}$.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	Midpoint=(8,6). AB gradient=(6)/8; perpendicular gradient=-8/(6). Equation: y-(6)=-8/(6)(x-8).	**5**	M1 setup; M2–M4 development; A1 conclusion.
10	Cosine rule: $BC^2=(13)^2+(16)^2-2(13)(16)\cos(78°)$, so BC=18.4.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

FLUENCY CHECK - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	2a-b=(4,6)-(-3,2)=(7,4).	**2**	M1 method; A1 answer.
2	AP:PB=3:4; OP=[4a+3b]/(7).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.
4	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	PQ=OQ-OP=6/13(b-a)=6/13AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	OQ=3(a+7b)=3OP. One position vector is a scalar multiple of the other, so O,P,Q are collinear.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	Let AB=b and AC=c. AP=⅔b and AQ=⅔c. PQ=AQ-AP=⅔(c-b)=⅔BC, so PQ is parallel to BC.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	XY is parallel to AB. If k>0 directions agree; if k<0 directions are opposite. If |k|=1 lengths are equal; otherwise |XY|=|k||AB|.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

STANDARD HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	2a-b=(8,12)-(-6,4)=(14,8).	**2**	M1 method; A1 answer.
2	AP:PB=5:7; OP=[7a+5b]/(12).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.
4	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	PQ=OQ-OP=2/7(b-a)=2/7AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	OQ=6(a+3b)=6OP. One position vector is a scalar multiple of the other, so O,P,Q are collinear.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

STANDARD HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
7	Let AB=b and AC=c. AP=⅔b and AQ=⅔c. PQ=AQ-AP=⅔(c-b)=⅔BC, so PQ is parallel to BC.	**5**	M1 setup; M2–M4 development; A1 conclusion.
8	XY is parallel to AB. If k>0 directions agree; if k<0 directions are opposite. If |k|=1 lengths are equal; otherwise |XY|=|k||AB|.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
9	2a-b=(12,8)-(-4,6)=(16,2).	**2**	M1 method; A1 answer.
10	AP:PB=7:5; OP=[5a+7b]/(12).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.
12	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

MULTI-STEP HIGHER • PART A - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	PQ=OQ-OP=4/7(b-a)=4/7AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	OQ=4(a+5b)=4OP. One position vector is a scalar multiple of the other, so O,P,Q are collinear.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	Let AB=b and AC=c. AP=⅔b and AQ=⅔c. PQ=AQ-AP=⅔(c-b)=⅔BC, so PQ is parallel to BC.	**5**	M1 setup; M2–M4 development; A1 conclusion.
4	XY is parallel to AB. If k>0 directions agree; if k<0 directions are opposite. If |k|=1 lengths are equal; otherwise |XY|=|k||AB|.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
5	2a-b=(4,14)-(-7,2)=(11,12).	**2**	M1 method; A1 answer.
6	AP:PB=3:3; OP=[3a+3b]/(6).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
7	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.
8	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

MULTI-STEP HIGHER • PART B - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
9	PQ=OQ-OP=6/12(b-a)=6/12AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	OQ=7(a+7b)=7OP. One position vector is a scalar multiple of the other, so O,P,Q are collinear.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
11	Let AB=b and AC=c. AP=⅔b and AQ=⅔c. PQ=AQ-AP=⅔(c-b)=⅔BC, so PQ is parallel to BC.	**5**	M1 setup; M2–M4 development; A1 conclusion.
12	XY is parallel to AB. If k>0 directions agree; if k<0 directions are opposite. If |k|=1 lengths are equal; otherwise |XY|=|k||AB|.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
13	2a-b=(8,10)-(-5,4)=(13,6).	**2**	M1 method; A1 answer.
14	AP:PB=5:6; OP=[6a+5b]/(11).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
15	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.

MODULE 09 / G24,G25

Vectors and Geometric Proof

GRADE 7-9 CHALLENGE - ANSWERS

HIGHER TIER • SHOW CLEAR WORKING • KEEP EXACT VALUES WHERE REQUIRED

Name: _____ Date: _____ Score: _____ / _____

Use the formula/method guides in the consolidated solutions section. Award method marks where the setup is correct.

