GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 1 | NUMBER

TEACH-FROM-SCRATCH • PUBLICATION EDITION

10 detailed chapters • original diagrams • worked examples • guided practice • answers

COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Practice third.

FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each worked solution and reproduce it. 3 Complete practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final statement when interpretation is required.

Error loop

REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

Higher success requires connected reasoning

OCR Higher questions combine exact manipulation, proof and unfamiliar contexts. Grades 7–9 depend on selecting efficient structures, linking several ideas and justifying conclusions.

COMPREHENSIVE LESSON NOTEBOOK

Unit 1 coverage map

OCR J560 Number content organised into a teachable sequence.

CH 01	1.01a–1.01d	Number structure and exact calculation
CH 02	1.02a–1.02c	Prime factors, divisibility, HCF and LCM
CH 03	2.01	Fractions and exact proportional calculation
CH 04	2.02–2.04	Decimals, percentages and recurring values
CH 05	3.01a–3.01b	Integer, negative and fractional indices
CH 06	3.01c	Surds and exact simplification
CH 07	3.02	Standard form and orders of magnitude
CH 08	4.01a	Rounding, estimation and significant figures
CH 09	4.01b–4.01c	Bounds, error intervals and limits of accuracy
CH 10	1–4 synthesis	Higher Number problem solving

IMPORTANT TIER NOTE

This notebook covers OCR J560 sections 1–4 at Higher Tier. Ratio and proportion are developed separately in Unit 2. Specification references are identifiers, not copied specification wording.

COMPREHENSIVE LESSON NOTEBOOK

Fast-reference method bank

Understand each relationship before memorising it.

Recurring decimal	align repeating tails, subtract, then solve
Compound change	original × multiplier^n
Reverse percentage	known amount ÷ combined multiplier
Fractional index	a^(m/n) = (nth root of a)^m
Negative index	a^(-n) = 1/a^n
Surd product	√a × √b = √(ab)
Conjugates	$(a+b)(a-b)=a^2-b^2$
Standard form	$A \times 10^n$, $1 \le A < 10$
HCF / LCM	minimum / maximum prime exponents
Error interval	rounded value ± half the rounding unit

CURRENT ASSESSMENT NOTE

OCR has announced Foundation and Higher formula sheets for GCSE Mathematics examinations in 2025, 2026 and 2027. Use the current official OCR sheet alongside this learning bank and check OCR for later exam-series arrangements.

COMPREHENSIVE LESSON NOTEBOOK

Number structure and exact calculation

Integers, rational and irrational numbers; operation hierarchy; proof and counterexample.

SPECIFICATION FOCUS

OCR reference 1.01a–1.01d. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

The real-number system contains rational values, which can be expressed as a fraction of integers, and irrational values, whose decimal expansions neither terminate nor repeat. Higher-tier questions test classification together with exact manipulation, structural reasoning and proof.

KEY VOCABULARY

integer • rational • irrational • real • operation • proof • counterexample

rational numbers ∪ irrational numbers = real numbers

exact classification • operation structure • proof or counterexample

Why the method works

- Definitions make classification reliable.
- An identity or parity proof must cover every allowed case.
- One valid counterexample is enough to disprove an “always” statement.

CHECK BEFORE MOVING ON

Answer mentally: Evaluate $6 - 3[2 - 5(1 - 4)]^2$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Number structure and exact calculation — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Classify numbers by definitions, not appearance.
- Apply brackets and indices before equal-priority operations from left to right.
- For a proof, begin with a general form; to disprove, one valid counterexample is enough.

WORKED EXAMPLE 1

Evaluate $6 - 3[2 - 5(1 - 4)]^2$.

SOLUTION

1. Evaluate the inner bracket: $1 - 4 = -3$.
2. Then $2 - 5(-3) = 17$.
3. Calculate $6 - 3(17^2) = 6 - 867 = -861$.
4. Check: $3 \times 289 = 867$.

ANSWER: -861

WORKED EXAMPLE 2

Prove that the square of any odd integer is odd.

SOLUTION

1. Write an arbitrary odd integer as $2n+1$.
2. Expand its square.
3. Factor out 2 from all but the final 1.
4. Conclude using the definition of odd.

ANSWER: $(2n+1)^2 = 2(2n^2+2n)+1$, so it is odd

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Number structure and exact calculation — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Exact number structure supports cryptography, computer arithmetic and error-free symbolic models.

Common misconceptions

- Assuming every square root is irrational.
- Treating a calculator approximation as an exact value.
- Checking a few examples and calling that a proof.

GUIDED TRY

Arrange $\sqrt{50}$, 7.05, 22/3 and $2\pi+1$ in ascending order. Show sufficient comparisons. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Insert one pair of brackets into $18 \div 3 + 3 \times 2$ so that the value is 2. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Number structure and exact calculation

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Evaluate $6 - 3[2 - 5(1 - 4)]^2$.** **[4]**

2. **Prove that the square of any odd integer is odd.** **[4]**

3. **Arrange $\sqrt{50}$, 7.05, 22/3 and $2\pi+1$ in ascending order. Show sufficient comparisons.** **[4]**

4. **Find an irrational number strictly between 3.1 and 3.2 and justify your choice.** **[3]**

5. **A student claims: "the product of two irrational numbers is irrational." Decide whether this is always true.** **[3]**

6. **Evaluate exactly $(3/4 - 5/6) \div (7/9 + 1/3)$.** **[4]**

7. **The integers a and b satisfy $a+b$ odd. Prove that a^2+b^2 is odd.** **[5]**

SHOW ALL WORKING

8. **Insert one pair of brackets into $18 \div 3 + 3 \times 2$ so that the value is 2.** **[3]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Prime factors, divisibility, HCF and LCM

Prime decomposition, divisibility arguments and linked HCF/LCM problems.

SPECIFICATION FOCUS

OCR reference 1.02a–1.02c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Prime factorisation is a coordinate system for whole numbers: each prime has an exponent. Divisibility, greatest common factors, least common multiples and perfect-square or perfect-cube conditions become comparisons of those exponents.

KEY VOCABULARY

prime • divisor • congruence • HCF • LCM • perfect power

2^4	3^2	5^1
max → LCM	compare powers	min → HCF

Why the method works

- Unique prime factorisation makes exponent comparison decisive.
- A sixth power is simultaneously a square and cube.
- Modular arithmetic records remainders efficiently.

