

GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 5 | PROBABILITY

TEACH-FROM-SCRATCH • FREE WEBSITE EDITION

17 detailed chapters • original diagrams • worked examples • guided practice • answers

Student: _____ Target grade: _____

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START HERE

STUDY ROUTINE

How to learn from this notebook

Meaning first. Method second. Practice third.

THE FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each solution and reproduce it. 3 Complete the practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final sentence when the result must be interpreted.

Error log

REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

Higher probability requires connected reasoning

Higher Tier rewards secure fluency and connected reasoning. Grade 4–5 questions often combine several familiar skills, hide the required operation inside a context, or ask whether an answer is reasonable. The notebook therefore explains why methods work as well as how to use them.

SPECIFICATION MAP

PEARSON EDEXCEL 1MA1

Unit 5 coverage map

Probability content organised into a teachable sequence.

CH 01	P3	Probability language and the 0-to-1 scale
CH 02	P2, P7	Theoretical probability and equally likely outcomes
CH 03	P6, P7	Sample spaces and systematic listing
CH 04	P4	Mutually exclusive events and complements
CH 05	P3, P5	Experimental probability and relative frequency
CH 06	P2	Expected frequency and prediction
CH 07	P6, P8	Combined events using tables and lists
CH 08	P8	Tree diagrams with and without replacement
CH 09	P1, P6	Two-way tables, frequency trees and Venn diagrams
CH 10	P9	Conditional probability: the meaning of given
CH 11	P1	Frequency trees and analysing experimental outcomes
CH 12	P6, P7	Counting principles, sets and systematic enumeration
CH 13	P4, P8	The general AND and OR rules
CH 14	P8, P9	Dependent probability trees without replacement
CH 15	P9	Conditional probability from two-way and frequency tables
CH 16	P6, P9	Venn diagrams, set notation and conditional regions
CH 17	P8, P9	Reverse probability, algebraic branches and model evaluation

IMPORTANT TIER NOTE

This notebook follows the complete Higher Tier content identified in the Pearson Edexcel GCSE Mathematics (1MA1) specification. Shared foundations and Higher-only P9 conditional-probability content are integrated into the sequence.

REFERENCE

Fast-reference method bank

PROBABILITY FACTS AND METHODS

Understand each formula before memorising it.

Probability range

$$0 \leq P(A) \leq 1$$

Equally likely model

$$\text{favourable outcomes} \div \text{total outcomes}$$

Complement

$$P(A') = 1 - P(A)$$

Experimental probability

$$\text{event frequency} \div \text{trials}$$

Expected frequency

$$\text{probability} \times \text{trials}$$

Independent A and B

$$P(A) \times P(B)$$

Mutually exclusive A or B

$$P(A) + P(B)$$

Tree route

$$\text{multiply along branches}$$

Several routes

$$\text{add terminal route probabilities}$$

Conditional probability

$$\text{required frequency} \div \text{restricted total}$$

CHAPTER 01 • P3

P3

Probability language and the 0-to-1 scale

Turn everyday likelihood into precise mathematical statements.

SPECIFICATION FOCUS

Pearson Edexcel content references P3. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Probability measures long-run likelihood from 0 to 1 inclusive. An impossible event has probability 0, a certain event has probability 1, and an even chance has probability 0.5. Fractions, decimals and percentages describe the same scale, but words such as “unlikely” describe only approximate regions.

KEY VOCABULARY

outcome • event • impossible • unlikely • even chance • likely • certain • probability



Why the method works

- Every probability lies in the interval $0 \leq P(A) \leq 1$.
- Events nearer 1 are more likely than events nearer 0.
- Probability describes uncertainty before an outcome; it does not guarantee what happens once.
- Equivalent fractions, decimals and percentages occupy the same position on the scale.

CHECK BEFORE MOVING ON

Answer mentally: 1) Write impossible event probability. 2) Write certain event probability. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 01 • P3 • METHODS

P3

Probability language and the 0-to-1 scale — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Identify the event precisely.
- Decide whether it is impossible, certain or between these extremes.
- Convert all given values to one representation before comparing.
- Use the context to reject impossible values below 0 or above 1.

WORKED EXAMPLE 1

Place probabilities 0.15, $\frac{3}{5}$ and 85% in increasing order.

1. Convert: $\frac{3}{5}=0.60$ and $85\%=0.85$.
2. Compare on the 0-to-1 scale.

ANSWER: 0.15, $\frac{3}{5}$, 85%

WORKED EXAMPLE 2

A forecast gives probability of rain 1.2. Is it valid?

1. A probability cannot exceed 1.

ANSWER: No

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Convert 0.35 to percentage. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 01 • P3 • APPLICATION

P3

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Weather, medical risk and quality control use probability to communicate uncertainty. “A 20% chance of rain” does not mean rain for exactly 20% of one day; it summarises the frequency expected across comparable forecasts.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why probability 1.04 is invalid. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 01 • P3 • PRACTICE

P3

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Write impossible event probability.

2. Write certain event probability.

3. Convert 0.35 to percentage.

4. Convert 72% to decimal.

5. Order $\frac{1}{4}$, 0.2, 30%.

6. Which is more likely: 0.62 or $\frac{3}{5}$?

7. Give a probability for an even chance.

8. Explain why probability 1.04 is invalid.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 02 • P2, P7

P2, P7

Theoretical probability and equally likely outcomes

Count favourable outcomes only after confirming fairness.

SPECIFICATION FOCUS

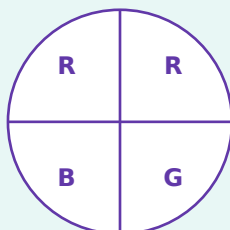
Pearson Edexcel content references P2, P7. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

For a finite set of equally likely outcomes, $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total outcomes}}$. The denominator counts all possible outcomes once; the numerator counts those satisfying the event. Fair coins, unbiased dice and equal spinner sectors support equal-likelihood models. Unequal sectors or biased devices require different probabilities.

KEY VOCABULARY

favourable • possible • equally likely • fair • unbiased • theoretical probability



equal sectors -> equally likely outcomes

Why the method works

- Equal counting works only when individual outcomes have equal probability.
- The favourable outcomes form a subset of the complete outcome set.
- Simplifying a fraction changes its form, not its probability.
- Symmetry can justify equal likelihood when the physical model is fair.

CHECK BEFORE MOVING ON

Answer mentally: 1) Fair die: $P(\text{odd})$. 2) Fair die: $P(\text{number} > 4)$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 02 • P2, P7 • METHODS

P2, P7

Theoretical probability and equally likely outcomes — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- List the complete outcome set.
- Check that outcomes are equally likely.
- Count favourable and total outcomes.
- Write, simplify and sense-check the fraction.

WORKED EXAMPLE 1

A fair die is rolled. Find $P(\text{multiple of 3})$.

1. Outcomes are 1,2,3,4,5,6.
2. Favourable outcomes are 3 and 6.
3. Probability= $\frac{2}{6}$.

ANSWER: $\frac{1}{3}$

WORKED EXAMPLE 2

A bag has 5 red, 3 blue and 2 green counters. Find $P(\text{not blue})$.

1. Total=10.
2. Not blue means red or green: $5+2=7$.

ANSWER: $\frac{7}{10}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Bag 4 red,6 blue: $P(\text{red})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 02 • P2, P7 • APPLICATION

P2, P7

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Games are fair only if the assigned prizes reflect the event probabilities. A spinner with differently sized sectors cannot be analysed by counting colours alone; sector angles or areas determine likelihood.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain when favourable/total is valid. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 02 • P2, P7 • PRACTICE

P2, P7

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Fair die: $P(\text{odd})$.

2. Fair die: $P(\text{number} > 4)$.

3. Bag 4 red, 6 blue: $P(\text{red})$.

4. Cards 1-10: $P(\text{prime})$.

5. Alphabet tiles A,A,B,C: $P(A)$.

6. Spinner equal sectors R,R,B,G: $P(\text{not R})$.

7. Can probability $\frac{3}{7}$ be based on 21 equally likely outcomes? Give favourable count.

8. Explain when favourable/total is valid.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 03 • P6, P7

P6, P7

Sample spaces and systematic listing

Organise every outcome once so the denominator is proved complete.

SPECIFICATION FOCUS

Pearson Edexcel content references P6, P7. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A sample space lists all possible outcomes of an experiment. For two stages, an ordered pair records the first and second result. A grid or systematic list prevents omission and duplication. If stages and their outcomes are equally likely, each cell has the same probability; otherwise the grid organises outcomes but not equal weights.

KEY VOCABULARY

sample space • ordered pair • outcome • event • systematic • grid

	systematic outcome grid			

Why the method works

- A complete sample space is evidence that every possible result was considered.
- Order matters when first and second stages have distinct roles.
- For independent fair stages, the number of combined outcomes is the product of stage counts.
- Restrictions are applied after creating or carefully pruning the complete structure.

