GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 3 | ALGEBRA

TEACH-FROM-SCRATCH • PUBLICATION EDITION

10 detailed chapters • original diagrams • worked examples • guided practice • answers

COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Practice third.

FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each worked solution and reproduce it. 3 Complete practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final statement when interpretation is required.

Error loop

REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

Higher success requires connected reasoning

OCR Higher algebra requires exact manipulation, proof, equation selection and interpretation. Grades 7–9 depend on connecting representations and communicating why each transformation is valid.

COMPREHENSIVE LESSON NOTEBOOK

Unit 3 coverage map

OCR J560 Algebra content organised into a teachable sequence.

CH 01	OCR 6.01a–6.01c	Algebraic structure, indices and proof
CH 02	OCR 6.01d–6.01e	Expansion and factorisation
CH 03	OCR 6.01f; 6.03b	Completing the square and quadratic form
CH 04	OCR 6.01g	Algebraic fractions
CH 05	OCR 6.02a–6.02e	Formulae, rearrangement and kinematics
CH 06	OCR 6.03a–6.03b	Linear and quadratic equations
CH 07	OCR 6.03c	Simultaneous equations: linear and quadratic
CH 08	OCR 6.03d–6.03e	Approximation and iteration
CH 09	OCR 6.04a–6.04b	Linear, quadratic and graphical inequalities
CH 10	OCR 6.05a–6.06b	Functions and advanced sequences

IMPORTANT TIER NOTE

This notebook covers OCR J560 section 6 at Higher Tier: expressions, formulae, equations, inequalities and sequences. Specification references are identifiers, not copied specification wording.

COMPREHENSIVE LESSON NOTEBOOK

Fast-reference method bank

Understand each relationship before memorising it.

Index quotient	$a^m \div a^n = a^{(m-n)}$
Quadratic formula	$x = (-b \pm \sqrt{b^2-4ac}) / 2a$
Discriminant	b^2-4ac: positive 2 roots; zero 1; negative 0
Complete square	$x^2+bx = (x+b/2)^2-(b/2)^2$
Algebraic fraction	factor first; cancel common factors only
Change subject	collect subject terms, factor, then divide
Quadratic inequality	find roots, then test sign intervals
Iteration	$x_{n+1}=g(x_n)$; retain unrounded values
Quadratic sequence	second difference = 2a
Kinematics	$v=u+at$; $s=ut+\frac{1}{2}at^2$; $v^2=u^2+2as$

CURRENT ASSESSMENT NOTE

OCR has announced Foundation and Higher formula sheets for GCSE Mathematics examinations in 2025, 2026 and 2027. Use the current official OCR sheet alongside this learning bank and check OCR for later exam-series arrangements.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic structure, indices and proof

Use precise algebraic language, simplify structure and construct general arguments.

SPECIFICATION FOCUS

OCR reference OCR 6.01a–6.01c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Algebra expresses general structure. Higher work requires exact simplification and proof that covers every allowed value, not numerical pattern spotting.

KEY VOCABULARY

identity • equation • coefficient • index • parity • proof

words	expression	equation

seven more than n → n + 7

Why the method works

- Index laws follow repeated-factor structure.
- Parity forms 2n and 2n+1 cover all integers.
- An identity remains true throughout its domain.

CHECK BEFORE MOVING ON

Answer mentally: Simplify $18a^5b^{-2} \div 6a^{-1}b^3$. Give positive indices. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic structure, indices and proof — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Separate coefficients from powers of each variable.
- An identity is true for every permitted value; an equation only for its solutions.
- A proof begins with a general algebraic form, not selected examples.

WORKED EXAMPLE 1

Simplify $18a^5b^{-2} \div 6a^{-1}b^3$. Give positive indices.

SOLUTION

1. Divide coefficients.
2. Subtract indices: a^(5-(-1))b^(-2-3).
3. Rewrite with positive indices.

ANSWER: $3a^6/b^5$

WORKED EXAMPLE 2

Prove that the product of two consecutive integers is even.

SOLUTION

1. Let the integers be n and n+1.
2. One of any two consecutive integers is even.
3. Therefore their product has factor 2.

ANSWER: n(n+1) is even

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic structure, indices and proof — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

General algebra validates algorithms and mathematical claims.

Common misconceptions

- Calling examples a proof.
- Combining unlike terms.
- Losing negative indices.

GUIDED TRY

Show that $(3x-2)^2-(3x+2)^2$ simplifies to $-24x$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

State whether $4x(x-3)=4x^2-12x$ is an equation or identity, and justify. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Algebraic structure, indices and proof

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Simplify $18a^5b^{-2} \div 6a^{-1}b^3$. Give positive indices.** [4]

2. **Prove that the product of two consecutive integers is even.** [4]

3. **Show that $(3x-2)^2-(3x+2)^2$ simplifies to $-24x$.** [3]

4. **Disprove: "n^2+n+41 is prime for every positive integer n."** [4]

5. **Simplify $(12x^3y^2)^2/(18x^4y^5)$.** [4]

6. **Prove the difference of the squares of consecutive integers is odd.** [4]

7. **If $x+y$ is odd, prove x^2+y^2 is odd for integer x,y.** [5]
SHOW ALL WORKING

8. **State whether $4x(x-3)=4x^2-12x$ is an equation or identity, and justify.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Expansion and factorisation

Expand multiple brackets and factorise monic and non-monic quadratics.