CHECK BEFORE MOVING ON

Answer mentally: Write 756 as a product of prime factors. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Prime factors, divisibility, HCF and LCM — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Express each integer as a product of prime powers.
- Use minimum exponents for HCF and maximum exponents for LCM.
- Translate context: largest equal grouping suggests HCF; first coincidence suggests LCM.

WORKED EXAMPLE 1

Write 756 as a product of prime factors.

SOLUTION

1. Repeatedly divide by primes.
2. $756=4\times189=2^2\times3^3\times7$.

ANSWER: $2^2 \times 3^3 \times 7$

WORKED EXAMPLE 2

Given $A=2^4\times3^2\times5$ and $B=2^2\times3\times7$, find HCF(A,B) and LCM(A,B).

SOLUTION

1. HCF uses shared primes with smaller powers: $2^2\times3$.
2. LCM uses all primes with larger powers: $2^4\times3^2\times5\times7$.
3. Evaluate to 12 and 5040.

ANSWER: HCF=12, LCM=5040

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Prime factors, divisibility, HCF and LCM — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Repeating schedules, equal batch sizes and data-cycle synchronisation all rely on common multiples and divisors.

Common misconceptions

- Using maximum exponents for an HCF.
- Assuming HCF×LCM works for more than two numbers without care.
- Not checking a contextual answer is the least or greatest possible.

GUIDED TRY

Find the least positive integer n such that 180n is a perfect square. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Show that the sum of three consecutive integers is divisible by 3. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Prime factors, divisibility, HCF and LCM

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Write 756 as a product of prime factors.** [3]

2. **Given $A=2^4\times3^2\times5$ and $B=2^2\times3\times7$, find HCF(A,B) and LCM(A,B).** [4]

3. **Find the least positive integer n such that 180n is a perfect square.** [4]

4. **Two positive integers have HCF 18 and LCM 756. One integer is 108. Find the other.** [4]

5. **Prove that 7^n-1 is divisible by 6 for every positive integer n.** [4]

6. **Three beacons flash every 42 s, 70 s and 90 s. They flash together at noon. When next?** [5]
SHOW ALL WORKING

7. **The number 48p is both a square and a cube. Find the least positive integer p.** [5]
SHOW ALL WORKING

8. **Show that the sum of three consecutive integers is divisible by 3.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Fractions and exact proportional calculation

Complex fractions, exact arithmetic and reasoning with rational values.

SPECIFICATION FOCUS

OCR reference 2.01. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Fractions preserve exactness and reveal multiplicative structure. Complex fraction problems are safest when each numerator and denominator is simplified separately before the main division is changed into multiplication by a reciprocal.

KEY VOCABULARY

numerator • denominator • reciprocal • complex fraction • restriction • exact value

complex fraction → simplify top and bottom → multiply by reciprocal

keep exact values • cancel factors only • record denominator restrictions

Why the method works

- Equivalent fractions multiply by a disguised 1.
- Common denominators create equal-sized parts.
- Division by a non-zero fraction is multiplication by its reciprocal.

CHECK BEFORE MOVING ON

Answer mentally: Simplify (5/6 - 1/4) ÷ (7/9). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Fractions and exact proportional calculation — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Convert mixed numbers before multiplying or dividing.
- Use a common denominator only for addition or subtraction.
- Cancel factors, never terms joined by addition.

WORKED EXAMPLE 1

Simplify (5/6 - 1/4) ÷ (7/9).

SOLUTION

1. 5/6-1/4=10/12-3/12=7/12.
2. Divide by 7/9: 7/12×9/7.
3. Cancel and simplify to 9/12=3/4.

ANSWER: 3/4

WORKED EXAMPLE 2

Solve 3/5 of x plus 7/12 of x equals 71.

SOLUTION

1. Combine coefficients: 3/5+7/12=36/60+35/60=71/60.
2. So (71/60)x=71.
3. Multiply by 60/71 to obtain x=60.

ANSWER: x=60

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Fractions and exact proportional calculation — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Mixtures, scale factors and rates often produce ratios that should remain exact until the final interpretation.

Common misconceptions

- Cancelling across addition.
- Losing restrictions when algebraic denominators are present.
- Turning exact fractions into decimals too early.

GUIDED TRY

A tank is 3/8 full. After 84 litres are added it is 5/6 full. Find its capacity. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

The fraction (n+1)/(n+3) equals 4/5. Find n and check the denominator restriction. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Fractions and exact proportional calculation

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Simplify (5/6 - 1/4) ÷ (7/9).** **[4]**

2. **Solve 3/5 of x plus 7/12 of x equals 71.** **[4]**

3. **A tank is 3/8 full. After 84 litres are added it is 5/6 full. Find its capacity.** **[4]**

4. **Simplify the complex fraction (2/x + 1/3) ÷ (1/x - 1/6).** **[5]**

SHOW ALL WORKING

5. **Without a calculator, show that 37/60 lies between 0.61 and 0.62.** **[3]**

6. **A recipe uses flour:sugar = 7:3. Of the flour, 2/5 is wholemeal. In a 1.5 kg flour-and-sugar mixture, how much white flour is used?** **[5]**

SHOW ALL WORKING

7. **Find the exact value of 1/(1-2/3) - 1/(1+2/3).** **[4]**

8. **The fraction (n+1)/(n+3) equals 4/5. Find n and check the denominator restriction.** **[4]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Decimals, percentages and recurring values

Exact conversions, recurring-decimal algebra and compound percentage models.

SPECIFICATION FOCUS

OCR reference 2.02–2.04. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Recurring decimals are rational because their repetition can be eliminated algebraically. Percentage multipliers model the proportion retained after each change; repeated changes therefore compound multiplicatively.

KEY VOCABULARY

terminating • recurring • multiplier • reverse percentage • compound change

$$x = 0.1818\ldots \qquad 100x = 18.1818\ldots$$

aligned recurring tails subtract exactly: $99x = 18$

Why the method works

- Shifting by the length of a repeating block aligns identical decimal tails.
- Subtracting then removes the infinite recurring part.
- Successive percentage changes act on changing bases.

CHECK BEFORE MOVING ON

Answer mentally: Write 0.18 (0.181818...) as a fraction in simplest form. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Decimals, percentages and recurring values — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Use algebraic subtraction to eliminate a recurring block.
- Percentage multipliers compose by multiplication, not addition.
- Reverse change divides by the multiplier acting on the original.

WORKED EXAMPLE 1

Write 0.18 (0.181818...) as a fraction in simplest form.