CHECK BEFORE MOVING ON

Answer mentally: 1) List outcomes for coin and die 1–3. 2) Two coins: P(two tails). 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 03 • P6, P7 • METHODS

P6, P7

Sample spaces and systematic listing — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Write outcomes of stage one in rows and stage two in columns.
- Fill every cell with an ordered pair or combined value.
- Mark cells satisfying the event.
- Count marked cells only when all cells are equally likely.

WORKED EXAMPLE 1

Two fair coins are tossed. Find $P(\text{exactly one head})$.

1. Sample space: HH, HT, TH, TT.
2. Favourable: HT, TH.
3. Probability = $\frac{2}{4}$.

ANSWER: $\frac{1}{2}$

WORKED EXAMPLE 2

Two fair dice are rolled. Find $P(\text{total } 9)$.

1. There are 36 ordered pairs.
2. Favourable: (3,6), (4,5), (5,4), (6,3).

ANSWER: $\frac{1}{9}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Two dice: $P(\text{total } 7)$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 03 • P6, P7 • APPLICATION

P6, P7

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Product menus and PIN construction use the same multiplication principle as sample spaces. Four starters and three mains create $4 \times 3 = 12$ pairs, provided every starter can be combined with every main.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why (1,2) differs from (2,1) for two dice. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 03 • P6, P7 • PRACTICE

P6, P7

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. List outcomes for coin and die 1-3.

2. Two coins: $P(\text{two tails})$.

3. Two dice: $P(\text{total } 7)$.

4. Two dice: $P(\text{double})$.

5. Cards A,B then 1,2,3: number of pairs.

6. Digits 2,4,6 without repetition: two-digit outcomes.

7. Two spinners with 3 and 5 equal sectors: total outcomes.

8. Explain why (1,2) differs from (2,1) for two dice.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 04 • P4

P4

Mutually exclusive events and complements

Use a complete total of 1 to find missing probability.

SPECIFICATION FOCUS

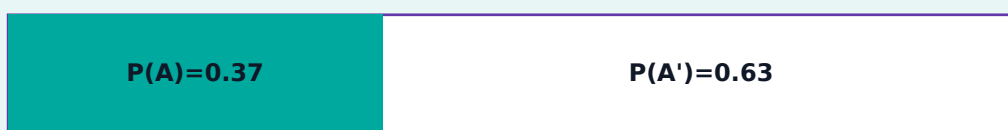
Pearson Edexcel content references P4. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Events are mutually exclusive when they cannot occur together in one trial. Their probabilities may be added for “A or B”. Exhaustive events cover every possible outcome, so their probabilities total 1. The complement A' means “not A”, giving $P(A')=1-P(A)$. Events can be exhaustive without being equally likely.

KEY VOCABULARY

mutually exclusive • exhaustive • complement • intersection • union



event + complement = 1

Why the method works

- The total probability of all mutually exclusive exhaustive outcomes is 1.
- An event and its complement are always mutually exclusive and exhaustive.
- Addition for A or B is valid directly when overlap is impossible.
- If events overlap, adding both probabilities double-counts their intersection.

CHECK BEFORE MOVING ON

Answer mentally: 1) $P(A)=0.37$: find $P(\text{not } A)$. 2) Outcomes probabilities 0.2, 0.35, x: find x. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 04 • P4 • METHODS

P4

Mutually exclusive events and complements — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Decide whether the events can happen together.
- Identify whether the listed outcomes cover the whole sample space.
- Write a probability equation before substituting.
- Subtract from 1 for a complement or missing exhaustive outcome.

WORKED EXAMPLE 1

$P(\text{red})=0.28$, $P(\text{blue})=0.47$ and only red, blue or green are possible. Find $P(\text{green})$.

1. The colour events are mutually exclusive and exhaustive.
2. $P(\text{green})=1-0.28-0.47$.

ANSWER: 0.25

WORKED EXAMPLE 2

$P(\text{bus late})=0.12$. Find $P(\text{bus not late})$.

1. Late and not late are complements.
2. $1-0.12=0.88$.

ANSWER: 0.88

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Fair die: $P(1 \text{ or } 6)$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 04 • P4 • APPLICATION

P4

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Insurance categories must be defined so cases do not overlap accidentally. “Under 25” and “student” can overlap, while “under 25” and “25 or older” are complementary.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why mutually exclusive events cannot be independent when both have positive probability. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 04 • P4 • PRACTICE

P4

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. $P(A)=0.37$: find $P(\text{not } A)$.

2. Outcomes probabilities 0.2,0.35,x: find x.

3. Fair die: $P(1 \text{ or } 6)$.

4. Are odd and even on a die mutually exclusive?

5. Are prime and odd mutually exclusive on a die?

6. $P(R)=0.4, P(B)=0.15$, disjoint: $P(R \text{ or } B)$.

7. Events A and not A total what?

8. Explain why mutually exclusive events cannot be independent when both have positive probability.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 05 • P3, P5

P3, P5

Experimental probability and relative frequency

Use observed frequency to estimate an unknown long-run probability.

SPECIFICATION FOCUS

Pearson Edexcel content references P3, P5. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Experimental probability, or relative frequency, is event frequency divided by number of trials. It estimates the underlying probability from data. Small samples fluctuate substantially; as trials increase, relative frequency often stabilises near the true probability, although it need not move closer on every step. A biased sample or changing conditions can still produce a poor estimate.

KEY VOCABULARY

trial • frequency • relative frequency • experimental probability • estimate • bias



Why the method works

- Relative frequency = event frequency ÷ number of trials.
- Larger representative samples usually reduce random variation.
- An estimate is not proof of the exact theoretical probability.
- Combining compatible trials increases the evidence base by adding frequencies and totals.

CHECK BEFORE MOVING ON

Answer mentally: 1) 18 heads in 30 tosses: estimate. 2) 72 sixes in 420 rolls: estimate. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 05 • P3, P5 • METHODS

P3, P5

Experimental probability and relative frequency — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Confirm the event count and total number of trials.
- Divide frequency by trials and convert form if requested.
- Assess whether the sample is large and representative.
- Compare estimates using both sample size and experimental conditions.

WORKED EXAMPLE 1

A drawing pin lands point-up 138 times in 240 trials. Estimate probability.

1. Relative frequency= $138 \div 240$.
2. Simplify or convert.

ANSWER: 0.575

WORKED EXAMPLE 2

Trials give 42 successes in 60, then 65 in 100. Find combined estimate.

1. Combined successes= 107 .
2. Combined trials= 160 .
3. Estimate= $107/160$.

ANSWER: 0.66875

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

A colour occurs 45 of 150: percentage. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 05 • P3, P5 • APPLICATION

P3, P5

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Manufacturers test product defects using samples. Selecting only one machine or one time of day may introduce bias even when the sample is large.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why experimental and theoretical probabilities may differ. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 05 • P3, P5 • PRACTICE

P3, P5

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. 18 heads in 30 tosses: estimate.

2. 72 sixes in 420 rolls: estimate.

3. A colour occurs 45 of 150: percentage.

4. Combine 12/20 and 35/50 successes.

5. Which is more reliable: 8/10 or 790/1000?

6. Estimate failures if 23 of 500 fail.

7. State effect of more trials.

8. Explain why experimental and theoretical probabilities may differ.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 06 • P2

P2

Expected frequency and prediction

Scale a probability to a number of repeated trials, then interpret sensibly.

SPECIFICATION FOCUS

Pearson Edexcel content references P2. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Expected frequency is probability multiplied by number of trials. It is a long-run model, not a promise. The result can be non-integer because it represents an average across many repetitions. Conversely, an observed frequency and total can be used to estimate probability before predicting a larger population.

KEY VOCABULARY

expected frequency • prediction • trial • population • long-run average

probability p

$\times$ number of trials

expected count

expectation is a long-run average, not a guarantee

Why the method works

- Expected frequency $= n \times p$ because p is the expected fraction of n trials.
- Predictions inherit uncertainty from both chance and any estimated probability.
- Expected counts across exhaustive events add to the total number of trials.
- A non-integer expected value is valid as a statistical expectation.

CHECK BEFORE MOVING ON

Answer mentally: 1) $p=0.3$ over 200 trials: expected. 2) $p=3/8$ over 480 trials. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 06 • P2 • METHODS

P2

Expected frequency and prediction — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Identify or estimate the probability.
- Multiply by the future number of trials.
- Round only when the context requires a whole count.
- Describe the answer as an estimate, not certainty.

WORKED EXAMPLE 1

A seed germinates with probability 0.82. Predict germinations from 350 seeds.

1. Expected frequency = 350×0.82 .

ANSWER: 287

WORKED EXAMPLE 2

In 80 checks, 6 items fail. Predict failures among 1,200.

1. Estimated probability = $6/80 = 0.075$.
2. Expected failures = 1200×0.075 .

ANSWER: 90

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

12 defective of 300: predict in 2000. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 06 • P2 • APPLICATION

P2

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Staffing forecasts may use expected customer arrivals, but managers need contingency because the actual count varies. The mathematical expectation supports planning rather than guaranteeing demand.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why expected frequency need not occur exactly. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 06 • P2 • PRACTICE

P2

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. $p=0.3$ over 200 trials: expected.