Q	Answer / concise method	Marks	Check
1	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
2	PQ=OQ-OP=2/6(b-a)=2/6AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
3	OQ=5(a+3b)=5OP. One position vector is a scalar multiple of the other, so O,P,Q are collinear.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
4	Let AB=b and AC=c. AP=⅔b and AQ=⅔c. PQ=AQ-AP=⅔(c-b)=⅔BC, so PQ is parallel to BC.	**5**	M1 setup; M2–M4 development; A1 conclusion.
5	XY is parallel to AB. If k>0 directions agree; if k<0 directions are opposite. If |k|=1 lengths are equal; otherwise |XY|=|k||AB|.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
6	2a-b=(12,6)-(-3,6)=(15,0).	**2**	M1 method; A1 answer.
7	AP:PB=7:4; OP=[4a+7b]/(11).	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
8	OA=a, OB=b, so AB=b-a. OM=OA+AM=a+½(b-a)=½(a+b).	**3**	M1 setup; M1 process; A1 conclusion.
9	AC=a+b. Midpoint of AC has position ½(a+b). BD goes from b to a, whose midpoint is b+½(a-b)=½(a+b). Equal midpoint proves bisection.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.
10	PQ=OQ-OP=4/11(b-a)=4/11AB. It is a scalar multiple of AB, so PQ∥AB.	**4**	M1 setup; M1 development; A1 accuracy; A1 conclusion.

U4-M10-Q01 [4 marks]

Arc length=(80/360)×2π×6=2.66667π cm. Area=(80/360)×π×6²=8π cm².

U4-M10-Q02 [4 marks]

Cosine rule: $c^2=7^2+10^2-2(7)(10)\cos 50°$. Hence c=7.68 cm.

U4-M10-Q03 [4 marks]

Sine rule: sin B/10=sin 35°/8, so sin B=0.71697. The acute solution is B=45.8°.

U4-M10-Q04 [4 marks]

Area factor=1.4^2=1.96; volume factor=1.4^3=2.744. Larger volume=250×2.744=686.000 cm³.

U4-M10-Q05 [4 marks]

OP=OA+[2/(5)]AB=a+[2/5](b−a)=[3a+2b]/5.

U4-M10-Q06 [5 marks]

Midpoint=(5,3). Segment gradient=-0.666667, so perpendicular gradient=1.5. Therefore y=1.5x-4.5.

U4-M10-Q07 [4 marks]

TSA=$2\pi r^2+2\pi rh=2\pi(3)^2+2\pi(3)(8)=66\pi$=207 cm².

U4-M10-Q08 [4 marks]

AB=$\sqrt{(4^2+7^2)}$=8.06 km. tan θ=4/7, so θ=29.7°. Bearing=029.7°.

U4-M10-Q09 [4 marks]

Exterior=360/8=45°. Interior=180−45=135°. The vertex angle of the isosceles triangle is the interior angle, so each base angle=(180−135)/2=22.5°.

U4-M10-Q10 [4 marks]

Arc length=(85/360)×2π×7=3.30556π cm. Area=(85/360)×π×7^2=11.5694π cm^2.

U4-M10-Q11 [4 marks]

Cosine rule: $c^2=8^2+11^2-2(8)(11)\cos 53°$. Hence c=8.89 cm.

U4-M10-Q12 [4 marks]

Sine rule: sin B/11=sin 37°/9, so sin B=0.73555. The acute solution is B=47.4°.

U4-M10-Q13 [4 marks]

Area factor=1.5^2=2.25; volume factor=1.5^3=3.375. Larger volume=300×3.375=1012.500 cm^3.

U4-M10-Q14 [4 marks]

OP=OA+[3/(6)]AB=a+[3/6](b−a)=[3a+3b]/6.

U4-M10-Q15 [5 marks]

Midpoint=(6,3). Segment gradient=-1, so perpendicular gradient=1. Therefore y=1x-3.

U4-M10-Q16 [4 marks]

TSA=$2\pi r^2+2\pi rh=2\pi(4)^2+2\pi(4)(9)=104\pi$=327 cm^2.

U4-M10-Q17 [4 marks]

AB=√(5²+8²)=9.43 km. tan θ=5/8, so θ=32.0°. Bearing=032.0°.

U4-M10-Q18 [4 marks]

Exterior=360/9=40°. Interior=180−40=140°. The vertex angle of the isosceles triangle is the interior angle, so each base angle=(180−140)/2=20°.

U4-M10-Q19 [4 marks]

Arc length=(90/360)×2π×8=4π cm. Area=(90/360)×π×8²=16π cm².