SOLUTION

1. Let x=0.181818...
2. Then 100x=18.181818...
3. Subtract: 99x=18.
4. So x=18/99=2/11.

ANSWER: 2/11

WORKED EXAMPLE 2

Write 0.23 (0.2333...) as a fraction in simplest form.

SOLUTION

1. Let x=0.2333... and 10x=2.333...
2. Also 100x=23.333...
3. Subtract: 90x=21.
4. Thus x=7/30.

ANSWER: 7/30

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Decimals, percentages and recurring values — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Finance, depreciation and population models depend on multiplicative change and careful interpretation of the base value.

Common misconceptions

- Using 10x when two recurring digits require 100x.
- Adding percentage changes across successive stages.
- Dividing by the percentage rather than its multiplier.

GUIDED TRY

A value rises by 12% and then falls by 12%. Find the overall percentage change. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A shop increases a price by p% and later restores the original price by reducing the new price by 20%. Find p. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Decimals, percentages and recurring values

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Write $0.\dot{1}\dot{8}$ (0.181818...) as a fraction in simplest form.** [4]

2. **Write $0.2\dot{3}$ (0.2333...) as a fraction in simplest form.** [4]

3. **A value rises by 12% and then falls by 12%. Find the overall percentage change.** [4]

4. **After a 15% discount and then 20% VAT, an item costs £918. Find its original marked price.** [5]
SHOW ALL WORKING

5. **A population grows by 3.5% each year. It is initially 48,000. Find the population after 6 years.** [4]

6. **A machine loses 18% of its value each year. After 4 years it is worth £13,562.40. Find its original value.** [5]
SHOW ALL WORKING

7. **Prove that $0.\dot{9}=1$.** [4]

8. **A shop increases a price by p% and later restores the original price by reducing the new price by 20%. Find p.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Integer, negative and fractional indices

Index laws extended to reciprocals, roots and algebraic simplification.

SPECIFICATION FOCUS

OCR reference 3.01a–3.01b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Index laws extend repeated multiplication consistently to zero, negative and fractional exponents. A negative exponent reverses a scale; a denominator in a fractional exponent specifies a root.

KEY VOCABULARY

base • index • reciprocal • radical • fractional power • exponent equation

$$a^{-n} = 1/a^n \qquad a^{(m/n)} = (\sqrt[n]{a})^m$$

negative exponent → reciprocal • denominator → root • numerator → power

Why the method works

- Subtracting equal exponents gives $a^0=1$.
- Continuing the exponent pattern downward produces reciprocals.
- $a^{(m/n)}$ is the nth root of a raised to the mth power.

CHECK BEFORE MOVING ON

Answer mentally: Simplify $(6x^5y^{-2})(3x^{-7}y^4)$. Give positive indices. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Integer, negative and fractional indices — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Use laws only for a common base.
- Interpret a negative index as a reciprocal.
- Interpret a fractional index as a root and power.

WORKED EXAMPLE 1

Simplify $(6x^5y^{-2})(3x^{-7}y^4)$. Give positive indices.

SOLUTION

1. Multiply coefficients.
2. Add indices for each common base: $x^{-2}y^2$.
3. Rewrite x^{-2} as $1/x^2$.

ANSWER: $18y^2/x^2$

WORKED EXAMPLE 2

Evaluate 81^(3/4) without a calculator.

SOLUTION

1. Fourth root of 81 is 3.
2. Then cube: $3^3=27$.
3. Equivalently cube first and fourth-root.

ANSWER: 27

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Integer, negative and fractional indices — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Exponential scales occur in science, compound growth, algorithms and models spanning very large ranges.

Common misconceptions

- Applying a law across addition.
- Forgetting that a negative exponent does not make the value negative.
- Ignoring domain restrictions for even roots.

GUIDED TRY

Simplify $(16a^8b^{-4})^{(1/2)}$, assuming $a,b>0$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Find k if $(27x^6)^{(2/3)}=kx^4$ for $x>0$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Integer, negative and fractional indices

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Simplify $(6x^5y^{-2})(3x^{-7}y^4)$. Give positive indices.** **[4]**

2. **Evaluate 81^(3/4) without a calculator.** **[4]**

3. **Simplify $(16a^8b^{-4})$^(1/2), assuming a,b>0.** **[4]**

4. **Solve 2^(3x-1)=32.** **[4]**

5. **Show that (a^(1/2)-b^(1/2))(a^(1/2)+b^(1/2))=a-b.** **[3]**

6. **Simplify (x^(2/3) × x^(5/6)) ÷ x^(1/2).** **[4]**

7. **Evaluate 32^(-2/5).** **[4]**

8. **Find k if $(27x^6)$^(2/3)=kx^4 for x>0.** **[4]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Surds and exact simplification

Simplifying, combining and rationalising surd expressions.

SPECIFICATION FOCUS

OCR reference 3.01c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A surd is an exact irrational root. Simplification extracts square factors, while rationalising rewrites a quotient with a rational denominator without changing its value.

KEY VOCABULARY

surd • irrational • simplify • conjugate • rationalise • exact

$$\sqrt{72} = \sqrt{(36\times2)} = 6\sqrt{2}$$

extract square factors • combine only like surds • preserve exactness

Why the method works

- $\sqrt{(ab)}=\sqrt{a}\sqrt{b}$ for non-negative a and b.
- Conjugate products form a difference of squares.
- Like surds combine exactly as like algebraic terms.

CHECK BEFORE MOVING ON

Answer mentally: Simplify $\sqrt{180} - 2\sqrt{20} + \sqrt{45}$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Surds and exact simplification — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Extract square factors before combining surds.
- Only like surds add or subtract.
- Rationalise by multiplying by a suitable surd or conjugate.

WORKED EXAMPLE 1

Simplify $\sqrt{180} - 2\sqrt{20} + \sqrt{45}$.

SOLUTION

1. $\sqrt{180}=6\sqrt{5}$, $\sqrt{20}=2\sqrt{5}$, $\sqrt{45}=3\sqrt{5}$.
2. So $6\sqrt{5}-4\sqrt{5}+3\sqrt{5}=5\sqrt{5}$.

ANSWER: $5\sqrt{5}$

WORKED EXAMPLE 2

Expand and simplify $(3+\sqrt{5})(4-\sqrt{5})$.

SOLUTION

1. Expand four terms.
2. $12-3\sqrt{5}+4\sqrt{5}-5$.
3. Combine to $7+\sqrt{5}$.

ANSWER: $7+\sqrt{5}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Surds and exact simplification — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Exact lengths in geometry and trigonometry often contain roots; surd form protects exactness.