2. $p=3/8$ over 480 trials.

3. 12 defective of 300: predict in 2000.

4. 54 rainy days of 180: predict in 365.

5. Expected red 45 of 150: inferred p .

6. $p=0.07$ over 250 gives non-integer expected value. Find it.

7. Expected successes 96 with $p=0.4$: trials.

8. Explain why expected frequency need not occur exactly.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 07 • P6, P8

P6, P8

Combined events using tables and lists

Use addition for alternatives and multiplication for successive independent choices.

SPECIFICATION FOCUS

Pearson Edexcel content references P6, P8. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Combined-event problems contain more than one stage or category. A two-way outcome table displays all combinations. For independent stages, probabilities along a route multiply because the second-stage probability is unaffected by the first. Probabilities of separate favourable routes add when those complete routes are mutually exclusive.

KEY VOCABULARY

combined event • independent • route • product rule • sum rule • outcome table

	systematic outcome grid			

Why the method works

- “And” along a complete independent route leads to multiplication.
- “Or” between mutually exclusive complete routes leads to addition.
- A table protects against reversed or missing ordered pairs.
- Independence is a model statement, not something to assume from different labels.

CHECK BEFORE MOVING ON

Answer mentally: 1) Coin and die: $P(T \text{ and even})$. 2) Two dice: $P(\text{both six})$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 07 • P6, P8 • METHODS

P6, P8

Combined events using tables and lists — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Represent every first-stage and second-stage combination.
- Calculate the probability of each required route.
- Add routes satisfying the event.
- Check all terminal route probabilities total 1.

WORKED EXAMPLE 1

A fair coin and fair die are used. Find $P(\text{head and number} > 4)$.

1. $P(\text{head}) = 1/2$; $P(\text{number} > 4) = 2/6$.
2. Independent, so multiply.

ANSWER: $1/6$

WORKED EXAMPLE 2

Two fair coins: find $P(\text{at least one head})$.

1. Use complement: $1 - P(TT)$.
2. $1 - (1/2 \times 1/2)$.

ANSWER: $3/4$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Two dice: $P(\text{at least one six})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 07 • P6, P8 • APPLICATION

P6, P8

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

A café can model drink-and-snack choices with a table, but multiplication of observed proportions is justified only if snack choice is independent of drink choice.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain “at least one” complement. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 07 • P6, P8 • PRACTICE

P6, P8

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Coin and die: $P(T \text{ and even})$.

2. Two dice: $P(\text{both six})$.

3. Two dice: $P(\text{at least one six})$.

4. Spinner $p(\text{red})=0.4$ twice independently: $P(\text{two red})$.

5. Same spinner: $P(\text{exactly one red})$.

6. Choose letter A/B and digit 1/2/3: outcomes.

7. Independent $P(A)=0.3, P(B)=0.5$: $P(A \text{ and } B)$.

8. Explain “at least one” complement.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 08 • P8

P8

Tree diagrams with and without replacement

Follow each route in order and update probabilities when the contents change.

SPECIFICATION FOCUS

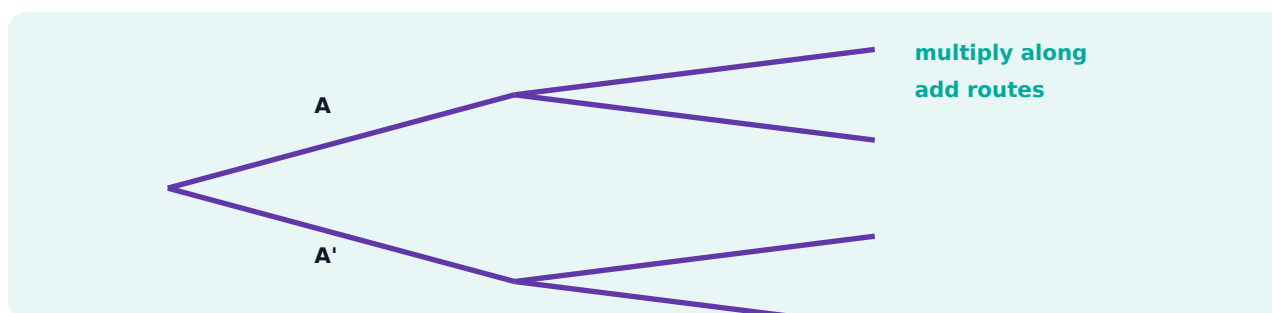
Pearson Edexcel content references P8. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A probability tree displays sequential events. Branches from the same node total 1. Multiply along a route and add separate terminal routes. With replacement, probabilities remain constant and repeated selections are independent. Without replacement, totals and relevant category counts change after the first selection.

KEY VOCABULARY

branch • stage • route • replacement • without replacement • dependent



Why the method works

- Every split represents a complete set of next-stage possibilities, so branches total 1.
- A route is an intersection of stage events, so its probabilities multiply.
- Different complete routes are mutually exclusive, so their probabilities add.
- Without replacement, the second denominator is one smaller and the numerator may change.

CHECK BEFORE MOVING ON

Answer mentally: 1) Fair coin twice: $P(HH)$. 2) $p(\text{success})=0.7$ twice independent: $P(\text{exactly one})$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 08 • P8 • METHODS

P8

Tree diagrams with and without replacement — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Label events and complements on every split.
- Write branch probabilities before calculating.
- Multiply along each required route.
- Add required routes and check against a complement where possible.

WORKED EXAMPLE 1

Bag has 3 red, 2 blue. Two selected without replacement. Find $P(\text{two red})$.

1. First red = $\frac{3}{5}$.
2. Then red = $\frac{2}{4}$.
3. Multiply = $\frac{6}{20}$.

ANSWER: $\frac{3}{10}$

WORKED EXAMPLE 2

Same bag: find $P(\text{one of each colour})$.

1. Routes RB and BR.
2. $\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{12}{20}$.

ANSWER: $\frac{3}{5}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Bag 4R,1B without replacement: $P(\text{B then R})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 08 • P8 • APPLICATION

P8

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Medical screening and reliability models use tree diagrams to separate stages. Conditional branch probabilities must refer to the population reaching that node, not the original total.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain denominator change without replacement. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 08 • P8 • PRACTICE

P8

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Fair coin twice: $P(HH)$.

2. $p(\text{success})=0.7$ twice independent: $P(\text{exactly one})$.

3. Bag 4R,1B without replacement: $P(B \text{ then } R)$.

4. Bag 4R,1B: $P(\text{two } R)$.

5. With replacement 4R,1B: $P(\text{two } R)$.

6. Branches from node 0.36 and x : x .

7. $P(\text{no success})$ if $p(\text{success})=0.2$ twice independently.

8. Explain denominator change without replacement.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 09 • P1, P6

P1, P6

Two-way tables, frequency trees and Venn diagrams

Separate frequencies from probabilities and make every region explicit.

SPECIFICATION FOCUS

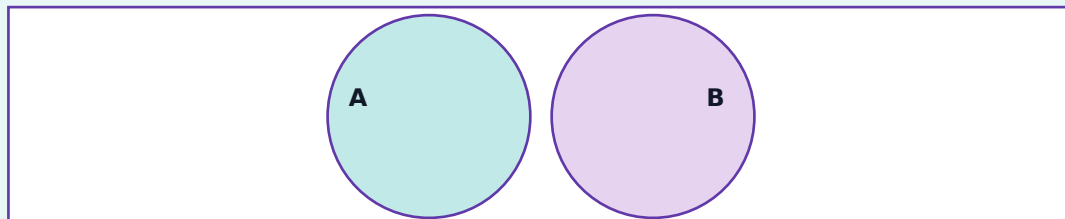
Pearson Edexcel content references P1, P6. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Two-way tables classify each individual by two categorical variables; row and column totals expose missing frequencies. Frequency trees show successive subgroup counts. Venn diagrams display sets: the overlap represents both, the non-overlap regions represent exactly one, and the exterior represents neither. Fill an overlap before subtracting from totals.

KEY VOCABULARY

set • union • intersection • complement • overlap • neither • frequency • two-way table



fill the intersection before the outer regions

Why the method works

- A grand total must equal every set of row totals and every set of column totals.
- In a Venn diagram, set totals include the intersection.
- $A \cap B$ means both; $A \cup B$ means at least one.
- Probability from a frequency diagram divides the required region by the universal total.

CHECK BEFORE MOVING ON

Answer mentally: 1) $U=50, A=27, B=21, \text{both}=8$: A only. 2) Same data: B only. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 09 • P1, P6 • METHODS

P1, P6

Two-way tables, frequency trees and Venn diagrams — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Place the intersection first.
- Subtract the intersection from each set total.
- Find the outside region from the universal total.
- Check all disjoint regions add to the universal total.

WORKED EXAMPLE 1

In 40 students, 22 study French, 18 Spanish and 9 both. Find neither.

1. French only=13; Spanish only=9.
2. Union=13+9+9=31.
3. Neither=40–31.

ANSWER: 9

WORKED EXAMPLE 2

A table has 60 pupils: 28 boys; 35 travel by bus; 16 boys use bus. Find girls not using bus.

