

GCSE MATHEMATICS

HIGHER TIER

# COMPREHENSIVE LESSON NOTEBOOK

## UNIT 2 | ALGEBRA

TEACH-FROM-SCRATCH • FREE WEBSITE EDITION

20 detailed chapters • original diagrams • worked examples • guided practice • answers

Student: \_\_\_\_\_ Target grade: \_\_\_\_\_

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START HERE

STUDY ROUTINE

# How to learn from this notebook

Meaning first. Method second. Practice third.

## THE FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each solution and reproduce it. 3 Complete the practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

## What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final sentence when the result must be interpreted.

## Error log

### REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

## Higher Tier requires connected depth

Higher Tier rewards secure fluency and connected reasoning. Grade 4–5 questions often combine several familiar skills, hide the required operation inside a context, or ask whether an answer is reasonable. The notebook therefore explains why methods work as well as how to use them.

## SPECIFICATION MAP

## PEARSON EDEXCEL 1MA1

# Unit 2 coverage map

Algebra content organised into a teachable sequence.

|       |                  |                                                           |
|-------|------------------|-----------------------------------------------------------|
| CH 01 | A1, A2, A3       | Algebraic notation, structure and substitution            |
| CH 02 | A4               | Collecting terms and algebraic index laws                 |
| CH 03 | A4               | Expanding products and factorising quadratics             |
| CH 04 | A4               | Algebraic fractions and domain restrictions               |
| CH 05 | A5               | Rearranging complex formulae                              |
| CH 06 | A6               | Identities and algebraic proof                            |
| CH 07 | A7               | Functions, composites and inverses                        |
| CH 08 | A8, A9, A10      | Coordinates, gradient and equations of straight lines     |
| CH 09 | A11, A12         | Quadratic graphs, roots and turning points                |
| CH 10 | A12, A13         | Cubic, reciprocal, exponential and transformed graphs     |
| CH 11 | A14, A15         | Real-context graphs, gradients and areas under graphs     |
| CH 12 | A16              | Circle equations and tangents                             |
| CH 13 | A17, A21         | Linear equations and algebraic modelling                  |
| CH 14 | A18              | Quadratic equations: factorisation and the formula        |
| CH 15 | A11, A18         | Completing the square and optimisation                    |
| CH 16 | A19              | Simultaneous equations including linear-quadratic systems |
| CH 17 | A20              | Iteration and numerical solutions                         |
| CH 18 | A22              | Linear and quadratic inequalities                         |
| CH 19 | A23, A24, A25    | Arithmetic, geometric and quadratic sequences             |
| CH 20 | A1-A25 synthesis | Higher Algebra modelling and connected problem solving    |

## IMPORTANT TIER NOTE

This notebook follows the complete Higher Tier Algebra content identified in the Pearson Edexcel GCSE Mathematics (1MA1) specification, including shared foundations and Higher-only extensions.

## REFERENCE

# Fast-reference method bank

## ALGEBRA FACTS AND METHODS

Understand each formula before memorising it.

**Straight line**

$$y=mx+c$$

**Gradient**

$$\Delta y/\Delta x$$

**Perpendicular gradients**

$$m_1m_2=-1$$

**Quadratic formula**

$$x=(-b\pm\sqrt{(b^2-4ac)})/(2a)$$

**Discriminant**

$$b^2-4ac$$

**Circle at origin**

$$x^2+y^2=r^2$$

**Composite function**

$$fg(x)=f(g(x))$$

**Iteration**

$$x_{n+1}=g(x_n)$$

**Arithmetic nth term**

$$\text{difference}\times n+\text{adjustment}$$

**Quadratic second difference**

$$2\times\text{coefficient of }n^2$$

## CHAPTER 01 • A1-A3

A1, A2, A3

# Algebraic notation, structure and substitution

Read an expression structurally before attempting to manipulate it.

## SPECIFICATION FOCUS

Pearson Edexcel content references A1, A2, A3. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Higher algebra depends on recognising terms, factors, coefficients, powers and grouped subexpressions. An expression has no equality claim; an equation is true for solution values; a formula links quantities; an identity is true for every permitted value. Substitution must preserve brackets, especially for negative or fractional inputs.

## KEY VOCABULARY

term • factor • coefficient • expression • equation • formula • identity • substitution



Division/multiplication share a level; addition/subtraction share a level. Work left to right.

## Why the method works

- Terms are separated only by top-level addition or subtraction.
- A power applies to its complete base.
- Fraction bars and roots group their complete contents.
- Equivalent expressions have the same value for every permitted input.

## CHECK BEFORE MOVING ON

Answer mentally: 1) State coefficient of  $x^2$  in  $7-5x^2$ . 2) How many terms in  $3x^2-2xy+y$ ? 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 01 • A1-A3 • METHODS

A1, A2, A3

# Algebraic notation, structure and substitution — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Evaluate  $2x^3 - 3xy$  for  $x = -2, y = 5$ .

1. Substitute with brackets:  $2(-2)^3 - 3(-2)(5)$ .
2. Calculate  $-16 + 30$ .

**ANSWER: 14**

### WORKED EXAMPLE 2

Classify  $3(x+1) \equiv 3x+3$ .

1. Both sides are equivalent for every  $x$ .

**ANSWER: Identity**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Evaluate  $a^2/b$  for  $a = -6, b = 9$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 01 • A1-A3 • APPLICATION

A1, A2, A3

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Scientific and financial formulas require careful substitution with units and restrictions; a correct calculator entry begins with correct algebraic grouping.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain why  $-3^2$  differs from  $(-3)^2$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 01 • A1-A3 • PRACTICE

A1, A2, A3

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. State coefficient of  $x^2$  in  $7-5x^2$ .

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---

2. How many terms in  $3x^2-2xy+y$ ?

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---

3. Evaluate  $a^2/b$  for  $a=-6, b=9$ .

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4. Evaluate  $\sqrt{(p+5)}$  for  $p=11$ .

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5. Classify  $4x-1=15$ .

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6. Classify  $V=\pi r^2 h$ .

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7. Substitute  $x=-3$  into  $(x-2)^2$ .

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8. Explain why  $-3^2$  differs from  $(-3)^2$ .

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 02 • A4

## A4

# Collecting terms and algebraic index laws

Combine only identical algebraic structures and count repeated factors.

## SPECIFICATION FOCUS

Pearson Edexcel content references A4. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Like terms have the same variable part with identical powers. Coefficients then add or subtract. Index laws apply to products and quotients of the same base: multiplying adds indices, dividing subtracts, and a power of a power multiplies. Negative and fractional numerical ideas extend directly to algebraic bases, subject to non-zero and real-domain restrictions.

## KEY VOCABULARY

like term • coefficient • base • index • zero index • negative index • reciprocal

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

base = repeated factor    index = number of factors

## Why the method works

- $ax^k + bx^k = (a+b)x^k$ .
- $x^m x^n = x^{m+n}$  and  $x^m / x^n = x^{m-n}$  for  $x \neq 0$ .
- $(x^m)^n = x^{mn}$ .
- $x^{-n} = 1/x^n$  and  $x^0 = 1$  for  $x \neq 0$ .

## CHECK BEFORE MOVING ON

Answer mentally: 1) Simplify  $7x^2 - 3x + 5x^2 + x$ . 2) Simplify  $4a^3 \times 3a^5$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 02 • A4 • METHODS

A4

# Collecting terms and algebraic index laws — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**Simplify  $6a^3b^2 \times (-2a^4b)$ .**

1. Coefficients  $6 \times -2 = -12$ .
2. Add indices for each base.

**ANSWER:  $-12a^7b^3$**

### WORKED EXAMPLE 2

**Simplify  $15x^8y^3/(5x^2y^5)$ .**

1. Coefficients give 3; subtract indices.

**ANSWER:  $3x^6/y^2$**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Simplify  $18p^7/(6p^2)$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 02 • A4 • APPLICATION

## A4

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Algebraic scaling formulas use index laws to show how area, volume and physical quantities change with dimension.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain why  $x^2 + x^3$  cannot collect. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 02 • A4 • PRACTICE

A4

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Simplify  $7x^2 - 3x + 5x^2 + x$ .

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---

2. Simplify  $4a^3 \times 3a^5$ .

---



---

3. Simplify  $18p^7 / (6p^2)$ .

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---

4. Simplify  $(2x^3)^4$ .

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5. Write  $y^{-4}$  with positive index.

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6. Simplify  $a^3b^{-2} \times a^{-1}b^5$ .

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7. Evaluate  $x^0$  when  $x \neq 0$ .

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8. Explain why  $x^2 + x^3$  cannot collect.

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 03 • A4

## A4

# Expanding products and factorising quadratics

Use distribution in both directions and verify every factorisation by expansion.

## SPECIFICATION FOCUS

Pearson Edexcel content references A4. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Expanding two binomials creates four products. Factorising reverses this structure. For  $x^2+bx+c$ , choose numbers with product  $c$  and sum  $b$ . For  $ax^2+bx+c$ , split the middle term or use a structured factor-pair method. Difference of two squares follows  $a^2-b^2=(a-b)(a+b)$ .

## KEY VOCABULARY

binomial • quadratic • expand • factorise • middle term • difference of squares

Brackets

Indices

÷ and ×

+ and −

Division/multiplication share a level; addition/subtraction share a level. Work left to right.

## Why the method works

- Every term in one bracket multiplies every term in the other.
- For  $ax^2+bx+c$ , split  $b$  using numbers whose product is  $ac$ .
- A common factor should be removed before quadratic factorisation.
- Expansion is a decisive verification.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Expand  $(x-4)(x+7)$ . 2) Expand  $(3x+2)(2x-5)$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 03 • A4 • METHODS

A4

# Expanding products and factorising quadratics — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**Factorise  $6x^2+11x+3$ .**

1.  $ac=18$ ; split  $11x$  as  $9x+2x$ .
2.  $3x(2x+3)+1(2x+3)$ .