Common misconceptions

- Claiming $\sqrt{(a+b)}=\sqrt{a}+\sqrt{b}$.
- Combining unlike surds.
- Multiplying only the denominator during rationalisation.

GUIDED TRY

Rationalise $7/\sqrt{3}$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Prove that $3+2\sqrt{2}=(1+\sqrt{2})^2$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Surds and exact simplification

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Simplify $\sqrt{180} - 2\sqrt{20} + \sqrt{45}$.** [4]

2. **Expand and simplify $(3+\sqrt{5})(4-\sqrt{5})$.** [4]

3. **Rationalise $7/\sqrt{3}$.** [3]

4. **Rationalise and simplify $5/(2+\sqrt{3})$.** [5]

SHOW ALL WORKING

5. **Show that $\sqrt{48}/\sqrt{3}$ is an integer.** [3]

6. **Solve $(x+\sqrt{2})(x-\sqrt{2})=23$.** [4]

7. **The area of a square is 72 cm^2. Give its perimeter in simplest surd form.** [4]

8. **Prove that $3+2\sqrt{2}=(1+\sqrt{2})^2$.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Standard form and orders of magnitude

Multi-step standard-form arithmetic and interpretation of scale.

SPECIFICATION FOCUS

OCR reference 3.02. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Standard form separates significant digits from order of magnitude. Higher problems combine arithmetic, units and interpretation, so the coefficient and power of ten must be handled independently and then normalised.

KEY VOCABULARY

standard form • coefficient • exponent • magnitude • normalise • scale

$$(6.4\times10^{7})(3.5\times10^{-3}) = 22.4\times10^{4} = 2.24\times10^{5}$$

calculate coefficient and power separately, then normalise

Why the method works

- Multiplying powers adds exponents.
- Division subtracts exponents.
- Addition requires a common place-value scale.

CHECK BEFORE MOVING ON

Answer mentally: Calculate $(6.4\times10^{7})(3.5\times10^{-3})$. Give standard form. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Standard form and orders of magnitude — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Normalise coefficients so $1 \le A < 10$.
- Multiply/divide coefficients and powers separately.
- For addition/subtraction, first express terms with the same power.

WORKED EXAMPLE 1

Calculate $(6.4\times10^{7})(3.5\times10^{-3})$. Give standard form.

SOLUTION

1. $6.4\times3.5=22.4$.
2. $10^{7}\times10^{-3}=10^{4}$.
3. $22.4\times10^{4}=2.24\times10^{5}$.

ANSWER: 2.24×10^{5}

WORKED EXAMPLE 2

Calculate $(8.1\times10^{-5}) \div (2.7\times10^{3})$.

SOLUTION

1. Divide coefficients: $8.1\div2.7=3$.
2. Subtract exponents: $-5-3=-8$.

ANSWER: 3×10^{-8}

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Standard form and orders of magnitude — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Scientific measurements—from cells to astronomical distances—need compact notation and meaningful comparison.

Common misconceptions

- Adding exponents during addition.
- Leaving a coefficient outside the interval $1 \le A < 10$.
- Losing units when interpreting a scientific result.

GUIDED TRY

Calculate $7.2\times10^{6} + 4.85\times10^{5}$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A cube has side 2.5×10^{-4} m. Find its volume in standard form. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Standard form and orders of magnitude

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Calculate $(6.4\times10^{7})(3.5\times10^{-3})$. Give standard form.** [4]

2. **Calculate $(8.1\times10^{-5}) \div (2.7\times10^{3})$.** [4]

3. **Calculate $7.2\times10^{6} + 4.85\times10^{5}$.** [4]

4. **A signal travels 3.0×10^{8} m/s for 2.4×10^{-3} s. Find the distance in standard form.** [4]

5. **One cell has mass 4.2×10^{-12} g. Find the mass of 7.5×10^{8} cells.** [4]

6. **How many times larger is 5.6×10^{9} than 7×10^{5}?** [4]

7. **Estimate $(3.98\times10^{5})(6.12\times10^{-2})/(1.97\times10^{3})$ to one significant figure.** [4]

8. **A cube has side 2.5×10^{-4} m. Find its volume in standard form.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Rounding, estimation and significant figures

Accuracy choices, error awareness and reasonableness checks.

SPECIFICATION FOCUS

OCR reference 4.01a. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Rounding communicates precision; estimation checks plausibility. The requested accuracy determines the rounding unit, while an estimate deliberately selects nearby values that make mental structure visible.

KEY VOCABULARY

decimal place • significant figure • estimate • approximation • order of magnitude

0.00496781 → 0.00497 (3 significant figures)

locate the first non-zero digit • retain requested digits • inspect the next digit

Why the method works

- Rounding depends on the first discarded digit.
- Significant figures begin at the first non-zero digit.
- A good estimate preserves the scale of the original calculation.

CHECK BEFORE MOVING ON

Answer mentally: Round 0.00496781 to 3 significant figures. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Rounding, estimation and significant figures — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Identify the requested place before rounding.
- Use compatible one-significant-figure values for estimation.
- State whether an estimate is likely an over- or underestimate when asked.

WORKED EXAMPLE 1

Round 0.00496781 to 3 significant figures.

SOLUTION

1. The first significant digit is 4.
2. The fourth significant digit is 7, so 6 rounds up to 7.

ANSWER: 0.00497

WORKED EXAMPLE 2

Estimate $(19.8^2 - 4.9^2)/0.203$.

SOLUTION

1. Use 20^2-5^2 over 0.2.
2. This is $(400-25)/0.2=1875$.
3. A one-significant-figure estimate is 2000.

ANSWER: about 1875 or 2000

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Rounding, estimation and significant figures — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Engineering decisions and data summaries require stated precision and rapid detection of impossible calculator results.

Common misconceptions

- Counting leading zeros as significant.
- Rounding intermediate calculator values.
- Choosing approximations that make mental arithmetic no easier.

GUIDED TRY

A calculator gives 0.000739584. Write this to 2 significant figures and 5 decimal places. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Without calculating exactly, decide whether 49.8×0.203 is above or below 10. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Rounding, estimation and significant figures

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Round 0.00496781 to 3 significant figures.** [2]

2. **Estimate $(19.8^2 - 4.9^2)/0.203$.** [4]

3. **A calculator gives 0.000739584. Write this to 2 significant figures and 5 decimal places.** [3]

4. **Estimate $\sqrt{(158\times39.6)}$. State suitable rounded values.** [3]

5. **A journey is 286 km and takes 3 h 47 min. Estimate the average speed.** [4]

6. **Explain why rounding every intermediate value can make a final answer unreliable.** [3]

7. **The value of $(7.84\times3.17)/0.491$ is required to 3 significant figures. Give the answer.** [3]

8. **Without calculating exactly, decide whether 49.8×0.203 is above or below 10.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Bounds, error intervals and limits of accuracy

Error intervals and worst-case calculations for measured quantities.