1. Girls=32.
2. Girls using bus=35–16=19.
3. Girls not bus=32–19.

ANSWER: 13

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Same data: neither. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 09 • P1, P6 • APPLICATION

P1, P6

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Customer data can be classified by membership and purchase. Overlapping categories must not be added without removing the double-counted intersection.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why A+B double-counts overlap. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 09 • P1, P6 • PRACTICE

P1, P6

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. $U=50, A=27, B=21, \text{both}=8$: A only.

2. Same data: B only.

3. Same data: neither.

4. $U=80, A=45, B=38, \text{both}=20$: union.

5. Two-way total 100, male 46, glasses 39, male glasses 18: female glasses.

6. From previous: female without glasses.

7. Write symbol for both A and B.

8. Explain why $A+B$ double-counts overlap.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 10 • P9

Conditional probability: the meaning of given

P9

Restrict the sample space before forming the probability.

SPECIFICATION FOCUS

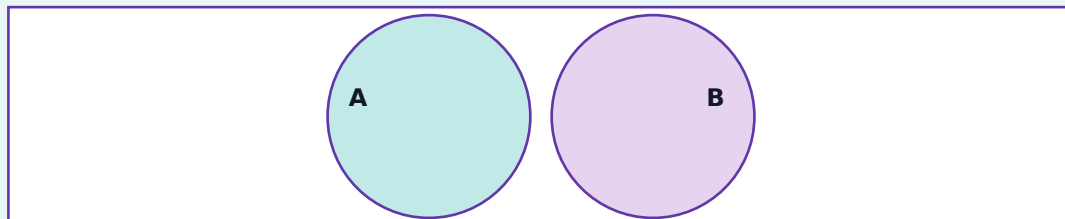
Pearson Edexcel content references P9. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A conditional probability answers a question inside a restricted population. $P(A \text{ given } B)$ means: assume B has happened, discard outcomes outside B , then find the proportion of the remaining B outcomes that are also in A . In frequency form, $P(A \text{ given } B) = \frac{\text{frequency}(A \text{ and } B)}{\text{frequency}(B)}$. The order matters: $P(A \text{ given } B)$ and $P(B \text{ given } A)$ usually use different denominators and answer different questions.

KEY VOCABULARY

conditional • given • restricted sample space • intersection • denominator • reverse condition



fill the intersection before the outer regions

Why the method works

- The words after given define the new universe and therefore the denominator.
- The numerator must satisfy both the condition and the requested event.
- $P(A \text{ given } B)$ is generally not equal to $P(B \text{ given } A)$.
- Independence is equivalent to $P(A \text{ given } B) = P(A)$, provided $P(B) > 0$.

CHECK BEFORE MOVING ON

Answer mentally: 1) Group B has 45 people, 27 succeed: $P(\text{success}|B)$. 2) 18 women among 30 engineers: $P(\text{woman}|\text{engineer})$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 10 • P9 • METHODS

P9

Conditional probability: the meaning of given — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Circle or underline the event after given.
- Restrict the table, tree or Venn diagram to that event.
- Count or calculate the part also satisfying the requested event.
- Divide by the restricted total and compare with the unconditional probability.

WORKED EXAMPLE 1

Of 80 students, 32 study music and 14 of those study drama. Find $P(\text{drama given music})$.

1. Restrict to the 32 music students.
2. 14 also study drama.

ANSWER: 7/16

WORKED EXAMPLE 2

$P(A)=0.45$, $P(B)=0.60$ and $P(A \text{ and } B)=0.27$. Are A and B independent?

1. $P(A)P(B)=0.45 \times 0.60=0.27$.
2. This equals $P(A \text{ and } B)$.

ANSWER: Yes

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

12 engineers among 18 women: $P(\text{engineer}|\text{woman})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 10 • P9 • APPLICATION

P9

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Medical-test and risk questions often reverse conditions. Sensitivity $P(\text{positive given illness})$ is not the same as $P(\text{illness given positive})$; the second also depends on prevalence.

Common mistakes and how to repair them

MISTAKE 1

Dividing by the overall total after a condition has restricted the population. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Reversing the events around the word given. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Assuming independence merely because two event names sound unrelated. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain why reversing a condition changes a question. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 10 • P9 • PRACTICE

P9

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Group B has 45 people, 27 succeed: $P(\text{success}|\text{B})$.

2. 18 women among 30 engineers: $P(\text{woman}|\text{engineer})$.

3. 12 engineers among 18 women: $P(\text{engineer}|\text{woman})$.

4. $P(A)=.4, P(B)=.5, P(A \cap B)=.2$: independent?

5. $P(A)=.4, P(B)=.5, P(A \cap B)=.1$: $P(A|B)$.

6. Explain the denominator in $P(A|B)$.

7. State an independence test using conditional probability.

8. Explain why reversing a condition changes a question.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 11 • P1

P1

Frequency trees and analysing experimental outcomes

Carry counts through branches and check that every split preserves its total.

SPECIFICATION FOCUS

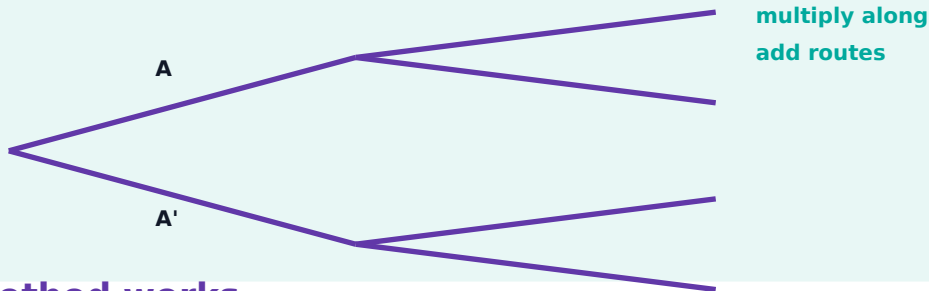
Pearson Edexcel content references P1. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A frequency tree records numbers of observations through successive categories. Unlike a probability tree, branches contain frequencies. The two child frequencies from any node add to the parent frequency. Missing values are found by subtraction, and probabilities are calculated only after the relevant final frequency and denominator are identified.

KEY VOCABULARY

frequency tree • parent total • branch frequency • outcome • experimental data



Why the method works

- Child frequencies add to their parent frequency.
- The first split totals the complete sample.
- Frequency and probability labels must not be mixed.
- An experimental probability divides the required final count by the appropriate overall trial count.

CHECK BEFORE MOVING ON

Answer mentally: 1) Using branch probabilities instead of counts. 2) Subtracting from the grand total instead of the parent node. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 11 • P1 • METHODS

P1

Frequency trees and analysing experimental outcomes — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Place the overall total at the start.
- Complete each split using subtraction.
- Check terminal frequencies add to the original total.
- Convert to probability only when requested.

WORKED EXAMPLE 1

Of 120 items, 70 are type A. Of type A, 18 fail. Find A items passing.

1. $70 - 18$.

ANSWER: 52

WORKED EXAMPLE 2

There are 50 type B items and 6 fail. Find P(random item is B and fails) if total is 120.

1. The terminal frequency is 6 of the complete 120.

ANSWER: 1/20

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Dividing by a subgroup when an overall probability is requested. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 11 • P1 • APPLICATION

P1

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Factories and surveys use frequency trees to preserve subgroup counts through stages. The tree reveals which denominator belongs to an overall proportion.

Common mistakes and how to repair them

MISTAKE 1

65 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

54 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

39 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 4

200 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 5

81/200 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 6

18 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 7

60 Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 8

CHAPTER 11 • P1 • PRACTICE

P1

Independent practice

Complete without notes, then use the answer section to diagnose errors.

- 1. Using branch probabilities instead of counts.**

- 2. Subtracting from the grand total instead of the parent node.**

- 3. Dividing by a subgroup when an overall probability is requested.**

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 12 • P6-P7

P6, P7

Counting principles, sets and systematic enumeration

Prove that every permitted outcome has been counted exactly once.

SPECIFICATION FOCUS

Pearson Edexcel content references P6, P7. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Systematic enumeration is not merely listing until the pattern looks complete. A product rule counts a sequence of choices when every option at one stage can be paired with every permitted option at the next. Restrictions such as no repetition, order requirements or incompatible choices change later counts. Tables, grids and trees reveal whether order matters and whether outcomes are equally likely.

KEY VOCABULARY

product rule • permutation • combination • ordered outcome • restriction • repetition

	systematic outcome grid			

Why the method works

- If stage counts are a,b,c with unrestricted choices, total outcomes=abc.
- Without repetition, the number of available choices falls at later stages.
- AB and BA are different only when order or roles matter.
- Counting outcomes does not make them equally likely.

CHECK BEFORE MOVING ON

Answer mentally: 1) 3 shirts, 5 trousers: outfits. 2) Digits 1-6: two-digit codes with repetition. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 12 • P6-P7 • METHODS

P6, P7

Counting principles, sets and systematic enumeration — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Define the stages and restrictions.
- Decide whether order matters.
- Count choices at each stage after earlier decisions.
- Multiply stage counts, then test with a short systematic list.