**ANSWER:  $(3x+1)(2x+3)$**

### WORKED EXAMPLE 2

**Expand and simplify  $(2x-3)(x+5)-x(x-1)$ .**

1. First product  $= 2x^2+7x-15$ .
2. Subtract  $x^2-x$ .

**ANSWER:  $x^2+8x-15$**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Factorise  $x^2+3x-28$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 03 • A4 • APPLICATION

## A4

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Area models and projectile equations produce quadratic expressions; factor form reveals zeros while expanded form reveals coefficients.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain difference of squares. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 03 • A4 • PRACTICE

A4

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Expand  $(x-4)(x+7)$ .

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---

2. Expand  $(3x+2)(2x-5)$ .

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---

3. Factorise  $x^2+3x-28$ .

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4. Factorise  $2x^2+7x+3$ .

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5. Factorise  $12x^2-27$ .

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6. Factorise  $5x^2-20x$ .

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7. Expand  $(x+3)^2$ .

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8. Explain difference of squares.

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 04 • A4

A4

# Algebraic fractions and domain restrictions

Factor first, cancel factors only, and preserve excluded values.

## SPECIFICATION FOCUS

Pearson Edexcel content references A4. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

An algebraic fraction is defined only where its denominator is non-zero. Simplification uses factorisation and cancellation of common factors. Terms joined by addition cannot cancel. Addition and subtraction require a common algebraic denominator. A simplified expression remains subject to restrictions inherited from the original.

## KEY VOCABULARY

algebraic fraction • denominator • restriction • common factor • common denominator • excluded value



3 shaded equal parts out of 8 =  $\frac{3}{8}$

## Why the method works

- Cancellation divides a common non-zero factor from numerator and denominator.
- Original denominator zeros remain excluded after simplification.
- A common denominator is the least expression containing every denominator factor.
- Factorisation often reveals cancellation invisible in expanded form.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Simplify  $\frac{6x^2}{9x}$ . 2) Simplify  $\frac{x^2-16}{x-4}$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 04 • A4 • METHODS

A4

# Algebraic fractions and domain restrictions — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**Simplify  $(x^2-9)/(x^2+x-6)$ .**

1. Factor:  $(x-3)(x+3)/(x+3)(x-2)$ .
2. Cancel  $x+3$ , retaining  $x \neq -3, 2$ .

**ANSWER:**  $(x-3)/(x-2)$

### WORKED EXAMPLE 2

**Simplify  $2/x+3/(2x)$ .**

1. Common denominator  $2x$ .
2.  $4/(2x)+3/(2x)$ .

**ANSWER:**  $7/(2x)$ ,  $x \neq 0$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Simplify  $(x^2+5x+6)/(x+2)$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 04 • A4 • APPLICATION

## A4

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Rational formulas in rates and geometry can have forbidden inputs; simplification must not erase physical or algebraic restrictions.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Simplify  $(2x^2 - 8)/(x^2 - 4)$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 04 • A4 • PRACTICE

A4

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Simplify  $6x^2/(9x)$ .

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---

2. Simplify  $(x^2-16)/(x-4)$ .

---



---

3. Simplify  $(x^2+5x+6)/(x+2)$ .

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---

4. Simplify  $3/x-1/(2x)$ .

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5. Add  $1/(x+1)+2/(x+1)$ .

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6. State restrictions for  $(x+2)/(x^2-4)$ .

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7. Why cannot  $x$  cancel in  $(x+3)/x$ ?

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---

8. Simplify  $(2x^2-8)/(x^2-4)$ .

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 05 • A5

A5

# Rearranging complex formulae

Treat the required subject as one object and undo operations across complete expressions.

## SPECIFICATION FOCUS

Pearson Edexcel content references A5. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Changing the subject produces an equivalent formula with the chosen variable isolated. Variables may appear in fractions, powers, roots or on both sides. Factorising the subject can collect repeated occurrences. Squaring or square-rooting may introduce sign considerations, so domain and context determine the valid branch.

## KEY VOCABULARY

subject • rearrange • isolate • factorise • reciprocal • branch • restriction



Reverse the machine with inverse operations:  $-7$ , then divide by  $3$

## Why the method works

- Every operation must be applied to both complete sides.
- If the subject appears in several terms, collect it by factorisation.
- Rearranging a denominator often begins by multiplying through.
- From  $x^2=k$ , algebra gives  $x=\pm\sqrt{k}$  unless context restricts the sign.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Make  $x$  subject:  $y=5x-8$ . 2) Make  $h$  subject:  $A=\pi r^2+2rh$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 05 • A5 • METHODS

A5

# Rearranging complex formulae — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Make  $x$  the subject of  $y = (3x - 2)/(x + 4)$ .

1.  $y(x + 4) = 3x - 2$ .
2.  $yx + 4y = 3x - 2$ .
3.  $x(y - 3) = -2 - 4y$ .

**ANSWER:**  $x = (2 + 4y)/(3 - y)$

### WORKED EXAMPLE 2

Make  $r$  the subject of  $V = 4\pi r^3/3$ .

1.  $r^3 = 3V/(4\pi)$ .
2. Take cube root.

**ANSWER:**  $r = \sqrt[3]{3V/(4\pi)}$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Make  $x$  subject:  $y = 1/x + 3$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 05 • A5 • APPLICATION

A5

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Physics and engineering formulas are rearranged to match the measured and unknown quantities; units help verify the result.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain why  $\pm$  may appear. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 05 • A5 • PRACTICE

A5

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Make  $x$  subject:  $y=5x-8$ .

---



---

2. Make  $h$  subject:  $A=\pi r^2+2rh$ .

---



---

3. Make  $x$  subject:  $y=1/x+3$ .

---



---

4. Make  $a$  subject:  $v^2=u^2+2as$ .

---



---

5. Make  $t$  subject:  $s=ut+\frac{1}{2}at^2$  when  $u=0$ .

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---

6. Make  $x$  subject:  $p=qx/(x+r)$ .

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---

7. Make  $r$  subject:  $A=\pi r^2$ ,  $r>0$ .

---



---

8. Explain why  $\pm$  may appear.

---



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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 06 • A6

A6

# Identities and algebraic proof

Represent a general case and transform it into the required form.

## SPECIFICATION FOCUS

Pearson Edexcel content references A6. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

An identity is true for every permitted value; an equation is true only for its solutions. Proof uses general representations rather than selected examples. Even integers are  $2n$ , odd integers  $2n+1$ , consecutive integers  $n, n+1, \dots$  and multiples of  $k$  are  $kn$ . A counterexample disproves a universal assertion.

## KEY VOCABULARY

identity • proof • general integer • parity • consecutive • counterexample

Brackets

Indices

÷ and ×

+ and −

Division/multiplication share a level; addition/subtraction share a level. Work left to right.

## Why the method works

- Examples can suggest a statement but cannot establish every case.
- A factor  $k$  proves an expression is divisible by  $k$ .
- The square of an odd integer can be written in the form  $8q+1$ .
- One valid counterexample disproves an “always” statement.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Show  $\text{even} \times \text{integer}$  is even. 2) Show  $\text{odd} + \text{odd}$  is even. 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 06 • A6 • METHODS

# Identities and algebraic proof — method <sup>A6</sup> and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**Prove square of an odd integer is odd.**

1. Let odd integer  $= 2n+1$ .
2. Square  $= 4n^2+4n+1 = 2(2n^2+2n)+1$ .

**ANSWER: Odd**

### WORKED EXAMPLE 2

**Show  $n^2+n$  is even for integer  $n$ .**

1.  $n^2+n = n(n+1)$ .
2. Consecutive integers include one even factor.

**ANSWER: Even**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Show three consecutive integers sum to multiple of 3. First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 06 • A6 • APPLICATION

A6

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Number theory and algorithm verification require universal reasoning, not a list of successful tests.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain role of counterexample. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 06 • A6 • PRACTICE

A6

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Show  $\text{even} \times \text{integer}$  is even.

---

---

2. Show  $\text{odd} + \text{odd}$  is even.

---

---

3. Show three consecutive integers sum to multiple of 3.

---

---

4. Prove difference of consecutive squares is odd.

---

---

5. Disprove  $x^2 \geq x$  for every real  $x$ .

---

---

6. Show  $(n+1)^2 - n^2 = 2n+1$ .

---

---

7. Is  $2(x+3) \equiv 2x+6$ ?

---

---

8. Explain role of counterexample.

---

---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 07 • A7

A7

# Functions, composites and inverses

Track mappings in order and distinguish an inverse from a reciprocal.

## SPECIFICATION FOCUS

Pearson Edexcel content references A7. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

A function assigns one output to each permitted input. Formal notation  $f(x)$  names the output of  $f$  at  $x$ . A composite  $fg(x)$  means  $f(g(x))$  under the usual Edexcel convention: apply  $g$  first, then  $f$ . An inverse function reverses a one-to-one mapping and may require a restricted domain.

## KEY VOCABULARY

function • notation • composite • inverse • domain • range • one-to-one



Reverse the machine with inverse operations:  $-7$ , then divide by  $3$

## Why the method works

- $f(a)$  means substitute  $a$  for every  $x$  in the rule.
- $fg(x)=f(g(x))$ ; order generally matters.
- An inverse satisfies  $f^{-1}(f(x))=x$  on the permitted domain.
- A quadratic needs a restricted domain to have a single-valued inverse.

## CHECK BEFORE MOVING ON

Answer mentally: 1)  $f(x)=2x+3$ :  $f(5)$ . 2)  $g(x)=x^2-1$ :  $g(-3)$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 07 • A7 • METHODS

A7

# Functions, composites and inverses — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

$f(x)=3x-4, g(x)=x^2+1$ . Find  $fg(2)$ .