SPECIFICATION FOCUS

OCR reference 4.01b–4.01c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

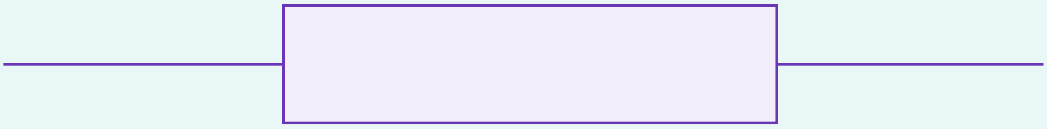
Understand the idea

A rounded measurement represents an interval, not a single exact value. Bounds questions ask for the worst permitted combination. The correct endpoint choices depend on how the target expression changes when each input changes.

KEY VOCABULARY

lower bound • upper bound • error interval • tolerance • maximum • minimum

rounded value represents every point in an interval



lower bound **upper bound**

Why the method works

- Nearest-unit rounding creates half-unit boundaries.
- Positive products increase with either factor.
- A positive quotient increases with its numerator but decreases with its denominator.

CHECK BEFORE MOVING ON

Answer mentally: A length is 8.4 cm correct to the nearest 0.1 cm. Write its error interval. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Bounds, error intervals and limits of accuracy — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Half the rounding unit gives each boundary.
- For positive quantities: maximum product uses upper bounds; minimum quotient uses numerator lower and denominator upper.
- Do not round a bound prematurely; preserve strict or inclusive endpoints.

WORKED EXAMPLE 1

A length is 8.4 cm correct to the nearest 0.1 cm. Write its error interval.

SOLUTION

1. Half of 0.1 is 0.05.
2. Subtract and add 0.05.
3. Lower bound is included; upper bound is excluded.

ANSWER: $8.35 \le L < 8.45$

WORKED EXAMPLE 2

A rectangle measures 12.6 cm by 7.2 cm, each correct to the nearest 0.1 cm. Find the upper bound for its area.

SOLUTION

1. Upper bounds are 12.65 and 7.25.
2. Multiply: $12.65 \times 7.25 = 91.7125$ cm².

ANSWER: 91.7125 cm²

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Bounds, error intervals and limits of accuracy — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Manufacturing tolerances, experimental measurements and safety margins require guaranteed limits rather than typical values.

Common misconceptions

- Including an open upper endpoint.
- Using all upper bounds automatically in a quotient.
- Rounding bounds before completing the calculation.

GUIDED TRY

A distance is 240 m correct to the nearest 10 m and time is 32 s correct to the nearest second. Find the upper bound for speed. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A cylinder’s measured radius is 4.0 cm and height is 10.0 cm, both to the nearest 0.1 cm. Give an exact expression for the upper bound of its volume. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Bounds, error intervals and limits of accuracy

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A length is 8.4 cm correct to the nearest 0.1 cm. Write its error interval.** **[3]**

2. **A rectangle measures 12.6 cm by 7.2 cm, each correct to the nearest 0.1 cm. Find the upper bound for its area.** **[4]**

3. **A distance is 240 m correct to the nearest 10 m and time is 32 s correct to the nearest second. Find the upper bound for speed.** **[5]**
SHOW ALL WORKING

4. **The mass of 18 identical components is 2.7 kg correct to the nearest 0.1 kg. Find the lower bound for one component's mass.** **[4]**

5. **x=5.2 correct to 1 decimal place and y=1.8 correct to 1 decimal place. Find the upper bound of (x+y)/(x-y).** **[5]**
SHOW ALL WORKING

6. **A square has area 50 cm² correct to the nearest cm². Find bounds for its side length.** **[5]**
SHOW ALL WORKING

7. **A rounded value is 0.036 correct to 2 significant figures. Write its error interval.** **[4]**

8. **A cylinder's measured radius is 4.0 cm and height is 10.0 cm, both to the nearest 0.1 cm. Give an exact expression for the upper bound of its volume.** **[5]**
SHOW ALL WORKING

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Higher Number problem solving

Connected problems requiring selection, justification and exact-to-approximate control.

SPECIFICATION FOCUS

OCR reference 1–4 synthesis. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Higher-tier Number questions frequently connect several ideas. Success depends on recognising structure—rather than matching a memorised surface pattern—and controlling the transition from exact work to justified approximation.

KEY VOCABULARY

model • exact • approximate • constraint • justification • synthesis

model → exact structure → controlled approximation → interpretation

check restrictions, units, magnitude and whether the conclusion answers the question

Why the method works

- A diagram, variable or multiplier exposes hidden relationships.
- Exact forms preserve information through multiple stages.
- A final reasonableness check tests magnitude, sign and units.

CHECK BEFORE MOVING ON

Answer mentally: Show that $(\sqrt{12}+\sqrt{27})^2$ is an integer and find it. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Higher Number problem solving — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Translate the context before choosing operations.
- Keep exact forms during reasoning, then round once if required.
- Check restrictions, units and whether a bound or ordinary value is needed.

WORKED EXAMPLE 1

Show that $(\sqrt{12}+\sqrt{27})^2$ is an integer and find it.

SOLUTION

1. Simplify $\sqrt{12}=2\sqrt{3}$ and $\sqrt{27}=3\sqrt{3}$.
2. Sum$=5\sqrt{3}$.
3. Square: $25\times3=75$.

ANSWER: 75

WORKED EXAMPLE 2

A quantity is written as 0.45 of N and also as 5×10^3. Find N exactly.

SOLUTION

1. $0.4545\ldots=5/11$.
2. So $(5/11)N=5000$.
3. $N=5000\times11/5=11000$.

ANSWER: 11,000

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Higher Number problem solving — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Real decisions combine growth, measurement, tolerance and scale; the mathematics must remain interpretable throughout.

Common misconceptions

- Starting arithmetic before modelling the question.
- Forcing every problem into one familiar method.
- Giving a value without answering the contextual decision.