WORKED EXAMPLE 1

How many 4-digit PINs use digits 0-9 if repetition is allowed?

1. Each of four positions has 10 choices.
2. 10^4 .

ANSWER: 10,000

WORKED EXAMPLE 2

How many 3-letter arrangements can be made from A,B,C,D without repetition?

1. Choices: 4 then 3 then 2.
2. $4 \times 3 \times 2$.

ANSWER: 24

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Same without repetition. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 12 • P6-P7 • APPLICATION

P6, P7

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Security codes, menus and experimental designs use counting models. A count is valid only if every listed combination is actually permitted.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

State one reason cells may not be equally likely. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 12 • P6-P7 • PRACTICE

P6, P7

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. 3 shirts, 5 trousers: outfits.

2. Digits 1-6: two-digit codes with repetition.

3. Same without repetition.

4. Arrange 4 distinct books.

5. Choose starter and main from 4 and 7.

6. Explain why unordered pairs need adjustment.

7. A grid has 6 by 8 cells: outcomes.

8. State one reason cells may not be equally likely.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 13 • P4, P8

P4, P8

The general AND and OR rules

Separate intersection from union and correct for overlap.

SPECIFICATION FOCUS

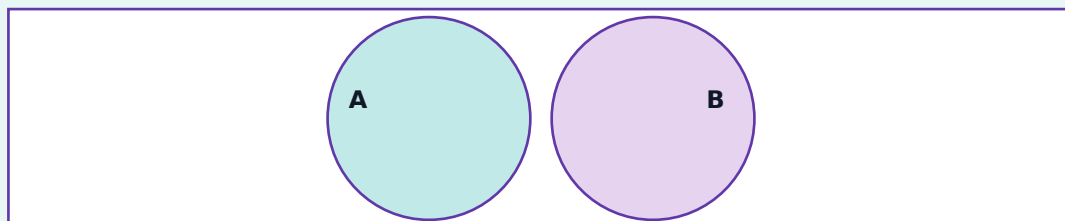
Pearson Edexcel content references P4, P8. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

The intersection $A \cap B$ contains outcomes where both events occur. The union $A \cup B$ contains outcomes in A or B or both. Therefore $P(A \cup B) = P(A) + P(B) - P(A \cap B)$: the overlap is subtracted once because it was counted twice. For any sequential events, $P(A \cap B) = P(A) \times P(B \text{ given } A)$. Only for independent events may the second factor be replaced by $P(B)$.

KEY VOCABULARY

intersection • union • overlap • AND rule • OR rule • independent • dependent



fill the intersection before the outer regions

Why the method works

- AND means intersection, not automatically multiplication of unchanged probabilities.
- OR includes both unless the wording says exactly one.
- The general OR rule subtracts the overlap once.
- For mutually exclusive events the overlap is zero.

CHECK BEFORE MOVING ON

Answer mentally: 1) $P(A) = .5, P(B) = .4, P(\text{both}) = .15$: union. 2) Disjoint probabilities $.32, .27$: union. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 13 • P4, P8 • METHODS

P4, P8

The general AND and OR rules — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Translate AND/OR into set notation.
- Identify or calculate the overlap.
- Use the general rule before simplifying for special cases.
- Check the result is at least each individual probability and no greater than 1.

WORKED EXAMPLE 1

$P(A)=.62, P(B)=.45, P(A \cap B)=.28$. Find $P(A \cup B)$.

1. $.62 + .45 - .28$.

ANSWER: 0.79

WORKED EXAMPLE 2

$P(A)=.7$ and $P(B|A)=.4$. Find $P(A \cap B)$.

1. Use general AND rule.
2. $.7 \times .4$.

ANSWER: 0.28

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

$P(A)=.6, P(B|A)=.25$: both. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 13 • P4, P8 • APPLICATION

P4, P8

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Insurance and diagnostic events overlap. Adding headline percentages without subtracting the intersection exaggerates the proportion affected.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain double-count correction. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 13 • P4, P8 • PRACTICE

P4, P8

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. $P(A)=.5, P(B)=.4, P(\text{both})=.15$: union.

2. Disjoint probabilities .32,.27: union.

3. $P(A)=.6, P(B|A)=.25$: both.

4. $P(A \cup B)=.8, P(A)=.55, P(B)=.4$: intersection.

5. State OR rule.

6. When may $P(A \cap B)=P(A)P(B)$?

7. Does OR normally include both?

8. Explain double-count correction.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 14 • P8-P9

P8, P9

Dependent probability trees without replacement

Change every branch from the population that actually reaches its node.

SPECIFICATION FOCUS

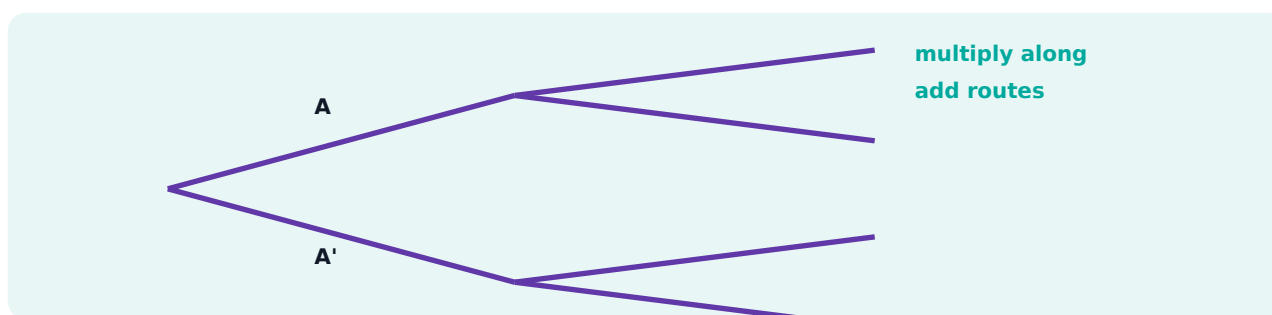
Pearson Edexcel content references P8, P9. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A tree diagram is a sequence of conditional models. Without replacement, the second-stage denominator is one smaller and its numerator depends on the first result. Each branch probability is conditional on the route taken to that node. Multiply along a complete route; add mutually exclusive terminal routes. A full terminal distribution must total 1.

KEY VOCABULARY

dependent • without replacement • conditional branch • route • denominator • terminal outcome



Why the method works

- Removing an item changes the composition and makes stages dependent.
- Second-stage branches from different nodes may have different probabilities.
- Multiply along one route and add separate routes.
- Terminal probabilities provide a powerful total-to-one check.

CHECK BEFORE MOVING ON

Answer mentally: 1) Bag4R,2B: $P(RR)$. 2) Same: $P(BB)$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 14 • P8-P9 • METHODS

P8, P9

Dependent probability trees without replacement — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Write the initial composition.
- After each first outcome, update both total and relevant counts.
- Label every second-stage branch from its own node.
- Calculate routes and check all terminals sum to 1.

WORKED EXAMPLE 1

A bag has 5 red and 3 blue. Find $P(\text{red then blue})$ without replacement.

1. First red = $\frac{5}{8}$.
2. Then blue = $\frac{3}{7}$.
3. Multiply.

ANSWER: $\frac{15}{56}$

WORKED EXAMPLE 2

Same bag: find $P(\text{exactly one red in two draws})$.

1. Routes RB and BR.
2. $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$.

ANSWER: $\frac{15}{28}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Same: $P(\text{one of each})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 14 • P8-P9 • APPLICATION

P8, P9

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Sampling without replacement, card problems and quality checks are dependent because the population composition changes after selection.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain what a branch probability is conditional on. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 14 • P8-P9 • PRACTICE

P8, P9

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Bag4R,2B: $P(RR)$.

2. Same: $P(BB)$.

3. Same: $P(\text{one of each})$.

4. Cards numbered1-5: $P(\text{two even without replacement})$.

5. Why second denominator falls.

6. When do branch probabilities stay unchanged?

7. State terminal total check.

8. Explain what a branch probability is conditional on.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 15 • P9

P9

Conditional probability from two-way and frequency tables

Use the conditioned row or column as a new total.

SPECIFICATION FOCUS

Pearson Edexcel content references P9. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A two-way table partitions a population by two properties. An overall probability uses the grand total; a conditional probability uses the total of the conditioned row or column. Complete missing cells and margins before forming ratios. Comparing $P(A \text{ given } B)$ with $P(A \text{ given not } B)$ can reveal association, but it does not by itself prove causation.

KEY VOCABULARY

two-way table • marginal total • joint frequency • conditional • association • causation

	systematic outcome grid			

Why the method works

- The cell at the intersection is the joint frequency.
- The conditioned margin supplies the denominator.
- Row and column totals must agree with the grand total.
- Different conditional rates can indicate association.