1.  $g(2)=5$ .
2.  $f(5)=11$ .

**ANSWER: 11**

### WORKED EXAMPLE 2

Find inverse of  $f(x)=(x+5)/2$ .

1. Let  $y=(x+5)/2$ .
2. Swap and solve:  $x=(y+5)/2$  gives  $y=2x-5$ .

**ANSWER:  $f^{-1}(x)=2x-5$**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Find  $fg(x)$  for  $f=2x+3, g=x-4$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 07 • A7 • APPLICATION

A7

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Encoding and conversion systems compose functions; reversing a process requires undoing the latest stage first.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Check  $f^{-1}(f(6))$  for  $f=2x-5$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 07 • A7 • PRACTICE

A7

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1.  $f(x)=2x+3$ :  $f(5)$ .

---



---

2.  $g(x)=x^2-1$ :  $g(-3)$ .

---



---

3. Find  $fg(x)$  for  $f=2x+3$ ,  $g=x-4$ .

---



---

4. Find  $gf(x)$ .

---



---

5. Inverse of  $f(x)=4x-7$ .

---



---

6. Solve  $f(x)=25$  for  $f=3x+1$ .

---



---

7. Why restrict  $y=x^2$  for inverse?

---



---

8. Check  $f^{-1}(f(6))$  for  $f=2x-5$ .

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 08 • A8-A10

A8, A9, A10

# Coordinates, gradient and equations of straight lines

Connect geometric movement with algebraic rate and intercept.

## SPECIFICATION FOCUS

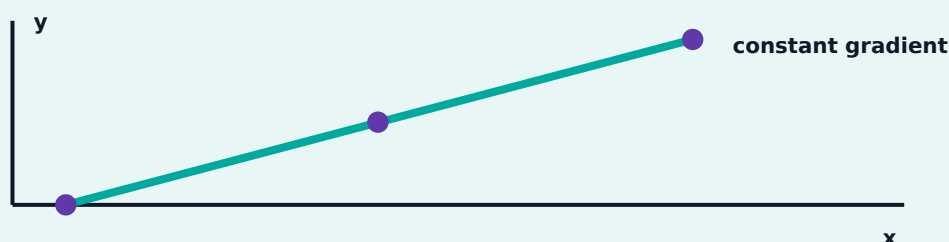
Pearson Edexcel content references A8, A9, A10. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

For two points, gradient  $m = (\text{change in } y) / (\text{change in } x)$ . In  $y = mx + c$ ,  $m$  is constant rate and  $c$  is the  $y$ -intercept. Parallel lines share gradient; perpendicular non-vertical lines have gradients whose product is  $-1$ . Point-gradient form  $y - y_1 = m(x - x_1)$  builds a line through a known point.

## KEY VOCABULARY

coordinate • gradient • intercept • parallel • perpendicular • point-gradient form



## Why the method works

- Gradient is a ratio of vertical to horizontal change.
- Parallel lines never meet because their gradients match.
- Perpendicular gradients are negative reciprocals.
- Substituting a point verifies membership of a line.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Gradient through  $(-2, 7), (4, -5)$ . 2) Equation gradient 5, intercept  $-3$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 08 • A8-A10 • METHODS

A8, A9, A10

# Coordinates, gradient and equations of straight lines — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Find equation through (2,5) and (6,13).

1.  $m = (13 - 5) / (6 - 2) = 2$ .
2. Use  $y - 5 = 2(x - 2)$ .

**ANSWER:**  $y = 2x + 1$

### WORKED EXAMPLE 2

Find line perpendicular to  $y = 3x - 4$  through (6,2).

1. Perpendicular gradient  $= -1/3$ .
2.  $y - 2 = -1/3(x - 6)$ .

**ANSWER:**  $y = -x/3 + 4$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Equation through (0,4),(3,10). First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 08 • A8-A10 • APPLICATION

A8, A9, A10

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Navigation, conversion and cost graphs use gradient as a rate; perpendicular models support normals and geometric construction.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain negative reciprocal. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 08 • A8-A10 • PRACTICE

A8, A9, A10

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Gradient through  $(-2, 7), (4, -5)$ .

---



---

2. Equation gradient 5, intercept  $-3$ .

---



---

3. Equation through  $(0, 4), (3, 10)$ .

---



---

4. Parallel to  $y = -2x + 1$  through  $(1, 5)$ .

---



---

5. Perpendicular to  $y = \frac{1}{2}x + 3$  gradient.

---



---

6. Does  $(4, 9)$  lie on  $y = 2x + 1$ ?

---



---

7. Find x-intercept  $y = 3x - 12$ .

---



---

8. Explain negative reciprocal.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 09 • A11-A12

A11, A12

# Quadratic graphs, roots and turning points

Read algebraic features from shape, intercepts and symmetry.

## SPECIFICATION FOCUS

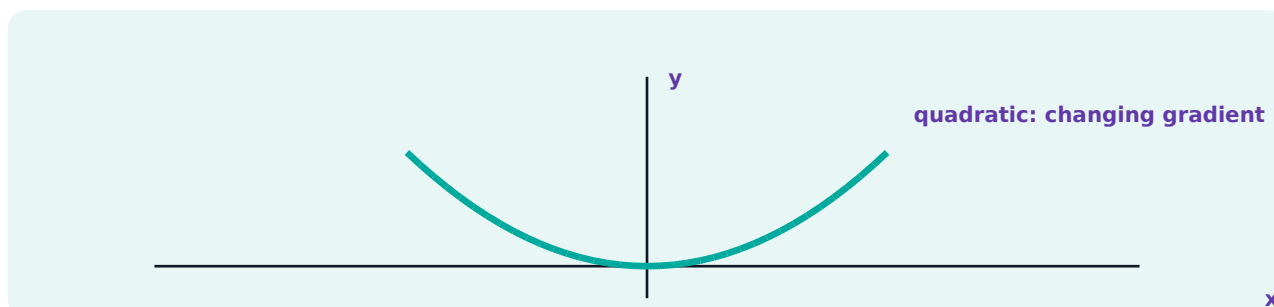
Pearson Edexcel content references A11, A12. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

A quadratic graph is a parabola. Roots are x-axis intersections, the y-intercept occurs at  $x=0$ , and the turning point is its maximum or minimum. Factor form reveals roots; completed-square form reveals turning point and symmetry line. A discriminant predicts the number of real roots.

## KEY VOCABULARY

parabola • root • turning point • axis of symmetry • discriminant • intercept



## Why the method works

- Factor form  $a(x-p)(x-q)$  shows roots  $p$  and  $q$ .
- Completed-square form  $a(x-h)^2+k$  shows turning point  $(h,k)$ .
- The axis of symmetry lies halfway between two roots.
- Discriminant  $b^2-4ac$  is positive, zero or negative for two, one or no real roots.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Roots of  $y=(x+1)(x-7)$ . 2) Symmetry line for roots  $-1,7$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 09 • A11-A12 • METHODS

A11, A12

# Quadratic graphs, roots and turning points — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

For  $y=(x-2)(x+6)$ , state roots and symmetry line.

1. Roots 2 and  $-6$ .
2. Midpoint  $= (2-6)/2 = -2$ .

**ANSWER:** roots 2,  $-6$ ;  $x=-2$

### WORKED EXAMPLE 2

For  $y=2(x-3)^2-5$ , state turning point.

1. Compare  $a(x-h)^2+k$ .

**ANSWER:** (3,  $-5$ )

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Turning point  $y=(x+4)^2-9$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 09 • A11-A12 • APPLICATION

A11, A12

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Projectile paths and optimisation models use turning points for maximum height or minimum cost.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain root meaning. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 09 • A11-A12 • PRACTICE

A11, A12

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Roots of  $y=(x+1)(x-7)$ .

---



---

2. Symmetry line for roots  $-1, 7$ .

---



---

3. Turning point  $y=(x+4)^2-9$ .

---



---

4. y-intercept  $y=2x^2-3x+5$ .

---



---

5. Discriminant  $x^2+4x+8$ .

---



---

6. Number roots  $3x^2-5x-2$ .

---



---

7. Solve  $x^2-9=0$  graphically/algebraically.

---



---

8. Explain root meaning.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 10 • A12-A13

A12, A13

# Cubic, reciprocal, exponential and transformed graphs

Predict a new graph from a known function without rebuilding every point.

## SPECIFICATION FOCUS

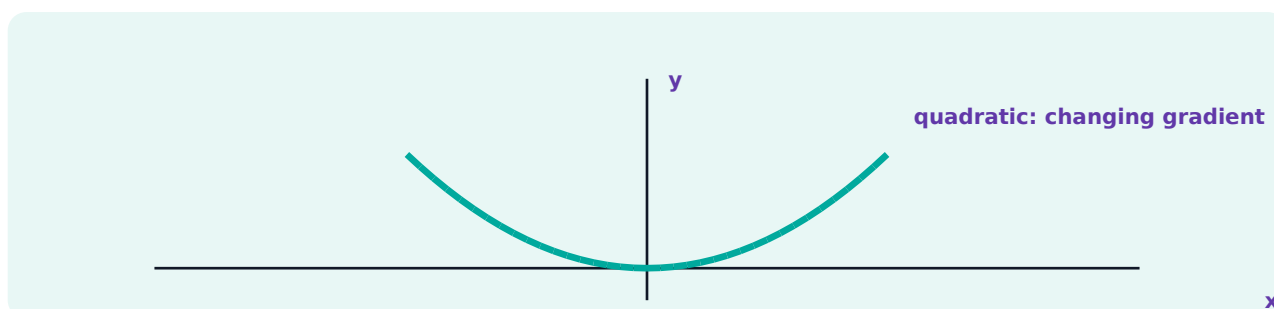
Pearson Edexcel content references A12, A13. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Higher Tier requires recognition of linear, quadratic, cubic, reciprocal and exponential shapes. Transformations follow coordinate effects:  $y=f(x)+a$  moves vertically,  $y=f(x-a)$  moves right,  $y=-f(x)$  reflects in the x-axis and  $y=f(-x)$  reflects in the y-axis. Horizontal changes act opposite to the sign inside the function.