GUIDED TRY

The sides of a rectangle are $(3+\sqrt{2})$ cm and $(5-\sqrt{2})$ cm. Find its exact area. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A value x is 4.8 correct to the nearest 0.1. Determine whether $1/x$ can round to 0.21 to 2 decimal places. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Higher Number problem solving

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Show that $(\sqrt{12}+\sqrt{27})^2$ is an integer and find it.** [5]

SHOW ALL WORKING

2. **A quantity is written as 0.45 of N and also as 5×10^3. Find N exactly.** [5]

SHOW ALL WORKING

3. **The sides of a rectangle are $(3+\sqrt{2})$ cm and $(5-\sqrt{2})$ cm. Find its exact area.** [4]

4. **A culture contains 3.2×10^6 cells and grows by 8% per hour. Estimate when it first exceeds 5×10^6 cells.** [5]

SHOW ALL WORKING

5. **A cube has measured side 6.2 cm to the nearest 0.1 cm. Find the upper bound of its surface area.** [5]

SHOW ALL WORKING

6. **Find the least integer k for which $3.7\times10^k > 8.1\times10^{12}$.** [4]

7. **Prove that the sum of the squares of two consecutive odd integers is 2 more than a multiple of 8.** [5]

SHOW ALL WORKING

8. **A value x is 4.8 correct to the nearest 0.1. Determine whether 1/x can round to 0.21 to 2 decimal places.** [5]

SHOW ALL WORKING

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Number structure and exact calculation

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Evaluate $6 - 3[2 - 5(1 - 4)]^2$.

ANSWER: -861

Method cue: Evaluate the inner bracket: $1 - 4 = -3$. Check sign, size, accuracy and units.

2. Prove that the square of any odd integer is odd.

ANSWER: $(2n+1)^2 = 2(2n^2+2n)+1$, so it is odd

Method cue: Write an arbitrary odd integer as $2n+1$. Check sign, size, accuracy and units.

3. Arrange $\sqrt{50}$, 7.05, 22/3 and $2\pi+1$ in ascending order. Show sufficient comparisons.

ANSWER: $7.05 < \sqrt{50} < 2\pi+1 < 22/3$

Method cue: Use reliable decimal estimates. Check sign, size, accuracy and units.

4. Find an irrational number strictly between 3.1 and 3.2 and justify your choice.

ANSWER: $\sqrt{10}$

Method cue: Square the endpoints: $3.1^2=9.61$ and $3.2^2=10.24$. Check sign, size, accuracy and units.

5. A student claims: "the product of two irrational numbers is irrational." Decide whether ...

ANSWER: False; $\sqrt{2} \times \sqrt{2} = 2$

Method cue: One counterexample disproves an universal claim. Check sign, size, accuracy and units.

6. Evaluate exactly $(3/4 - 5/6) \div (7/9 + 1/3)$.

ANSWER: -3/40

Method cue: First bracket: $9/12-10/12=-1/12$. Check sign, size, accuracy and units.

7. The integers a and b satisfy a+b odd. Prove that a^2+b^2 is odd.

ANSWER: One is even and one odd; their squares retain parity, so the sum is odd

Method cue: An odd sum requires opposite parity. Check sign, size, accuracy and units.

8. Insert one pair of brackets into $18 \div 3 + 3 \times 2$ so that the value is 2.

ANSWER: $18 \div (3+3\times2)=2$

Method cue: Work backwards: 18 divided by 9 gives 2. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Prime factors, divisibility, HCF and LCM

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Write 756 as a product of prime factors.

ANSWER: $2^2 \times 3^3 \times 7$

Method cue: Repeatedly divide by primes. Check sign, size, accuracy and units.

2. Given $A=2^4\times3^2\times5$ and $B=2^2\times3\times7$, find HCF(A,B) and LCM(A,B).

ANSWER: HCF=12, LCM=5040

Method cue: HCF uses shared primes with smaller powers: $2^2\times3$. Check sign, size, accuracy and units.

3. Find the least positive integer n such that 180n is a perfect square.

ANSWER: n=5

Method cue: $180=2^2\times3^2\times5$. Check sign, size, accuracy and units.

4. Two positive integers have HCF 18 and LCM 756. One integer is 108. Find the other.

ANSWER: 126

Method cue: Use HCF×LCM=product for two positive integers. Check sign, size, accuracy and units.

5. Prove that 7^n-1 is divisible by 6 for every positive integer n.

ANSWER: True

Method cue: Since $7\equiv1 \pmod 6$, $7^n\equiv1^n\equiv1 \pmod 6$. Check sign, size, accuracy and units.

6. Three beacons flash every 42 s, 70 s and 90 s. They flash together at noon. When next?

ANSWER: 12:10:30

Method cue: Find LCM. Check sign, size, accuracy and units.

7. The number 48p is both a square and a cube. Find the least positive integer p.

ANSWER: p=972

Method cue: A square and cube is a sixth power. Check sign, size, accuracy and units.

8. Show that the sum of three consecutive integers is divisible by 3.

ANSWER: $(n-1)+n+(n+1)=3n$

Method cue: Represent consecutive integers algebraically. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Fractions and exact proportional calculation

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Simplify (5/6 - 1/4) ÷ (7/9).

ANSWER: 3/4

Method cue: 5/6-1/4=10/12-3/12=7/12. Check sign, size, accuracy and units.

2. Solve 3/5 of x plus 7/12 of x equals 71.

ANSWER: x=60

Method cue: Combine coefficients: 3/5+7/12=36/60+35/60=71/60. Check sign, size, accuracy and units.

3. A tank is 3/8 full. After 84 litres are added it is 5/6 full. Find its capacity.

ANSWER: 183 3/11 litres

Method cue: Added fraction=5/6-3/8=20/24-9/24=11/24. Check sign, size, accuracy and units.

4. Simplify the complex fraction (2/x + 1/3) ÷ (1/x - 1/6).

ANSWER: 2(6+x)/(6-x)

Method cue: Combine the numerator over 3x: (6+x)/(3x). Check sign, size, accuracy and units.

5. Without a calculator, show that 37/60 lies between 0.61 and 0.62.

ANSWER: 0.61 < 37/60 < 0.62

Method cue: Convert decimal bounds to fractions with denominator 100. Check sign, size, accuracy and units.

6. A recipe uses flour:sugar = 7:3. Of the flour, 2/5 is wholemeal. In a 1.5 kg flour-and-s...

ANSWER: 630 g

Method cue: Flour is 7/10 of 1500 g=1050 g. Check sign, size, accuracy and units.