CHECK BEFORE MOVING ON

Answer mentally: 1) Sports club60, girls24; girls competing15: $P(\text{compete}|\text{girl})$. 2) Boys competing18: $P(\text{compete}|\text{boy})$. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: “The method works because ...” Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 15 • P9 • METHODS

P9

Conditional probability from two-way and frequency tables — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Complete all missing table entries.
- Locate the row or column named after given.
- Use its margin as denominator and the joint cell as numerator.
- Compare conditional rates only after using like-for-like denominators.

WORKED EXAMPLE 1

Of 120 patients, 50 received treatment; 35 of those improved. Find $P(\text{improved}|\text{treatment})$.

1. Restrict to 50 treated patients.
2. 35 improved.

ANSWER: 0.7

WORKED EXAMPLE 2

Of the 70 untreated patients, 35 improved. Compare improvement rates.

1. Untreated rate = $35/70 = .5$.
2. Treated rate = .7.

ANSWER: Treatment group is 0.2 higher

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Total employed 80, graduates 30, employed graduates 24: $P(\text{graduate}|\text{employed})$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 15 • P9 • APPLICATION

P9

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Clinical and survey tables support association comparisons, but hidden variables and allocation methods can affect interpretation.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

State correct denominator for $P(A|B)$. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 15 • P9 • PRACTICE

P9

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. Sports club60, girls24; girls competing15: $P(\text{compete}|\text{girl})$.

2. Boys competing18: $P(\text{compete}|\text{boy})$.

3. Total employed80,graduates30,employed graduates24: $P(\text{graduate}|\text{employed})$.

4. Why complete margins first?

5. What is joint frequency?

6. Can association prove causation?

7. Compare .6 and .45 conditional rates.

8. State correct denominator for $P(A|B)$.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 16 • P6, P9

P6, P9

Venn diagrams, set notation and conditional regions

Translate symbolic set language into disjoint diagram regions.

SPECIFICATION FOCUS

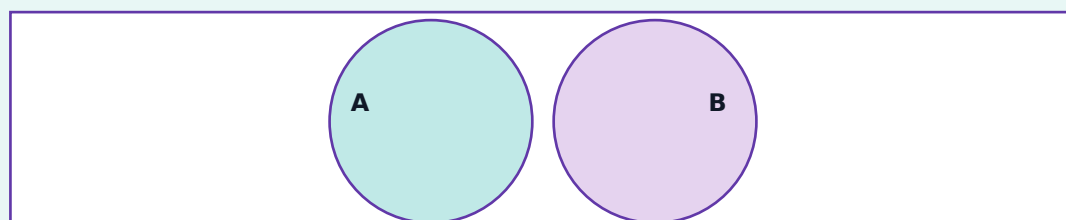
Pearson Edexcel content references P6, P9. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

A Venn diagram partitions the universal set into disjoint regions: A only, B only, $A \cap B$ and neither. Complements reverse membership; $A \cap B'$ means A but not B. Conditional probability restricts the denominator to the conditioning set, so $P(A|B) = P(A \cap B)/P(B)$. Algebraic region labels allow unknown frequencies or probabilities to be solved from totals.

KEY VOCABULARY

universal set • complement • intersection • union • conditional region • disjoint



fill the intersection before the outer regions

Why the method works

- Fill the intersection before set-only regions.
- A union includes the overlap; exactly one excludes it.
- A complement includes every region outside that set.
- Conditioning on B discards every region outside B.

CHECK BEFORE MOVING ON

Answer mentally: 1) U80,A45,B38,both20: A only. 2) Same: neither. 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 16 • P6, P9 • METHODS

P6, P9

Venn diagrams, set notation and conditional regions — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Translate the notation into words.
- Shade or identify the required disjoint regions.
- Add those regions for the numerator.
- For a condition, add only all regions inside the conditioning set for the denominator.

WORKED EXAMPLE 1

$U=100, A=58, B=47, A \cap B=25$. Find $P(A|B)$.

1. The restricted B total is 47.
2. 25 of B are also A.

ANSWER: 25/47

WORKED EXAMPLE 2

Using same data, find probability of exactly one of A,B.

1. A only=33; B only=22.
2. Exactly one=55.

ANSWER: 0.55

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Same: $P(A|B)$. First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 16 • P6, P9 • APPLICATION

P6, P9

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Customer segmentation and medical classification use overlapping sets. Conditional rates describe composition within one selected segment.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain fill-overlap-first. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 16 • P6, P9 • PRACTICE

P6, P9

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. U80,A45,B38,both20: A only.

2. Same: neither.

3. Same: $P(A|B)$.

4. Translate $A \cap B'$.

5. Translate $(A \cup B)'$.

6. What region is excluded by exactly one?

7. State conditional Venn formula.

8. Explain fill-overlap-first.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

CHAPTER 17 • P8-P9

P8, P9

Reverse probability, algebraic branches and model evaluation

Use a known final probability to solve backwards for an unknown branch.

SPECIFICATION FOCUS

Pearson Edexcel content references P8, P9. This lesson develops representation, accurate calculation and interpretation within Higher Tier problem solving.

Understand the idea

Higher probability models may contain an unknown probability x . Branch complements become $1-x$, and a stated terminal or combined probability creates an equation. Reverse trees can also infer an earlier composition from later information. Algebra is valid only after the routes are correctly identified. A complete answer should state assumptions such as independence, constant probability, random selection or replacement.

KEY VOCABULARY

unknown probability • algebraic tree • reverse process • equation • assumption • validation

define event

choose model

calculate

check total

interpret

The condition determines the denominator.

Why the method works

- Unknown complementary branches sum to 1.
- Terminal route expressions are products of their branches.
- Alternative routes are added only after each product is formed.
- A numerical solution outside 0 to 1 must be rejected.

CHECK BEFORE MOVING ON

Answer mentally: 1) x and $1-x$ are complementary: sum. 2) $x^2 = .36$: valid probability x . 3) State one definition or rule from this page in your own words.

EXPLAIN IT

Complete this sentence: "The method works because ..." Use the key vocabulary above and refer to the representation in the diagram.

CHAPTER 17 • P8-P9 • METHODS

P8, P9

Reverse probability, algebraic branches and model evaluation — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Label unknown branch x and complement $1-x$.
- Write expressions for every required route.
- Form and solve the probability equation.
- Reject invalid roots and interpret assumptions.

WORKED EXAMPLE 1

Independent trials have $P(\text{success})=x$. $P(\text{two successes})=0.49$. Find x .

1. $x^2=.49$.
2. Probability is non-negative.

ANSWER: 0.7

WORKED EXAMPLE 2

$P(A)=x, P(B)=.4$ independent and $P(A \text{ or } B)=.7$. Find x .

1. $.7=x+.4-.4x$.
2. $.6x=.3$.

ANSWER: 0.5

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes it easier to repair errors.

GUIDED TRY

Independent $P(A)=x, P(B)=.5, \text{both}=.2$: x . First predict the form or approximate size of the answer; then show a complete method.

CHAPTER 17 • P8-P9 • APPLICATION

P8, P9

Apply and reason

Connect the method to a context and communicate the conclusion.

Real-world model

CONTEXT

Reliability and risk models are simplified descriptions. Their calculations can be exact within the model while the model itself remains uncertain.

Common mistakes and how to repair them

MISTAKE 1

Treating outcomes as equally likely without evidence. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 2

Adding probabilities when events must both occur. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

MISTAKE 3

Giving a decimal without explaining the event or checking it lies from 0 to 1. Repair: return to the definition, write one complete mathematical line, then check the size or sign of the result.

Exam communication

- Write the operation or equation before the answer.
- Preserve exact values unless the question asks for rounding.
- Attach units and interpret the final value in the context.

REASONING PROMPT

Explain model versus calculation uncertainty. Write one sentence explaining why your calculation or conclusion is valid.

CHAPTER 17 • P8-P9 • PRACTICE

P8, P9

Independent practice

Complete without notes, then use the answer section to diagnose errors.

1. x and $1-x$ are complementary: sum.

2. $x^2 = .36$: valid probability x .

3. Independent $P(A)=x, P(B)=.5$, both $=.2$: x .

4. Disjoint $P(A)=x, P(B)=.25$, union $=.6$: x .

5. Why reject $x = -.6$?

6. State an independence assumption.

7. State a constant-rate assumption.

8. Explain model versus calculation uncertainty.

REFLECTION

Circle one: secure / almost / reteach. Write the question number of your most useful error and the rule that repairs it.

PRACTICE ANSWERS

CHAPTER 01

Answers, method guidance and repair notes

Probability language and the 0-to-1 scale

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Write impossible event probability.

ANSWER: 0

Method guidance: Identify the event precisely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Write certain event probability.

ANSWER: 1

Method guidance: Decide whether it is impossible, certain or between these extremes. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Convert 0.35 to percentage.

ANSWER: 35%

Method guidance: Convert all given values to one representation before comparing. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Convert 72% to decimal.

ANSWER: 0.72

Method guidance: Use the context to reject impossible values below 0 or above 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Order $\frac{1}{4}$, 0.2, 30%.

ANSWER: 0.2, $\frac{1}{4}$, 30%

Method guidance: Identify the event precisely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Which is more likely: 0.62 or $\frac{3}{5}$?