## KEY VOCABULARY

cubic • reciprocal • exponential • asymptote • translation • reflection • invariant



## Why the method works

- Outside addition changes y-coordinates.
- Replacing x by  $x-a$  shifts the graph right by a.
- Negating the function reflects outputs in the x-axis.
- Reciprocal graphs approach asymptotes without meeting them.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Describe  $y=f(x)-2$ . 2) Describe  $y=f(x-4)$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 10 • A12-A13 • METHODS

A12, A13

# Cubic, reciprocal, exponential and transformed graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Describe  $y=f(x)+5$ .

1. Every y-value increases by 5.

**ANSWER:** Translation vector (0,5)

### WORKED EXAMPLE 2

Describe  $y=f(x+3)$ .

1. Input  $-3$  now produces the old input 0 position.

**ANSWER:** Translation 3 left

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Describe  $y=-f(x)$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 10 • A12-A13 • APPLICATION

A12, A13

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Graph transformations model changing baselines, delays and sign conventions without recomputing an entire relationship.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain opposite horizontal sign. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 10 • A12-A13 • PRACTICE

A12, A13

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Describe  $y=f(x)-2$ .

---

---

2. Describe  $y=f(x-4)$ .

---

---

3. Describe  $y=-f(x)$ .

---

---

4. Describe  $y=f(-x)$ .

---

---

5. Name graph  $y=x^3$ .

---

---

6. State asymptotes of  $y=1/x$ .

---

---

7. Does  $y=2^x$  cross y-axis where?

---

---

8. Explain opposite horizontal sign.

---

---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 11 • A14-A15

A14, A15

# Real-context graphs, gradients and areas under graphs

Interpret slope and accumulated area using the units of both axes.

## SPECIFICATION FOCUS

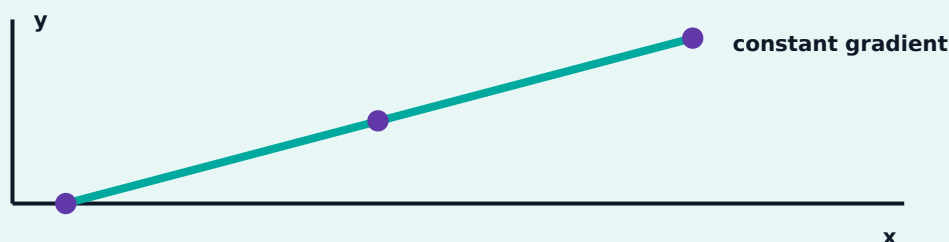
Pearson Edexcel content references A14, A15. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

A graph in context is interpreted through axis units. Gradient represents vertical-unit change per horizontal unit. On distance-time graphs it is speed; on velocity-time graphs it is acceleration. Area under a velocity-time graph represents displacement because velocity $\times$ time has distance units. Chords estimate average rate and tangents estimate instantaneous rate.

## KEY VOCABULARY

rate of change • chord • tangent • instantaneous • velocity • acceleration • displacement



## Why the method works

- Gradient units are vertical units divided by horizontal units.
- A chord gives average rate across an interval.
- A tangent gives instantaneous rate at one point.
- Signed area under velocity-time graph gives displacement.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Distance 150 km in 2.5 h: graph gradient. 2) Velocity 5 to 17 over 4s: acceleration. 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 11 • A14-A15 • METHODS

A14, A15

# Real-context graphs, gradients and areas under graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**Velocity rises linearly 4 to 16 m/s over 6 s. Find acceleration.**

1. Gradient =  $(16 - 4) / 6$ .

**ANSWER: 2 m/s<sup>2</sup>**

### WORKED EXAMPLE 2

**Velocity is 12 m/s for 8 s. Find displacement.**

1. Area rectangle =  $12 \times 8$ .

**ANSWER: 96 m**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Constant velocity 9 for 12s: displacement. First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 11 • A14-A15 • APPLICATION

A14, A15

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Transport and process-control graphs distinguish rate from accumulated quantity; units reveal the correct interpretation.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain area units  $m/s \times s$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 11 • A14-A15 • PRACTICE

A14, A15

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Distance 150 km in 2.5 h: graph gradient.

---

---

2. Velocity 5 to 17 over 4s: acceleration.

---

---

3. Constant velocity 9 for 12s: displacement.

---

---

4. Triangle velocity graph base 10, height 8: area.

---

---

5. Gradient of horizontal distance-time section.

---

---

6. Meaning negative velocity-time area.

---

---

7. Chord versus tangent.

---

---

8. Explain area units  $\text{m/s} \times \text{s}$ .

---

---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 12 • A16

## A16

# Circle equations and tangents

Use radius geometry to connect coordinate algebra and perpendicular lines.

## SPECIFICATION FOCUS

Pearson Edexcel content references A16. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

The circle centred at the origin with radius  $r$  has equation  $x^2+y^2=r^2$  by Pythagoras. A point lies on it when its coordinates satisfy the equation. The tangent at a point is perpendicular to the radius through that point, so its gradient is the negative reciprocal of the radius gradient.

## KEY VOCABULARY

circle • centre • radius • tangent • perpendicular • equation

Understand

Choose

Calculate

Check

Interpret

A reliable solution is a chain of decisions, not only a calculator display.

## Why the method works

- $x^2+y^2$  is the squared distance from the origin.
- The radius to a tangent point is perpendicular to the tangent.
- A coordinate satisfies the circle equation exactly when it lies on the circle.
- Gradient methods require care for horizontal or vertical radii.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Radius of  $x^2+y^2=49$ . 2) Does (5,12) lie on radius13 circle? 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 12 • A16 • METHODS

# Circle equations and tangents — method and examples <sup>A16</sup>

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Does (6,8) lie on  $x^2+y^2=100$ ?

$$1. 6^2+8^2=36+64=100.$$

ANSWER: Yes

### WORKED EXAMPLE 2

Find tangent gradient at (3,4) on  $x^2+y^2=25$ .

1. Radius gradient= $4/3$ .
2. Tangent gradient= $-3/4$ .

ANSWER:  $-3/4$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Find  $y$  if  $(8,y)$  on  $x^2+y^2=100, y>0$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 12 • A16 • APPLICATION

A16

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Circular tracks and rotating systems combine exact distance equations with tangent direction.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain Pythagoras derivation. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 12 • A16 • PRACTICE

A16

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Radius of  $x^2+y^2=49$ .

---



---

2. Does (5,12) lie on radius13 circle?

---



---

3. Find y if (8,y) on  $x^2+y^2=100,y>0$ .

---



---

4. Radius gradient to  $(-2,5)$ .

---



---

5. Tangent gradient there.

---



---

6. Tangent at (0,7): orientation.

---



---

7. Write circle radius  $\sqrt{30}$ .

---



---

8. Explain Pythagoras derivation.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 13 • A17, A21

A17, A21

# Linear equations and algebraic modelling

Clear structure efficiently and interpret the resulting value in context.

## SPECIFICATION FOCUS

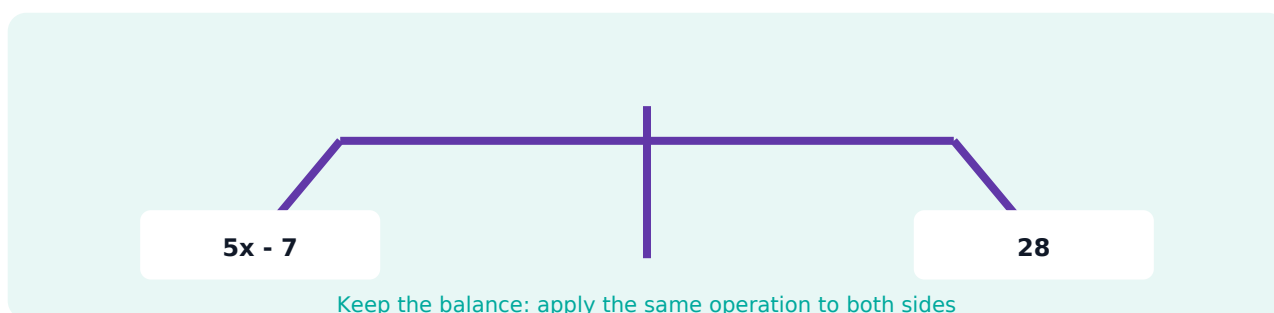
Pearson Edexcel content references A17, A21. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Linear equations may contain brackets, fractions and an unknown on both sides. Multiplying through by the lowest common denominator clears fractions while preserving equivalence. A contextual model begins by defining variables and translating each relationship before solving. Solutions must satisfy units, integer requirements and physical constraints.

## KEY VOCABULARY

linear equation • denominator • equivalent • model • constraint • verify



## Why the method works

- Multiplying every term by a common denominator preserves equality.
- Expanding precedes collection of like terms.
- A variable definition and unit make an equation interpretable.
- Substitution into the original verifies the result.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Solve  $5x - 7 = 3x + 19$ . 2) Solve  $4(2x - 3) = 5x + 9$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 13 • A17, A21 • METHODS

A17, A21

# Linear equations and algebraic modelling — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Solve  $(2x-1)/3 - (x+4)/2 = 5$ .