U4-M10-Q20 [4 marks]

Cosine rule: c²=9²+12²−2(9)(12)cos 56°. Hence c=10.2 cm.

U4-M10-Q21 [4 marks]

Sine rule: sin B/12=sin 39°/10, so sin B=0.75518. The acute solution is B=49.0°.

U4-M10-Q22 [4 marks]

Area factor=1.6²=2.56; volume factor=1.6³=4.096. Larger volume=350×4.096=1433.600 cm³.

U4-M10-Q23 [4 marks]

OP=OA+[4/(7)]AB=a+[4/7](b−a)=[3a+4b]/7.

U4-M10-Q24 [5 marks]

Midpoint=(7,3). Segment gradient=-1.33333, so perpendicular gradient=0.75. Therefore y=0.75x-2.25.

U4-M10-Q25 [4 marks]

TSA=2πr²+2πrh=2π(5)²+2π(5)(10)=150π=471 cm².

U4-M10-Q26 [4 marks]

AB=√(6²+9²)=10.8 km. tan θ=6/9, so θ=33.7°. Bearing=033.7°.

U4-M10-Q27 [4 marks]

Exterior=360/10=36°. Interior=180−36=144°. The vertex angle of the isosceles triangle is the interior angle, so each base angle=(180−144)/2=18°.

U4-M10-Q28 [4 marks]

Arc length=(95/360)×2π×9=4.75π cm. Area=(95/360)×π×9²=21.375π cm².

U4-M10-Q29 [4 marks]

Cosine rule: c²=10²+13²−2(10)(13)cos 59°. Hence c=11.6 cm.

U4-M10-Q30 [4 marks]

Sine rule: sin B/13=sin 41°/11, so sin B=0.77534. The acute solution is B=50.8°.

U4-M10-Q31 [4 marks]

Area factor=1.7²=2.89; volume factor=1.7³=4.913. Larger volume=400×4.913=1965.200 cm³.

U4-M10-Q32 [4 marks]

OP=OA+[5/(8)]AB=a+[5/8](b−a)=[3a+5b]/8.

U4-M10-Q33 [5 marks]

Midpoint=(8,3). Segment gradient=-1.66667, so perpendicular gradient=0.6. Therefore y=0.6x-1.8.

U4-M10-Q34 [4 marks]

TSA=$2\pi r^2+2\pi rh=2\pi(6)^2+2\pi(6)(11)=204\pi=641$ cm².

U4-M10-Q35 [4 marks]

AB=$\sqrt{(7^2+10^2)}=12.2$ km. $\tan\theta=7/10$, so $\theta=35.0°$. Bearing=035.0°.

U4-M10-Q36 [4 marks]

Exterior=360/11=32.7273°. Interior=180−32.7273=147.273°. The vertex angle of the isosceles triangle is the interior angle, so each base angle=(180−147.273)/2=16.3636°.

U4-M10-Q37 [4 marks]

Arc length=$(100/360)\times2\pi\times10=5.55556\pi$ cm. Area=$(100/360)\times\pi\times10^2=27.7778\pi$ cm².

U4-M10-Q38 [4 marks]

Cosine rule: $c^2=11^2+14^2-2(11)(14)\cos 62°$. Hence c=13.1 cm.

U4-M10-Q39 [4 marks]

Sine rule: $\sin B/14=\sin 43°/12$, so $\sin B=0.79566$. The acute solution is B=52.7°.

U4-M10-Q40 [4 marks]

Area factor=$1.8^2=3.24$; volume factor=$1.8^3=5.832$. Larger volume=$450\times5.832=2624.400$ cm³.

U4-M10-Q41 [4 marks]

OP=OA+[6/(9)]AB=a+[6/9](b−a)=[3a+6b]/9.

U4-M10-Q42 [5 marks]

Midpoint=(9,3). Segment gradient=-2, so perpendicular gradient=0.5. Therefore y=0.5x-1.5.

U4-M10-Q43 [4 marks]

TSA=2πr²+2πrh=2π(7)²+2π(7)(12)=266π=836 cm².

U4-M10-Q44 [4 marks]

AB=√(8²+11²)=13.6 km. tan θ=8/11, so θ=36.0°. Bearing=036.0°.

U4-M10-Q45 [4 marks]

Exterior=360/12=30°. Interior=180−30=150°. The vertex angle of the isosceles triangle is the interior angle, so each base angle=(180−150)/2=15°.