ANSWER: 0.62

Method guidance: Decide whether it is impossible, certain or between these extremes. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Give a probability for an even chance.

ANSWER: 0.5

Method guidance: Convert all given values to one representation before comparing. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why probability 1.04 is invalid.

ANSWER: greater than 1

Method guidance: Use the context to reject impossible values below 0 or above 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 02

Answers, method guidance and repair notes

Theoretical probability and equally likely outcomes

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Fair die: P(odd).

ANSWER: 1/2

Method guidance: List the complete outcome set. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Fair die: P(number >4).

ANSWER: 1/3

Method guidance: Check that outcomes are equally likely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Bag 4 red,6 blue: P(red).

ANSWER: 2/5

Method guidance: Count favourable and total outcomes. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Cards 1-10: P(prime).

ANSWER: 2/5

Method guidance: Write, simplify and sense-check the fraction. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Alphabet tiles A,A,B,C: P(A).

ANSWER: 1/2

Method guidance: List the complete outcome set. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Spinner equal sectors R,R,B,G: P(not R).

ANSWER: 1/2

Method guidance: Check that outcomes are equally likely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Can probability 3/7 be based on 21 equally likely outcomes? Give favourable count.

ANSWER: yes,9

Method guidance: Count favourable and total outcomes. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain when favourable/total is valid.

ANSWER: when outcomes equally likely

Method guidance: Write, simplify and sense-check the fraction. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 03

Answers, method guidance and repair notes

Sample spaces and systematic listing

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. List outcomes for coin and die 1-3.

ANSWER: H1,H2,H3,T1,T2,T3

Method guidance: Write outcomes of stage one in rows and stage two in columns. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Two coins: P(two tails).

ANSWER: 1/4

Method guidance: Fill every cell with an ordered pair or combined value. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Two dice: P(total 7).

ANSWER: 1/6

Method guidance: Mark cells satisfying the event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Two dice: P(double).

ANSWER: 1/6

Method guidance: Count marked cells only when all cells are equally likely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Cards A,B then 1,2,3: number of pairs.

ANSWER: 6

Method guidance: Write outcomes of stage one in rows and stage two in columns. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Digits 2,4,6 without repetition: two-digit outcomes.

ANSWER: 24,26,42,46,62,64

Method guidance: Fill every cell with an ordered pair or combined value. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Two spinners with 3 and 5 equal sectors: total outcomes.

ANSWER: 15

Method guidance: Mark cells satisfying the event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why (1,2) differs from (2,1) for two dice.

ANSWER: different die gives each value

Method guidance: Count marked cells only when all cells are equally likely. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 04

Answers, method guidance and repair notes

Mutually exclusive events and complements

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. $P(A)=0.37$: find $P(\text{not } A)$.

ANSWER: 0.63

Method guidance: Decide whether the events can happen together. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Outcomes probabilities 0.2,0.35,x: find x.

ANSWER: 0.45

Method guidance: Identify whether the listed outcomes cover the whole sample space. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Fair die: $P(1 \text{ or } 6)$.

ANSWER: 1/3

Method guidance: Write a probability equation before substituting. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Are odd and even on a die mutually exclusive?

ANSWER: yes

Method guidance: Subtract from 1 for a complement or missing exhaustive outcome. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Are prime and odd mutually exclusive on a die?

ANSWER: no

Method guidance: Decide whether the events can happen together. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. $P(R)=0.4, P(B)=0.15$, disjoint: $P(R \text{ or } B)$.

ANSWER: 0.55

Method guidance: Identify whether the listed outcomes cover the whole sample space. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Events A and not A total what?

ANSWER: 1

Method guidance: Write a probability equation before substituting. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why mutually exclusive events cannot be independent. **ANSWER: Events have positive probability, either not to occur**

Method guidance: Subtract from 1 for a complement or missing exhaustive outcome. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 05

Answers, method guidance and repair notes

Experimental probability and relative frequency

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. 18 heads in 30 tosses: estimate.

ANSWER: 0.6

Method guidance: Confirm the event count and total number of trials. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. 72 sixes in 420 rolls: estimate.

ANSWER: 6/35 or 0.1714

Method guidance: Divide frequency by trials and convert form if requested. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. A colour occurs 45 of 150: percentage.

ANSWER: 30%

Method guidance: Assess whether the sample is large and representative. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Combine 12/20 and 35/50 successes.

ANSWER: 47/70

Method guidance: Compare estimates using both sample size and experimental conditions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Which is more reliable: 8/10 or 790/1000?

ANSWER: 790/1000

Method guidance: Confirm the event count and total number of trials. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Estimate failures if 23 of 500 fail.

ANSWER: 0.046

Method guidance: Divide frequency by trials and convert form if requested. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. State effect of more trials.

ANSWER: usually more stable

Method guidance: Assess whether the sample is large and representative. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why experimental and theoretical probabilities may differ. **ANSWER: random variation/model imperfections**

Method guidance: Compare estimates using both sample size and experimental conditions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 06

Answers, method guidance and repair notes

Expected frequency and prediction

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. $p=0.3$ over 200 trials: expected.

ANSWER: 60

Method guidance: Identify or estimate the probability. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. $p=3/8$ over 480 trials.

ANSWER: 180

Method guidance: Multiply by the future number of trials. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. 12 defective of 300: predict in 2000.

ANSWER: 80

Method guidance: Round only when the context requires a whole count. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. 54 rainy days of 180: predict in 365.

ANSWER: 109.5 about 110

Method guidance: Describe the answer as an estimate, not certainty. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Expected red 45 of 150: inferred p .

ANSWER: 0.3

Method guidance: Identify or estimate the probability. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. $p=0.07$ over 250 gives non-integer expected value. Find it.

ANSWER: 17.5

Method guidance: Multiply by the future number of trials. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Expected successes 96 with $p=0.4$: trials.

ANSWER: 240

Method guidance: Round only when the context requires a whole count. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why expected frequency need not occur exactly.

ANSWER: chance causes variation

Method guidance: Describe the answer as an estimate, not certainty. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 07

Answers, method guidance and repair notes

Combined events using tables and lists

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Coin and die: $P(T \text{ and even})$.

ANSWER: $1/4$

Method guidance: Represent every first-stage and second-stage combination. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Two dice: $P(\text{both six})$.

ANSWER: $1/36$

Method guidance: Calculate the probability of each required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Two dice: $P(\text{at least one six})$.

ANSWER: $11/36$

Method guidance: Add routes satisfying the event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Spinner $p(\text{red})=0.4$ twice independently: $P(\text{two red})$.

ANSWER: 0.16

Method guidance: Check all terminal route probabilities total 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Same spinner: $P(\text{exactly one red})$.

ANSWER: 0.48

Method guidance: Represent every first-stage and second-stage combination. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Choose letter A/B and digit 1/2/3: outcomes.

ANSWER: A1,A2,A3,B1,B2,B3

Method guidance: Calculate the probability of each required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Independent $P(A)=0.3, P(B)=0.5$: $P(A \text{ and } B)$.

ANSWER: 0.15

Method guidance: Add routes satisfying the event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain “at least one” complement.

ANSWER: exclude only zero occurrences

Method guidance: Check all terminal route probabilities total 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 08

Answers, method guidance and repair notes

Tree diagrams with and without replacement

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Fair coin twice: $P(HH)$.

ANSWER: $1/4$

Method guidance: Label events and complements on every split. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. $p(\text{success})=0.7$ twice independent: $P(\text{exactly one})$.

ANSWER: 0.42

Method guidance: Write branch probabilities before calculating. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Bag 4R,1B without replacement: $P(B \text{ then } R)$.

ANSWER: $1/5$

Method guidance: Multiply along each required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Bag 4R,1B: $P(\text{two } R)$.

ANSWER: $3/5$

Method guidance: Add required routes and check against a complement where possible. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. With replacement 4R,1B: $P(\text{two } R)$.

ANSWER: $16/25$

Method guidance: Label events and complements on every split. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Branches from node 0.36 and x : x .

ANSWER: 0.64

Method guidance: Write branch probabilities before calculating. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. $P(\text{no success})$ if $p(\text{success})=0.2$ twice independently.

ANSWER: 0.64

Method guidance: Multiply along each required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain denominator change without replacement.

ANSWER: one item has been removed

Method guidance: Add required routes and check against a complement where possible. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 09

Answers, method guidance and repair notes

Two-way tables, frequency trees and Venn diagrams

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. $U=50, A=27, B=21, \text{both}=8$: A only.

ANSWER: 19

Method guidance: Place the intersection first. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Same data: B only.

ANSWER: 13

Method guidance: Subtract the intersection from each set total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Same data: neither.

ANSWER: 10

Method guidance: Find the outside region from the universal total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. $U=80, A=45, B=38, \text{both}=20$: union.

ANSWER: 63

Method guidance: Check all disjoint regions add to the universal total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Two-way total 100, male 46, glasses 39, male glasses 18: female glasses.

ANSWER: 21

Method guidance: Place the intersection first. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. From previous: female without glasses.