1. Multiply by 6:  $2(2x-1) - 3(x+4) = 30$ .
2.  $4x - 2 - 3x - 12 = 30$ .

**ANSWER:  $x=44$**

### WORKED EXAMPLE 2

Rectangle length  $x+3$ , width  $x-1$ , perimeter 40. Find dimensions.

1.  $2(x+3) + 2(x-1) = 40$ .
2.  $4x + 4 = 40$ , so  $x=9$ .

**ANSWER: 12 by 8**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Solve  $(x+1)/4 = 3$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 13 • A17, A21 • APPLICATION

A17, A21

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Pricing, mixtures and geometry become solvable when prose is converted into one consistent equation.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain clearing denominators. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 13 • A17, A21 • PRACTICE

A17, A21

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Solve  $5x - 7 = 3x + 19$ .

---



---

2. Solve  $4(2x - 3) = 5x + 9$ .

---



---

3. Solve  $(x + 1)/4 = 3$ .

---



---

4. Solve  $x/3 + (x - 1)/2 = 7$ .

---



---

5. Form equation: three consecutive integers sum 96.

---



---

6. Solve them.

---



---

7. Reject  $x = -4$  if  $x$  is length: why?

---



---

8. Explain clearing denominators.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 14 • A18

A18

# Quadratic equations: factorisation and the formula

Select a method from structure and retain both valid roots.

## SPECIFICATION FOCUS

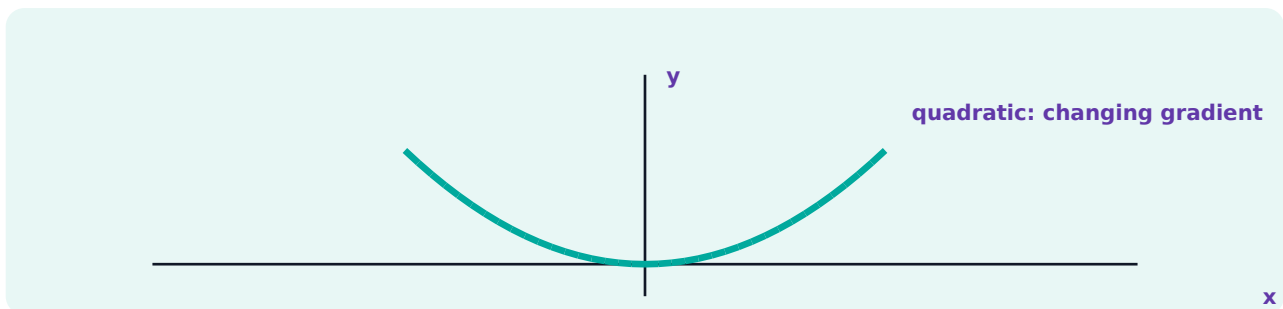
Pearson Edexcel content references A18. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Quadratic equations can be solved by factorisation, completing the square, the quadratic formula or graphically. The formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  applies to  $ax^2 + bx + c = 0$ . The discriminant controls the number and exact form of real solutions. Exact surd answers should be preserved unless decimals are requested.

## KEY VOCABULARY

quadratic formula • discriminant • root • exact solution • factorise



## Why the method works

- Factorisation is fastest when integer factors are visible.
- The  $\pm$  symbol represents two branches.
- A negative discriminant gives no real solutions.
- Substitution or sum/product checks validate roots.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Solve  $x^2 - 7x + 12 = 0$ . 2) Solve  $3x^2 + x - 2 = 0$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 14 • A18 • METHODS

A18

# Quadratic equations: factorisation and the formula — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Solve  $2x^2 - 5x - 3 = 0$ .

1. Formula or factor:  $(2x+1)(x-3)=0$ .

**ANSWER:**  $x=3$  or  $-1/2$

### WORKED EXAMPLE 2

Solve  $x^2 - 4x - 1 = 0$  exactly.

1.  $x = [4 \pm \sqrt{(16+4)}]/2$ .
2. Simplify  $\sqrt{20} = 2\sqrt{5}$ .

**ANSWER:**  $x = 2 \pm \sqrt{5}$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Solve  $x^2 + 6x + 1 = 0$  exactly. First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 14 • A18 • APPLICATION

A18

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Trajectory and design equations may give two mathematical times or dimensions; context determines which are usable.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain  $\pm$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 14 • A18 • PRACTICE

A18

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Solve  $x^2 - 7x + 12 = 0$ .

---



---

2. Solve  $3x^2 + x - 2 = 0$ .

---



---

3. Solve  $x^2 + 6x + 1 = 0$  exactly.

---



---

4. Discriminant  $2x^2 - 4x + 5$ .

---



---

5. Number real roots previous.

---



---

6. Solve  $4x^2 = 25$ .

---



---

7. Check roots sum for  $2x^2 - 5x - 3$ .

---



---

8. Explain  $\pm$ .

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 15 • A11, A18

A11, A18

# Completing the square and optimisation

Expose the turning point and solve quadratics through square structure.

## SPECIFICATION FOCUS

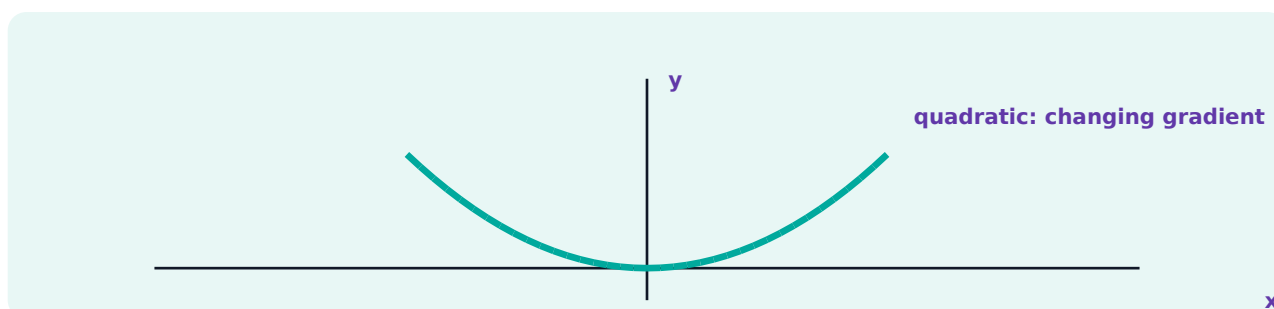
Pearson Edexcel content references A11, A18. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Completing the square rewrites  $x^2+bx+c$  as  $(x+b/2)^2$  plus an adjustment. For  $ax^2+bx+c$ , factor  $a$  from the  $x$  terms first. The completed form reveals turning point, symmetry line and minimum or maximum value, and can solve equations that do not factorise neatly.

## KEY VOCABULARY

complete square • turning point • minimum • maximum • symmetry • optimisation



## Why the method works

- Half the  $x$  coefficient enters the bracket.
- The added square must be compensated outside.
- $a(x-h)^2+k$  has turning point  $(h,k)$ .
- The sign of  $a$  determines minimum or maximum.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Complete  $x^2+6x+1$ . 2) Complete  $x^2-10x+7$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 15 • A11, A18 • METHODS

A11, A18

# Completing the square and optimisation — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Write  $x^2+8x+3$  in completed-square form.

1.  $(x+4)^2=x^2+8x+16$ .
2. Subtract 13.

**ANSWER:**  $(x+4)^2-13$

### WORKED EXAMPLE 2

Find minimum of  $2x^2-12x+25$ .

1.  $2(x^2-6x)+25=2[(x-3)^2-9]+25$ .
2.  $=2(x-3)^2+7$ .

**ANSWER:** minimum 7 at  $x=3$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Turning point  $y=(x-2)^2-5$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 15 • A11, A18 • APPLICATION

A11, A18

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Optimisation problems use the vertex to find best dimensions, minimum cost or maximum height.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain sign of  $a$ . Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 15 • A11, A18 • PRACTICE

A11, A18

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Complete  $x^2+6x+1$ .

---

---

2. Complete  $x^2-10x+7$ .

---

---

3. Turning point  $y=(x-2)^2-5$ .

---

---

4. Minimum of  $x^2+4x+9$ .

---

---

5. Complete  $3x^2+12x+1$ .

---

---

6. Solve  $x^2+4x-1=0$  via square.

---

---

7. Axis symmetry  $y=2(x+5)^2-3$ .

---

---

8. Explain sign of a.

---

---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 16 • A19

## A19

# Simultaneous equations including linear-quadratic systems

Substitute the linear relationship and expect multiple intersections.

## SPECIFICATION FOCUS

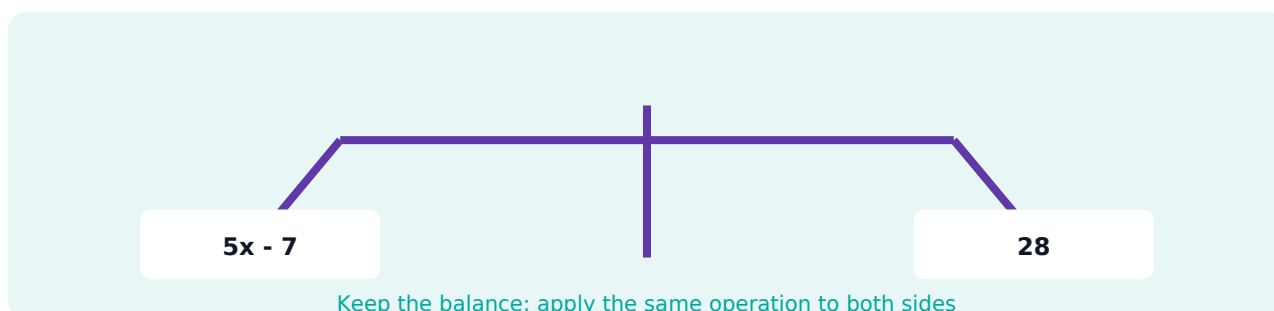
Pearson Edexcel content references A19. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Two equations are simultaneous when the same values satisfy both. Linear pairs use elimination or substitution. A linear-quadratic system commonly substitutes the linear equation into the quadratic, producing up to two solution pairs corresponding to graph intersections. Every pair must be checked in both originals.