7. Find the exact value of 1/(1-2/3) - 1/(1+2/3).

ANSWER: 12/5

Method cue: First denominator is 1/3, giving 3. Check sign, size, accuracy and units.

8. The fraction (n+1)/(n+3) equals 4/5. Find n and check the denominator restriction.

ANSWER: n=7

Method cue: Cross-multiply: 5(n+1)=4(n+3). Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Decimals, percentages and recurring values

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Write $0.\dot{1}\dot{8}$ (0.181818...) as a fraction in simplest form.

ANSWER: 2/11

Method cue: Let x=0.181818... Check sign, size, accuracy and units.

2. Write $0.2\dot{3}$ (0.2333...) as a fraction in simplest form.

ANSWER: 7/30

Method cue: Let x=0.2333... and 10x=2.333... Check sign, size, accuracy and units.

3. A value rises by 12% and then falls by 12%. Find the overall percentage change.

ANSWER: 1.44% decrease

Method cue: Use multipliers 1.12 and 0.88. Check sign, size, accuracy and units.

4. After a 15% discount and then 20% VAT, an item costs £918. Find its original marked pric...

ANSWER: £900

Method cue: Combined multiplier=0.85×1.20=1.02. Check sign, size, accuracy and units.

5. A population grows by 3.5% each year. It is initially 48,000. Find the population after ...

ANSWER: about 59,004

Method cue: Use 48000×1.035^6. Check sign, size, accuracy and units.

6. A machine loses 18% of its value each year. After 4 years it is worth £13,562.40. Find i...

ANSWER: £30,000

Method cue: Multiplier over four years is 0.82^4. Check sign, size, accuracy and units.

7. Prove that $0.\dot{9}=1$.

ANSWER: Let x=0.999...; 10x-x=9, so x=1

Method cue: Let x=0.999... Check sign, size, accuracy and units.

8. A shop increases a price by p% and later restores the original price by reducing the new...

ANSWER: 25%

Method cue: Restoration gives (1+p/100)×0.8=1. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Integer, negative and fractional indices

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Simplify $(6x^5y^{-2})(3x^{-7}y^4)$. Give positive indices.

ANSWER: $18y^2/x^2$

Method cue: Multiply coefficients. Check sign, size, accuracy and units.

2. Evaluate 81^(3/4) without a calculator.

ANSWER: 27

Method cue: Fourth root of 81 is 3. Check sign, size, accuracy and units.

3. Simplify $(16a^8b^{-4})$^(1/2), assuming a,b>0.

ANSWER: $4a^4/b^2$

Method cue: Square-root each factor. Check sign, size, accuracy and units.

4. Solve 2^(3x-1)=32.

ANSWER: x=2

Method cue: Write 32 as 2^5. Check sign, size, accuracy and units.

5. Show that (a^(1/2)-b^(1/2))(a^(1/2)+b^(1/2))=a-b.

ANSWER: Difference of two squares

Method cue: Recognise $(u-v)(u+v)=u^2-v^2$. Check sign, size, accuracy and units.

6. Simplify (x^(2/3) × x^(5/6)) ÷ x^(1/2).

ANSWER: x

Method cue: Add then subtract exponents. Check sign, size, accuracy and units.

7. Evaluate 32^(-2/5).

ANSWER: 1/4

Method cue: Fifth root of 32 is 2. Check sign, size, accuracy and units.

8. Find k if $(27x^6)$^(2/3)=kx^4 for x>0.

ANSWER: k=9

Method cue: Cube-root $27x^6$ to get $3x^2$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Surds and exact simplification

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Simplify $\sqrt{180} - 2\sqrt{20} + \sqrt{45}$.

ANSWER: $5\sqrt{5}$

Method cue: $\sqrt{180}=6\sqrt{5}$, $\sqrt{20}=2\sqrt{5}$, $\sqrt{45}=3\sqrt{5}$. Check sign, size, accuracy and units.

2. Expand and simplify $(3+\sqrt{5})(4-\sqrt{5})$.

ANSWER: $7+\sqrt{5}$

Method cue: Expand four terms. Check sign, size, accuracy and units.

3. Rationalise $7/\sqrt{3}$.

ANSWER: $7\sqrt{3}/3$

Method cue: Multiply numerator and denominator by $\sqrt{3}$. Check sign, size, accuracy and units.

4. Rationalise and simplify $5/(2+\sqrt{3})$.

ANSWER: $10-5\sqrt{3}$

Method cue: Multiply by conjugate $(2-\sqrt{3})/(2-\sqrt{3})$. Check sign, size, accuracy and units.

5. Show that $\sqrt{48}/\sqrt{3}$ is an integer.

ANSWER: 4

Method cue: Combine under one radical: $\sqrt{(48/3)}=\sqrt{16}$. Check sign, size, accuracy and units.

6. Solve $(x+\sqrt{2})(x-\sqrt{2})=23$.

ANSWER: $x=\pm5$

Method cue: Use difference of squares: $x^2-2=23$. Check sign, size, accuracy and units.

7. The area of a square is 72 cm^2. Give its perimeter in simplest surd form.

ANSWER: $24\sqrt{2}$ cm

Method cue: Side$=\sqrt{72}=6\sqrt{2}$ cm. Check sign, size, accuracy and units.

8. Prove that $3+2\sqrt{2}=(1+\sqrt{2})^2$.

ANSWER: Expansion gives $1+2\sqrt{2}+2$

Method cue: Expand the square. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Standard form and orders of magnitude

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Calculate $(6.4\times10^{7})(3.5\times10^{-3})$. Give standard form.

ANSWER: 2.24×10^{5}

Method cue: $6.4\times3.5=22.4$. Check sign, size, accuracy and units.

2. Calculate $(8.1\times10^{-5}) \div (2.7\times10^{3})$.

ANSWER: 3×10^{-8}

Method cue: Divide coefficients: $8.1\div2.7=3$. Check sign, size, accuracy and units.

3. Calculate $7.2\times10^{6} + 4.85\times10^{5}$.

ANSWER: 7.685×10^{6}

Method cue: Rewrite 4.85×10^{5} as 0.485×10^{6}. Check sign, size, accuracy and units.

4. A signal travels 3.0×10^{8} m/s for 2.4×10^{-3} s. Find the distance in standard form.

ANSWER: 7.2×10^{5} m

Method cue: Distance=speed×time. Check sign, size, accuracy and units.

5. One cell has mass 4.2×10^{-12} g. Find the mass of 7.5×10^{8} cells.

ANSWER: 3.15×10^{-3} g

Method cue: Multiply coefficients and powers. Check sign, size, accuracy and units.