ANSWER: 33

Method guidance: Subtract the intersection from each set total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Write symbol for both A and B.

ANSWER: $A \cap B$

Method guidance: Find the outside region from the universal total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why $A+B$ double-counts overlap.

ANSWER: intersection belongs to both totals

Method guidance: Check all disjoint regions add to the universal total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 10

Answers, method guidance and repair notes

Conditional probability: the meaning of given

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Group B has 45 people, 27 succeed: $P(\text{success}|\text{B})$.

ANSWER: 0.6

Method guidance: Circle or underline the event after given. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. 18 women among 30 engineers: $P(\text{woman}|\text{engineer})$.

ANSWER: 3/5

Method guidance: Restrict the table, tree or Venn diagram to that event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. 12 engineers among 18 women: $P(\text{engineer}|\text{woman})$.

ANSWER: 2/3

Method guidance: Count or calculate the part also satisfying the requested event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. $P(A)=.4, P(B)=.5, P(A \cap B)=.2$: independent?

ANSWER: yes

Method guidance: Divide by the restricted total and compare with the unconditional probability. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. $P(A)=.4, P(B)=.5, P(A \cap B)=.1$: $P(A|B)$.

ANSWER: 0.2

Method guidance: Circle or underline the event after given. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Explain the denominator in $P(A|B)$.

ANSWER: frequency/probability of B

Method guidance: Restrict the table, tree or Venn diagram to that event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. State an independence test using conditional probability.

ANSWER: $P(A|B)=P(A)$

Method guidance: Count or calculate the part also satisfying the requested event. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain why reversing a condition changes a question.

ANSWER: restricted populations differ

Method guidance: Divide by the restricted total and compare with the unconditional probability. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 11

Answers, method guidance and repair notes

Frequency trees and analysing experimental outcomes

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Using branch probabilities instead of counts.

ANSWER: Total 200, yes 135: no.

Method guidance: Place the overall total at the start. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Subtracting from the grand total instead of the parent node.

ANSWER: Yes 135, success 81: yes-fail.

Method guidance: Complete each split using subtraction. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Dividing by a subgroup when an overall probability is requested.

ANSWER: No 65, success 26: no-fail.

Method guidance: Check terminal frequencies add to the original total. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 12

Answers, method guidance and repair notes

Counting principles, sets and systematic enumeration

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. 3 shirts, 5 trousers: outfits.

ANSWER: 15

Method guidance: Define the stages and restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Digits 1-6: two-digit codes with repetition.

ANSWER: 36

Method guidance: Decide whether order matters. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Same without repetition.

ANSWER: 30

Method guidance: Count choices at each stage after earlier decisions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Arrange 4 distinct books.

ANSWER: 24

Method guidance: Multiply stage counts, then test with a short systematic list. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Choose starter and main from 4 and 7.

ANSWER: 28

Method guidance: Define the stages and restrictions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Explain why unordered pairs need adjustment.

ANSWER: AB and BA represent same selection

Method guidance: Decide whether order matters. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. A grid has 6 by 8 cells: outcomes.

ANSWER: 48

Method guidance: Count choices at each stage after earlier decisions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. State one reason cells may not be equally likely.

ANSWER: biased stages/unequal sector sizes

Method guidance: Multiply stage counts, then test with a short systematic list. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 13

Answers, method guidance and repair notes

The general AND and OR rules

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. $P(A)=.5, P(B)=.4, P(\text{both})=.15$: union.

ANSWER: 0.75

Method guidance: Translate AND/OR into set notation. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Disjoint probabilities .32,.27: union.

ANSWER: 0.59

Method guidance: Identify or calculate the overlap. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. $P(A)=.6, P(B|A)=.25$: both.

ANSWER: 0.15

Method guidance: Use the general rule before simplifying for special cases. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. $P(A \cup B)=.8, P(A)=.55, P(B)=.4$: intersection.

ANSWER: 0.15

Method guidance: Check the result is at least each individual probability and no greater than 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. State OR rule.

ANSWER: $P(A)+P(B)-P(A \cap B)$

Method guidance: Translate AND/OR into set notation. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. When may $P(A \cap B)=P(A)P(B)$?

ANSWER: when independent

Method guidance: Identify or calculate the overlap. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Does OR normally include both?

ANSWER: yes

Method guidance: Use the general rule before simplifying for special cases. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain double-count correction.

ANSWER: intersection appears in both totals

Method guidance: Check the result is at least each individual probability and no greater than 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 14

Answers, method guidance and repair notes

Dependent probability trees without replacement

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Bag4R,2B: P(RR).

ANSWER: 2/5

Method guidance: Write the initial composition. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Same: P(BB).

ANSWER: 1/15

Method guidance: After each first outcome, update both total and relevant counts. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Same: P(one of each).

ANSWER: 8/15

Method guidance: Label every second-stage branch from its own node. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Cards numbered1-5: P(two even without replacement).

ANSWER: 1/10

Method guidance: Calculate routes and check all terminals sum to 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Why second denominator falls.

ANSWER: one item removed

Method guidance: Write the initial composition. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. When do branch probabilities stay unchanged?

ANSWER: replacement/independent trials

Method guidance: After each first outcome, update both total and relevant counts. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. State terminal total check.

ANSWER: sum to1

Method guidance: Label every second-stage branch from its own node. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain what a branch probability is conditional on.

ANSWER: earlier route outcomes

Method guidance: Calculate routes and check all terminals sum to 1. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 15

Answers, method guidance and repair notes

Conditional probability from two-way and frequency tables

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. Sports club60, girls24; girls competing15: $P(\text{compete}|\text{girl})$.

ANSWER: 5/8

Method guidance: Complete all missing table entries. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Boys competing18: $P(\text{compete}|\text{boy})$.

ANSWER: 1/2

Method guidance: Locate the row or column named after given. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Total employed80,graduates30,employed graduates24: $P(\text{graduate}|\text{employed})$.

ANSWER: 3/10

Method guidance: Use its margin as denominator and the joint cell as numerator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Why complete margins first?

ANSWER: prevents wrong denominator

Method guidance: Compare conditional rates only after using like-for-like denominators. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. What is joint frequency?

ANSWER: cell satisfying both

Method guidance: Complete all missing table entries. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. Can association prove causation?

ANSWER: no

Method guidance: Locate the row or column named after given. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. Compare .6 and .45 conditional rates.

ANSWER: A is .15 higher under first condition

Method guidance: Use its margin as denominator and the joint cell as numerator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. State correct denominator for $P(A|B)$.

ANSWER: B total

Method guidance: Compare conditional rates only after using like-for-like denominators. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 16

Answers, method guidance and repair notes

Venn diagrams, set notation and conditional regions

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. U80,A45,B38,both20: A only.

ANSWER: 25

Method guidance: Translate the notation into words. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. Same: neither.

ANSWER: 17

Method guidance: Shade or identify the required disjoint regions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Same: $P(A|B)$.

ANSWER: 10/19

Method guidance: Add those regions for the numerator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Translate $A \cap B'$.

ANSWER: A but not B

Method guidance: For a condition, add only all regions inside the conditioning set for the denominator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Translate $(A \cup B)'$.

ANSWER: neither A nor B

Method guidance: Translate the notation into words. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. What region is excluded by exactly one?

ANSWER: intersection

Method guidance: Shade or identify the required disjoint regions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. State conditional Venn formula.

ANSWER: $P(A \cap B)/P(B)$

Method guidance: Add those regions for the numerator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain fill-overlap-first.

ANSWER: set totals include overlap

Method guidance: For a condition, add only all regions inside the conditioning set for the denominator. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

PRACTICE ANSWERS

CHAPTER 17

Answers, method guidance and repair notes

Reverse probability, algebraic branches and model evaluation

Attempt every question first. The answer confirms the outcome; the method guidance identifies the governing process. Rework an error from a blank line and show every algebraic or numerical stage.

1. x and $1-x$ are complementary: sum.

ANSWER: 1

Method guidance: Label unknown branch x and complement $1-x$. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

2. $x^2=.36$: valid probability x .

ANSWER: 0.6

Method guidance: Write expressions for every required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

3. Independent $P(A)=x, P(B)=.5, \text{both}=.2$: x .

ANSWER: 0.4

Method guidance: Form and solve the probability equation. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

4. Disjoint $P(A)=x, P(B)=.25, \text{union}=.6$: x .

ANSWER: 0.35

Method guidance: Reject invalid roots and interpret assumptions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

5. Why reject $x=-.6$?

ANSWER: probability cannot be negative

Method guidance: Label unknown branch x and complement $1-x$. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

6. State an independence assumption.

ANSWER: one event does not change other

Method guidance: Write expressions for every required route. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

7. State a constant-rate assumption.

ANSWER: probability unchanged across trials

Method guidance: Form and solve the probability equation. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

8. Explain model versus calculation uncertainty.

ANSWER: arithmetic may be exact but assumptions approximate

Method guidance: Reject invalid roots and interpret assumptions. Apply the rule to the stated values, record each transformation, then verify sign, size, restrictions, accuracy and units.

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