## KEY VOCABULARY

simultaneous • eliminate • substitute • intersection • solution pair



## Why the method works

- Each solution pair is an intersection of the two graphs.
- Substitution must replace every occurrence with brackets.
- A line can meet a parabola zero, one or two times.
- Back-substitution pairs each x-value with its correct y-value.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Solve  $x+y=10, x-y=4$ . 2) Solve  $2x+3y=13, x+y=5$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 16 • A19 • METHODS

A19

# Simultaneous equations including linear-quadratic systems — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Solve  $y=x+1$  and  $y=x^2-5$ .

- Set  $x+1=x^2-5$ :  $x^2-x-6=0$ .
- $(x-3)(x+2)=0$ .
- Use  $y=x+1$ .

**ANSWER: (3,4) and (-2,-1)**

### WORKED EXAMPLE 2

Solve  $2x+y=7$  and  $x-y=2$ .

- Add equations:  $3x=9$ .
- $x=3, y=1$ .

**ANSWER: (3,1)**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Solve  $y=2x, y=x^2-3$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 16 • A19 • APPLICATION

A19

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Break-even analysis and geometry intersections require all simultaneous solution pairs and contextual filtering.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain graphical meaning. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 16 • A19 • PRACTICE

A19

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Solve  $x+y=10, x-y=4$ .

---



---

2. Solve  $2x+3y=13, x+y=5$ .

---



---

3. Solve  $y=2x, y=x^2-3$ .

---



---

4. Solve  $y=x-2, y=x^2-8$ .

---



---

5. How many intersections possible line/parabola?

---



---

6. Check (2,4) in  $y=2x, y=x^2$ .

---



---

7. Why brackets in substitution?

---



---

8. Explain graphical meaning.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 17 • A20

A20

# Iteration and numerical solutions

Use a recurrence to approach a fixed point and report justified accuracy.

## SPECIFICATION FOCUS

Pearson Edexcel content references A20. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Some equations lack a convenient exact algebraic solution. Rearranging into  $x=g(x)$  creates an iterative rule  $x_{n+1}=g(x_n)$ . Starting from a given value, repeated substitution may converge to a fixed point. Convergence is not automatic; the question normally supplies a suitable rearrangement and starting value.

## KEY VOCABULARY

iteration • recurrence • fixed point • convergence • divergence • approximation

7

11

15

19

...

+4 each time -> nth term  $4n + 3$

## Why the method works

- A fixed point satisfies  $x=g(x)$ .
- Each new value uses the previous value without premature rounding.
- Agreement of successive values supports stated decimal accuracy.
- Different rearrangements can converge at different rates or diverge.

## CHECK BEFORE MOVING ON

Answer mentally: 1)  $x_{n+1}=(x_n+5)/3, x_1=2$ :  $x_2$ . 2) Then  $x_3$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 17 • A20 • METHODS

A20

# Iteration and numerical solutions — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Use  $x_{n+1} = \sqrt{10 - x_n}$ ,  $x_1 = 3$ . Find  $x_2, x_3$ .

1.  $x_2 = \sqrt{10 - 3} = 2.64575\dots$
2.  $x_3 = \sqrt{10 - 2.64575\dots} = 2.71187\dots$

**ANSWER: 2.646, 2.712**

### WORKED EXAMPLE 2

Values converge 2.70154, 2.70156. Give 3 d.p.

1. Both round to same 3 d.p.

**ANSWER: 2.702**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Fixed point of  $x = (x+5)/3$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 17 • A20 • APPLICATION

A20

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Numerical solvers in science and computing use iteration; convergence and stopping criteria matter as much as repeated calculation.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

State one convergence warning. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 17 • A20 • PRACTICE

A20

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1.  $x_{n+1} = (x_n + 5)/3, x_1 = 2$ :  $x_2$ .

---



---

2. Then  $x_3$ .

---



---

3. Fixed point of  $x = (x + 5)/3$ .

---



---

4. If values alternate growing magnitude, describe.

---



---

5. Why retain calculator accuracy?

---



---

6. Values 1.41420, 1.41422: 3 d.p.

---



---

7. Check fixed point 2.5 in rule  $(x + 5)/3$ .

---



---

8. State one convergence warning.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 18 • A22

A22

# Linear and quadratic inequalities

Use sign, critical values and interval testing to describe a solution region.

## SPECIFICATION FOCUS

Pearson Edexcel content references A22. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

A linear inequality is manipulated like an equation except multiplication or division by a negative reverses the sign. A quadratic inequality is solved by finding roots, sketching or analysing factor signs, then selecting intervals where the expression is positive or negative. Endpoints are included only for  $\leq$  or  $\geq$ .

## KEY VOCABULARY

inequality • critical value • interval • sign diagram • inclusive • strict

Position shows order: farther right means greater



## Why the method works

- Roots divide the number line into intervals of constant factor sign.
- An upward parabola is positive outside its roots and negative between them.
- A negative leading coefficient reverses that pattern.
- Multiplying an inequality by a negative reverses order.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Solve  $3x - 7 > 11$ . 2) Solve  $-2x \leq 8$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 18 • A22 • METHODS

A22

# Linear and quadratic inequalities — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Solve  $x^2 - 5x + 6 < 0$ .

1. Factor  $(x-2)(x-3)$ .
2. Upward parabola negative between roots.

**ANSWER:**  $2 < x < 3$

### WORKED EXAMPLE 2

Solve  $4 - x^2 \geq 0$ .

1.  $(2-x)(2+x) \geq 0$ .
2. Expression non-negative between roots including endpoints.

**ANSWER:**  $-2 \leq x \leq 2$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Solve  $x^2 - 9 > 0$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 18 • A22 • APPLICATION

A22

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Feasible design and profit regions often require intervals rather than one value.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain endpoint inclusion. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 18 • A22 • PRACTICE

A22

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Solve  $3x - 7 > 11$ .

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2. Solve  $-2x \leq 8$ .

---



---

3. Solve  $x^2 - 9 > 0$ .

---



---

4. Solve  $x^2 + x - 6 \leq 0$ .

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---

5. Solve  $(x - 1)(x + 5) \geq 0$ .

---



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6. Represent  $x < 2$  or  $x \geq 7$ .

---



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7. Why test intervals?

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8. Explain endpoint inclusion.

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 19 • A23-A25

A23, A24, A25

# Arithmetic, geometric and quadratic sequences

Use differences, ratios and position rules to identify and generalise patterns.

## SPECIFICATION FOCUS

Pearson Edexcel content references A23, A24, A25. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

Arithmetic sequences have constant first difference and linear  $n$ th term. Geometric sequences have constant ratio and form  $ar^{n-1}$ . Quadratic sequences have constant second difference; if  $n$ th term is  $an^2+bn+c$ , the second difference is  $2a$ . Fibonacci-type sequences use earlier terms rather than position alone.

## KEY VOCABULARY

arithmetic • geometric • quadratic • first difference • second difference • recurrence

7

11

15

19

...

+4 each time ->  $n$ th term  $4n + 3$

## Why the method works

- Constant first difference implies a linear  $n$ th term.
- Constant ratio implies geometric multiplication.
- Constant second difference  $2a$  identifies the  $n^2$  coefficient.
- A candidate  $n$ th term is verified against several positions.

## CHECK BEFORE MOVING ON

Answer mentally: 1)  $n$ th term 7,12,17,... 2)  $n$ th term 20,16,12,... 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 19 • A23-A25 • METHODS

A23, A24, A25

# Arithmetic, geometric and quadratic sequences — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

Find  $n$ th term 3,10,21,36,55.

1. First differences 7,11,15,19; second difference 4, so  $a=2$ .
2. Use  $2n^2+bn+c$  and first terms.

**ANSWER:**  $2n^2+n$

### WORKED EXAMPLE 2

Find  $n$ th term 5,15,45,135.

1. Ratio 3, first term 5.

**ANSWER:**  $5 \times 3^{n-1}$

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Next two 2,6,18,54. First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 19 • A23-A25 • APPLICATION

A23, A24, A25

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Population and finance patterns can be linear, geometric or quadratic; identifying the type prevents an inappropriate forecast.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

Explain difference between recurrence and nth term. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 19 • A23-A25 • PRACTICE

A23, A24, A25

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Nth term 7,12,17,... .

---



---

2. Nth term 20,16,12,... .

---



---

3. Next two 2,6,18,54.

---



---

4. Nth term previous geometric.

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---

5. Second difference if term  $3n^2$ .

---



---

6. Find nth term 4,11,22,37,56.

---



---

7. Is 202 in sequence  $3n^2+2$ ?

---



---

8. Explain difference between recurrence and nth term.

---



---

## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## CHAPTER 20 • A1-A25

## A1-A25 synthesis

# Higher Algebra modelling and connected problem solving

Select representations, preserve restrictions and communicate a defensible conclusion.

## SPECIFICATION FOCUS

Pearson Edexcel content references A1-A25 synthesis. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

## Understand the idea

A sustained Higher problem may combine functions, equations, graphs, proof and inequalities. Begin by defining variables and restrictions, select the representation that exposes the structure, and maintain equivalence through every step. Exact and approximate answers must be distinguished, and every surviving solution interpreted in context.

## KEY VOCABULARY

model • assumption • restriction • exact • approximate • proof • verify

Understand

Choose

Calculate

Check

Interpret

A reliable solution is a chain of decisions, not only a calculator display.