6. How many times larger is 5.6×10^{9} than 7×10^{5}?

ANSWER: 8000 times

Method cue: Form the quotient. Check sign, size, accuracy and units.

7. Estimate $(3.98\times10^{5})(6.12\times10^{-2})/(1.97\times10^{3})$ to one significant figure.

ANSWER: about 10

Method cue: Round to 4×10^{5}, 6×10^{-2} and 2×10^{3}. Check sign, size, accuracy and units.

8. A cube has side 2.5×10^{-4} m. Find its volume in standard form.

ANSWER: 1.5625×10^{-11} m³

Method cue: Cube coefficient and power. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Rounding, estimation and significant figures

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Round 0.00496781 to 3 significant figures.

ANSWER: 0.00497

Method cue: The first significant digit is 4. Check sign, size, accuracy and units.

2. Estimate $(19.8^2 - 4.9^2)/0.203$.

ANSWER: about 1875 or 2000

Method cue: Use 20^2-5^2 over 0.2. Check sign, size, accuracy and units.

3. A calculator gives 0.000739584. Write this to 2 significant figures and 5 decimal places...

ANSWER: 0.00074 in both cases

Method cue: For 2 s.f., retain 7 and 3 then round using 9. Check sign, size, accuracy and units.

4. Estimate $\sqrt{(158\times39.6)}$. State suitable rounded values.

ANSWER: about 80

Method cue: Round 158 to 160 and 39.6 to 40. Check sign, size, accuracy and units.

5. A journey is 286 km and takes 3 h 47 min. Estimate the average speed.

ANSWER: about 75 km/h

Method cue: Round distance to 300 km. Check sign, size, accuracy and units.

6. Explain why rounding every intermediate value can make a final answer unreliable.

ANSWER: Rounding errors accumulate and may change the final rounding

Method cue: Each rounded value replaces a range by one approximation. Check sign, size, accuracy and units.

7. The value of $(7.84\times3.17)/0.491$ is required to 3 significant figures. Give the answer.

ANSWER: 50.6

Method cue: Calculate with unrounded values: approximately 50.6179. Check sign, size, accuracy and units.

8. Without calculating exactly, decide whether 49.8×0.203 is above or below 10.

ANSWER: above 10

Method cue: 49.8 is slightly below 50; 0.203 is sufficiently above 0.2. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Bounds, error intervals and limits of accuracy

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A length is 8.4 cm correct to the nearest 0.1 cm. Write its error interval.

ANSWER: $8.35 \le L < 8.45$

Method cue: Half of 0.1 is 0.05. Check sign, size, accuracy and units.

2. A rectangle measures 12.6 cm by 7.2 cm, each correct to the nearest 0.1 cm. Find the upp...

ANSWER: 91.7125 cm^2

Method cue: Upper bounds are 12.65 and 7.25. Check sign, size, accuracy and units.

3. A distance is 240 m correct to the nearest 10 m and time is 32 s correct to the nearest ...

ANSWER: $245/31.5 \approx 7.78$ m/s

Method cue: Distance upper bound=245 m. Check sign, size, accuracy and units.

4. The mass of 18 identical components is 2.7 kg correct to the nearest 0.1 kg. Find the lo...

ANSWER: $2.65/18 \approx 0.1472$ kg

Method cue: Total mass lower bound=2.65 kg. Check sign, size, accuracy and units.

5. $x=5.2$ correct to 1 decimal place and $y=1.8$ correct to 1 decimal place. Find the upper bo...

ANSWER: $70/33 \approx 2.12$

Method cue: For positive $x>y$, the expression decreases as x increases and increases as y increases. Check sign, size, accuracy and units.

6. A square has area 50 cm^2 correct to the nearest cm^2. Find bounds for its side length.

ANSWER: $\sqrt{49.5} \le s < \sqrt{50.5}$

Method cue: Area interval is $49.5 \le A < 50.5$. Check sign, size, accuracy and units.

7. A rounded value is 0.036 correct to 2 significant figures. Write its error interval.

ANSWER: $0.0355 \le x < 0.0365$

Method cue: At this scale, 2 significant figures means nearest 0.001. Check sign, size, accuracy and units.

8. A cylinder's measured radius is 4.0 cm and height is 10.0 cm, both to the nearest 0.1 cm...

ANSWER: $\pi(4.05)^2(10.05)$ cm^3

Method cue: Cylinder volume is $\pi r^2 h$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Higher Number problem solving

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Show that $(\sqrt{12}+\sqrt{27})^2$ is an integer and find it.

ANSWER: 75

Method cue: Simplify $\sqrt{12}=2\sqrt{3}$ and $\sqrt{27}=3\sqrt{3}$. Check sign, size, accuracy and units.

2. A quantity is written as $0.4\dot{5}$ of N and also as 5×10^3. Find N exactly.

ANSWER: 11,000

Method cue: 0.4545...=5/11. Check sign, size, accuracy and units.

3. The sides of a rectangle are $(3+\sqrt{2})$ cm and $(5-\sqrt{2})$ cm. Find its exact area.

ANSWER: $13+2\sqrt{2}$ cm²

Method cue: Expand: $15-3\sqrt{2}+5\sqrt{2}-2$. Check sign, size, accuracy and units.

4. A culture contains 3.2×10^6 cells and grows by 8% per hour. Estimate when it first exceed...

ANSWER: after 6 hours

Method cue: Model $3.2\times10^6\times1.08^n$. Check sign, size, accuracy and units.

5. A cube has measured side 6.2 cm to the nearest 0.1 cm. Find the upper bound of its surfa...

ANSWER: $6(6.25)^2=234.375$ cm²

Method cue: Upper side bound=6.25 cm. Check sign, size, accuracy and units.

6. Find the least integer k for which $3.7\times10^k > 8.1\times10^{12}$.

ANSWER: k=13

Method cue: Compare orders of magnitude. Check sign, size, accuracy and units.

7. Prove that the sum of the squares of two consecutive odd integers is 2 more than a multi...

ANSWER: $(2n+1)^2+(2n+3)^2=8(n^2+2n+1)+2$

Method cue: Represent consecutive odd integers as $2n+1$ and $2n+3$. Check sign, size, accuracy and units.

8. A value x is 4.8 correct to the nearest 0.1. Determine whether 1/x can round to 0.21 to ...

ANSWER: Yes

Method cue: $4.75\le x<4.85$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

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