## Why the method works

- A model states how variables relate and what assumptions make that relationship plausible.
- Algebraic forms reveal different features: factors show roots, completed squares show vertices.
- Extraneous or context-invalid solutions must be rejected explicitly.
- A complete answer checks both mathematics and meaning.

## CHECK BEFORE MOVING ON

Answer mentally: 1) Solve  $x + 12/x = 7$ ,  $x \neq 0$ . 2) Find roots  $x^2 - 7x + 12$ . 3) State one definition or rule from this page in your own words.

## EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

## CHAPTER 20 • A1-A25 • METHODS

## A1-A25 synthesis

# Higher Algebra modelling and connected problem solving — method and examples

Follow the reasoning; do not copy steps mechanically.

## Reliable method

- Identify the algebraic structure and any restrictions.
- Choose a transformation that preserves equivalence.
- Show each manipulation on a complete mathematical line.
- Verify by expansion, substitution, graph or contextual check.

### WORKED EXAMPLE 1

**A rectangle has area 48 and length 2 more than width. Find dimensions.**

1. Let width  $x > 0$ ;  $x(x+2)=48$ .
2.  $x^2+2x-48=0=(x+8)(x-6)$ .
3. Reject negative width.

**ANSWER: 6 by 8**

### WORKED EXAMPLE 2

**$f(x)=x^2-4x-5$ . Find exact minimum and roots.**

1. Complete square:  $(x-2)^2-9$ , minimum  $-9$ .
2. Factor:  $(x-5)(x+1)$ .

**ANSWER: minimum  $-9$ ; roots  $5, -1$**

### SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

### GUIDED TRY

Complete square  $x^2-6x+2$ . First predict the form or approximate size of the answer; then show a complete method.

## CHAPTER 20 • A1-A25 • APPLICATION

## A1-A25 synthesis

# Apply and reason

Connect the method to a context and communicate the conclusion.

## Real-world model

### CONTEXT

Optimisation, pricing and scientific models reward a chain of justified decisions rather than isolated procedures.

## Common mistakes and how to repair them

### MISTAKE 1

Applying a rule to terms that do not satisfy its conditions. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 2

Losing a sign, bracket or excluded value during manipulation. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

### MISTAKE 3

Giving solutions without checking the original relationship. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

## Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

### REASONING PROMPT

State two final model checks. Write one sentence explaining why your calculation or conclusion is valid.

## CHAPTER 20 • A1-A25 • PRACTICE

## A1-A25 synthesis

# Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Solve  $x + 12/x = 7$ ,  $x \neq 0$ .

---



---

2. Find roots  $x^2 - 7x + 12$ .

---



---

3. Complete square  $x^2 - 6x + 2$ .

---



---

4. Line through (1,4), gradient -2.

---



---

5. Solve  $x^2 - 4x - 5 \geq 0$ .

---



---

6. Find inverse  $f = 3x + 8$ .

---



---

7. Quadratic sequence second difference 8: coefficient  $n^2$ .

---



---

8. State two final model checks.

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## REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

## PRACTICE ANSWERS

## CHAPTER 01

# Answers, method guidance and repair notes

## Algebraic notation, structure and substitution

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. State coefficient of $x^2$ in $7-5x^2$ .

**ANSWER: -5**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. How many terms in $3x^2-2xy+y$ ?

**ANSWER: 3**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Evaluate $a^2/b$ for $a=-6, b=9$ .

**ANSWER: 4**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Evaluate $\sqrt{p+5}$ for $p=11$ .

**ANSWER: 4**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Classify $4x-1=15$ .

**ANSWER: equation**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Classify $V=\pi r^2 h$ .

**ANSWER: formula**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Substitute $x=-3$ into $(x-2)^2$ .

**ANSWER: 25**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain why $-3^2$ differs from $(-3)^2$ .

**ANSWER: power acts before leading negative unless bracketed**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 02

# Answers, method guidance and repair notes

## Collecting terms and algebraic index laws

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Simplify $7x^2 - 3x + 5x^2 + x$ .

**ANSWER:  $12x^2 - 2x$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Simplify $4a^3 \times 3a^5$ .

**ANSWER:  $12a^8$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Simplify $18p^7 / (6p^2)$ .

**ANSWER:  $3p^5$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Simplify $(2x^3)^4$ .

**ANSWER:  $16x^{12}$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Write $y^{-4}$ with positive index.

**ANSWER:  $1/y^4$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Simplify $a^3b^{-2} \times a^{-1}b^5$ .

**ANSWER:  $a^2b^3$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Evaluate $x^0$ when $x \neq 0$ .

**ANSWER: 1**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain why $x^2 + x^3$ cannot collect.

**ANSWER: different powers are unlike terms**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 03

# Answers, method guidance and repair notes

## Expanding products and factorising quadratics

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Expand $(x-4)(x+7)$ .

**ANSWER:  $x^2+3x-28$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Expand $(3x+2)(2x-5)$ .

**ANSWER:  $6x^2-11x-10$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Factorise $x^2+3x-28$ .

**ANSWER:  $(x+7)(x-4)$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Factorise $2x^2+7x+3$ .

**ANSWER:  $(2x+1)(x+3)$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Factorise $12x^2-27$ .

**ANSWER:  $3(2x-3)(2x+3)$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Factorise $5x^2-20x$ .

**ANSWER:  $5x(x-4)$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Expand $(x+3)^2$ .

**ANSWER:  $x^2+6x+9$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain difference of squares.

**ANSWER: middle terms cancel in conjugates**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 04

# Answers, method guidance and repair notes

## Algebraic fractions and domain restrictions

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Simplify $6x^2/(9x)$ .

**ANSWER:  $2x/3$ ,  $x \neq 0$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Simplify $(x^2-16)/(x-4)$ .

**ANSWER:  $x+4$ ,  $x \neq 4$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Simplify $(x^2+5x+6)/(x+2)$ .

**ANSWER:  $x+3$ ,  $x \neq -2$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Simplify $3/x-1/(2x)$ .

**ANSWER:  $5/(2x)$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Add $1/(x+1)+2/(x+1)$ .

**ANSWER:  $3/(x+1)$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. State restrictions for $(x+2)/(x^2-4)$ .

**ANSWER:  $x \neq -2, 2$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Why cannot $x$ cancel in $(x+3)/x$ ?

**ANSWER:  $x$  is not factor of numerator**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Simplify $(2x^2-8)/(x^2-4)$ .

**ANSWER:  $2$ ,  $x \neq -2, 2$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 05

# Answers, method guidance and repair notes

## Rearranging complex formulae

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Make $x$ subject: $y=5x-8$ .

**ANSWER:  $x=(y+8)/5$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Make $h$ subject: $A=\pi r^2+2rh$ .

**ANSWER:  $h=(A-\pi r^2)/(2r)$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Make $x$ subject: $y=1/x+3$ .

**ANSWER:  $x=1/(y-3)$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Make $a$ subject: $v^2=u^2+2as$ .

**ANSWER:  $a=(v^2-u^2)/(2s)$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Make $t$ subject: $s=ut+\frac{1}{2}at^2$ when $u=0$ .

**ANSWER:  $t=\sqrt{(2s/a)}$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Make $x$ subject: $p=qx/(x+r)$ .

**ANSWER:  $x=pr/(q-p)$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Make $r$ subject: $A=\pi r^2$ , $r>0$ .

**ANSWER:  $r=\sqrt{(A/\pi)}$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain why $\pm$ may appear.

**ANSWER: square has two inverse branches**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 06

# Answers, method guidance and repair notes

## Identities and algebraic proof

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Show $even \times integer$ is even.

**ANSWER:  $2mn$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Show $odd + odd$ is even.

**ANSWER:  $2(m+n+1)$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Show three consecutive integers sum to multiple of 3.

**ANSWER:  $3(n+1)$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Prove difference of consecutive squares is odd.

**ANSWER:  $(n+1)^2 - n^2 = 2n+1$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Disprove $x^2 \geq x$ for every real $x$ .

**ANSWER:  $x = 1/2$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Show $(n+1)^2 - n^2 = 2n+1$ .

**ANSWER: shown by expansion**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Is $2(x+3) \equiv 2x+6$ ?

**ANSWER: yes**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain role of counterexample.

**ANSWER: disproves universal claim**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 07

# Answers, method guidance and repair notes

## Functions, composites and inverses

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. $f(x)=2x+3$ : $f(5)$ .

**ANSWER: 13**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. $g(x)=x^2-1$ : $g(-3)$ .

**ANSWER: 8**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Find $fg(x)$ for $f=2x+3$ , $g=x-4$ .

**ANSWER:  $2x-5$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Find $gf(x)$ .

**ANSWER:  $2x-1$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Inverse of $f(x)=4x-7$ .

**ANSWER:  $(x+7)/4$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Solve $f(x)=25$ for $f=3x+1$ .

**ANSWER: 8**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Why restrict $y=x^2$ for inverse?

**ANSWER: two inputs can share output**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Check $f^{-1}(f(6))$ for $f=2x-5$ .

**ANSWER: 6**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 08

# Answers, method guidance and repair notes

## Coordinates, gradient and equations of straight lines

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Gradient through $(-2,7),(4,-5)$ .

**ANSWER:  $-2$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Equation gradient 5, intercept $-3$ .

**ANSWER:  $y=5x-3$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Equation through $(0,4),(3,10)$ .

**ANSWER:  $y=2x+4$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Parallel to $y=-2x+1$ through $(1,5)$ .

**ANSWER:  $y=-2x+7$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Perpendicular to $y=\frac{1}{2}x+3$ gradient.

**ANSWER:  $-2$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Does $(4,9)$ lie on $y=2x+1$ ?

**ANSWER: yes**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Find x-intercept $y=3x-12$ .

**ANSWER: 4**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain negative reciprocal.

**ANSWER: product of perpendicular gradients  $-1$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 09

# Answers, method guidance and repair notes

## Quadratic graphs, roots and turning points

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Roots of $y=(x+1)(x-7)$ .

**ANSWER:  $-1, 7$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Symmetry line for roots $-1, 7$ .

**ANSWER:  $x=3$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Turning point $y=(x+4)^2-9$ .

**ANSWER:  $(-4, -9)$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. y-intercept $y=2x^2-3x+5$ .

**ANSWER: 5**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Discriminant $x^2+4x+8$ .

**ANSWER:  $-16$ , no real roots**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Number roots $3x^2-5x-2$ .

**ANSWER: two**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Solve $x^2-9=0$ graphically/algebraically.

**ANSWER:  $-3, 3$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain root meaning.

**ANSWER: x-value where  $y=0$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 10

# Answers, method guidance and repair notes

## Cubic, reciprocal, exponential and transformed graphs

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Describe $y=f(x)-2$ .

**ANSWER: 2 down**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Describe $y=f(x-4)$ .

**ANSWER: 4 right**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Describe $y=-f(x)$ .

**ANSWER: reflection x-axis**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Describe $y=f(-x)$ .

**ANSWER: reflection y-axis**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Name graph $y=x^3$ .

**ANSWER: cubic**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. State asymptotes of $y=1/x$ .

**ANSWER:  $x=0, y=0$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Does $y=2^x$ cross y-axis where?

**ANSWER: (0,1)**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain opposite horizontal sign.

**ANSWER: input shift must compensate**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 11

# Answers, method guidance and repair notes

## Real-context graphs, gradients and areas under graphs

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Distance 150 km in 2.5 h: graph gradient.

**ANSWER: 60 km/h**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Velocity 5 to 17 over 4s: acceleration.

**ANSWER: 3 m/s<sup>2</sup>**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Constant velocity 9 for 12s: displacement.

**ANSWER: 108 m**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Triangle velocity graph base 10, height 8: area.

**ANSWER: 40 m**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Gradient of horizontal distance-time section.

**ANSWER: 0, stationary**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Meaning negative velocity-time area.

**ANSWER: negative displacement**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Chord versus tangent.

**ANSWER: average versus instantaneous rate**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain area units m/s x s.

**ANSWER: metres**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 12

# Answers, method guidance and repair notes

## Circle equations and tangents

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Radius of $x^2+y^2=49$ .

**ANSWER: 7**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Does (5,12) lie on radius13 circle?

**ANSWER: yes**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Find y if (8,y) on $x^2+y^2=100, y>0$ .

**ANSWER: 6**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Radius gradient to $(-2,5)$ .

**ANSWER:  $-5/2$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Tangent gradient there.

**ANSWER:  $2/5$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Tangent at (0,7): orientation.

**ANSWER: horizontal**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Write circle radius $\sqrt{30}$ .

**ANSWER:  $x^2+y^2=30$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain Pythagoras derivation.

**ANSWER: coordinates form right triangle**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 13

# Answers, method guidance and repair notes

## Linear equations and algebraic modelling

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Solve $5x - 7 = 3x + 19$ .

**ANSWER: 13**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Solve $4(2x - 3) = 5x + 9$ .

**ANSWER: 7**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Solve $(x + 1)/4 = 3$ .

**ANSWER: 11**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Solve $x/3 + (x - 1)/2 = 7$ .

**ANSWER: 9**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Form equation: three consecutive integers sum 96.

**ANSWER:  $n + n + 1 + n + 2 = 96$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Solve them.

**ANSWER: 31, 32, 33**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Reject $x = -4$ if $x$ is length: why?

**ANSWER: negative length invalid**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain clearing denominators.

**ANSWER: multiplies every term equally**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 14

# Answers, method guidance and repair notes

## Quadratic equations: factorisation and the formula

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Solve $x^2 - 7x + 12 = 0$ .

**ANSWER: 3, 4**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Solve $3x^2 + x - 2 = 0$ .

**ANSWER:  $2/3, -1$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Solve $x^2 + 6x + 1 = 0$ exactly.

**ANSWER:  $-3 \pm 2\sqrt{2}$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Discriminant $2x^2 - 4x + 5$ .

**ANSWER:  $-24$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Number real roots previous.

**ANSWER: none**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Solve $4x^2 = 25$ .

**ANSWER:  $\pm 5/2$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Check roots sum for $2x^2 - 5x - 3$ .

**ANSWER:  $5/2$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain $\pm$ .

**ANSWER: square root has positive and negative branches**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 15

# Answers, method guidance and repair notes

## Completing the square and optimisation

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Complete $x^2+6x+1$ .

**ANSWER:  $(x+3)^2-8$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Complete $x^2-10x+7$ .

**ANSWER:  $(x-5)^2-18$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Turning point $y=(x-2)^2-5$ .

**ANSWER:  $(2,-5)$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Minimum of $x^2+4x+9$ .

**ANSWER: 5 at  $x=-2$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Complete $3x^2+12x+1$ .

**ANSWER:  $3(x+2)^2-11$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Solve $x^2+4x-1=0$ via square.

**ANSWER:  $-2\pm\sqrt{5}$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Axis symmetry $y=2(x+5)^2-3$ .

**ANSWER:  $x=-5$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain sign of a.

**ANSWER: positive minimum, negative maximum**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 16

# Answers, method guidance and repair notes

## Simultaneous equations including linear-quadratic systems

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Solve $x+y=10, x-y=4$ .

**ANSWER: (7,3)**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Solve $2x+3y=13, x+y=5$ .

**ANSWER: (2,3)**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Solve $y=2x, y=x^2-3$ .

**ANSWER:  $x=3, y=6$  or  $x=-1, y=-2$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Solve $y=x-2, y=x^2-8$ .

**ANSWER:  $x=3, y=1$  or  $x=-2, y=-4$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. How many intersections possible line/parabola?

**ANSWER: 0,1,2**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Check (2,4) in $y=2x, y=x^2$ .

**ANSWER: yes**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Why brackets in substitution?

**ANSWER: preserve complete expression**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain graphical meaning.

**ANSWER: common points**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 17

# Answers, method guidance and repair notes

## Iteration and numerical solutions

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1.  $x_{n+1} = (x_n + 5)/3, x_1 = 2: x_2.$

ANSWER: 7/3

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Then  $x_3.$

ANSWER: 22/9

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Fixed point of  $x = (x + 5)/3.$

ANSWER: 2.5

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. If values alternate growing magnitude, describe.

ANSWER: divergence

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Why retain calculator accuracy?

ANSWER: avoid rounding drift

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Values 1.41420, 1.41422: 3 d.p.

ANSWER: 1.414

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Check fixed point 2.5 in rule  $x = (x + 5)/3.$

ANSWER: yes

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. State one convergence warning.

ANSWER: not every rearrangement converges

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 18

# Answers, method guidance and repair notes

## Linear and quadratic inequalities

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Solve $3x - 7 > 11$ .

**ANSWER:  $x > 6$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Solve $-2x \leq 8$ .

**ANSWER:  $x \geq -4$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Solve $x^2 - 9 > 0$ .

**ANSWER:  $x < -3$  or  $x > 3$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Solve $x^2 + x - 6 \leq 0$ .

**ANSWER:  $-3 \leq x \leq 2$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Solve $(x - 1)(x + 5) \geq 0$ .

**ANSWER:  $x \leq -5$  or  $x \geq 1$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Represent $x < 2$ or $x \geq 7$ .

**ANSWER: two shaded rays**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Why test intervals?

**ANSWER: sign constant between roots**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain endpoint inclusion.

**ANSWER: equality allowed for  $\leq$  or  $\geq$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 19

# Answers, method guidance and repair notes

## Arithmetic, geometric and quadratic sequences

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Nth term 7,12,17,...

**ANSWER:  $5n+2$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Nth term 20,16,12,...

**ANSWER:  $24-4n$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Next two 2,6,18,54.

**ANSWER: 162,486**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Nth term previous geometric.

**ANSWER:  $2 \times 3^{n-1}$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Second difference if term $3n^2$ .

**ANSWER: 6**

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Find nth term 4,11,22,37,56.

**ANSWER:  $2n^2+n+1$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Is 202 in sequence $3n^2+2$ ?

**ANSWER: no,  $n^2=200/3$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. Explain difference between recurrence and nth term.

**ANSWER: previous terms versus direct position**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

## PRACTICE ANSWERS

## CHAPTER 20

# Answers, method guidance and repair notes

## Higher Algebra modelling and connected problem solving

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

### 1. Solve $x+12/x=7$ , $x \neq 0$ .

**ANSWER:  $x=3,4$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 2. Find roots $x^2-7x+12$ .

**ANSWER: 3,4**

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 3. Complete square $x^2-6x+2$ .

**ANSWER:  $(x-3)^2-7$** 

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 4. Line through (1,4), gradient -2.

**ANSWER:  $y=-2x+6$** 

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 5. Solve $x^2-4x-5 \geq 0$ .

**ANSWER:  $x \leq -1$  or  $x \geq 5$** 

Method guidance: Identify the algebraic structure and any restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 6. Find inverse $f=3x+8$ .

**ANSWER:  $(x-8)/3$** 

Method guidance: Choose a transformation that preserves equivalence. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 7. Quadratic sequence second difference 8: coefficient $n^2$ .

**ANSWER: 4**

Method guidance: Show each manipulation on a complete mathematical line. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

### 8. State two final model checks.

**ANSWER: restrictions, units, magnitude, assumptions**

Method guidance: Verify by expansion, substitution, graph or contextual check. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

EDITORIAL SIGN-OFF

PUBLICATION EDITION

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Unit 2: Algebra • Higher Tier

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