SPECIFICATION FOCUS

OCR reference OCR 6.01d–6.01e. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Expansion exposes coefficients; factorisation reverses the distributive law and exposes roots. Non-monic quadratics demand controlled factor pairs.

KEY VOCABULARY

binomial • trinomial • factor • root • discriminant

$$7x + 4x - 3x = 8x$$

combine coefficients only when the variable part matches

Why the method works

- Every term must multiply every term.
- Zero-product reasoning turns factors into solutions.
- Re-expansion verifies a factorisation.

CHECK BEFORE MOVING ON

Answer mentally: Expand and simplify (2x-3)(x+5)(x-1). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Expansion and factorisation — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Use every term in one factor with every term in the other.
- For ax^2+bx+c, split the middle term using factors of ac.
- Check factorisation by re-expanding.

WORKED EXAMPLE 1

Expand and simplify $(2x-3)(x+5)(x-1)$.

SOLUTION

1. First expand $(x+5)(x-1)=x^2+4x-5$.
2. Multiply by $2x-3$.
3. Collect terms: $-10x-12x=-22x$.

ANSWER: $2x^3+5x^2-22x+15$

WORKED EXAMPLE 2

Factorise completely $6x^2+x-2$.

SOLUTION

1. $ac=-12$; use 4 and -3.
2. $6x^2+4x-3x-2$.
3. Factor by grouping.

ANSWER: $(3x+2)(2x-1)$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Expansion and factorisation — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Quadratic structure models dimensions, trajectories and optimisation.

Common misconceptions

- Missing cross terms.
- Using factors of c but ignoring a.
- Stopping before full factorisation.

GUIDED TRY

Factorise $12x^3y-27xy^3$ completely. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Show $x^2+6x+11$ cannot be factorised into real linear factors. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Expansion and factorisation

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Expand and simplify $(2x-3)(x+5)(x-1)$.** [5]

SHOW ALL WORKING

2. **Factorise completely $6x^2+x-2$.** [4]

3. **Factorise $12x^3y-27xy^3$ completely.** [4]

4. **Find k if $(x+3)(2x+k)$ has x-coefficient 11.** [3]

5. **Factorise $8x^2-2$.** [3]

6. **Expand $(x-2)^3$.** [4]

7. **Solve by factorising $10x^2-17x+6=0$.** [5]

SHOW ALL WORKING

8. **Show $x^2+6x+11$ cannot be factorised into real linear factors.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Completing the square and quadratic form

Reveal turning points, ranges and solutions through completed-square form.

SPECIFICATION FOCUS

OCR reference OCR 6.01f; 6.03b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Completed-square form rewrites a quadratic as a translated square, making the turning point and range visible while supporting exact solution.

KEY VOCABULARY

completed square • vertex • axis • minimum • maximum

5(x + 3)

5x + 15

expand → **← factorise**

Why the method works

- A square is never negative.
- Half the linear coefficient locates the centre.
- Equivalent forms preserve the same graph.

CHECK BEFORE MOVING ON

Answer mentally: Write x^2-10x+7 as $(x\text{-}a)^2$+b. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Completing the square and quadratic form — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- For x^2+bx+c, add and subtract $(b/2)^2$.
- For ax^2+bx+c, factor a from x terms first.
- Completed-square form exposes the turning point immediately.

WORKED EXAMPLE 1

Write $x^2-10x+7$ as $(x-a)^2+b$.

SOLUTION

1. Half -10 to get -5.
2. $(x-5)^2=x^2-10x+25$.
3. Subtract 18.

ANSWER: $(x-5)^2-18$

WORKED EXAMPLE 2

Write $3x^2+12x-5$ in completed-square form.

SOLUTION

1. Factor 3 from x terms.
2. $x^2+4x=(x+2)^2-4$.
3. Multiply and combine constants.

ANSWER: $3(x+2)^2-17$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Completing the square and quadratic form — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Optimisation and graph analysis use vertex form.

Common misconceptions

- Forgetting an outer coefficient.
- Dropping ± when solving.
- Reading a minimum from a downward parabola.

GUIDED TRY

Find the minimum of $2x^2-8x+11$ and the x-value where it occurs. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Find exact roots of $2x^2-12x+7=0$ using completed square. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Completing the square and quadratic form

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Write $x^2-10x+7$ as $(x-a)^2+b$.** [3]

2. **Write $3x^2+12x-5$ in completed-square form.** [4]

3. **Find the minimum of $2x^2-8x+11$ and the x-value where it occurs.** [4]

4. **Solve $x^2+6x-4=0$ by completing the square.** [4]

5. **Find the range of $y=-(x-4)^2+9$.** [3]

6. **The graph $y=x^2+px+12$ has turning point $x=3$. Find p.** [4]

7. **Show x^2+4x+8 is always positive.** [4]

8. **Find exact roots of $2x^2-12x+7=0$ using completed square.** [5]

SHOW ALL WORKING

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic fractions

Simplify, combine and solve expressions containing algebraic denominators.

SPECIFICATION FOCUS

OCR reference OCR 6.01g. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Algebraic fractions behave like numerical fractions, but variable denominators create excluded values. Factorisation reveals legal cancellation.

KEY VOCABULARY

algebraic fraction • denominator • restriction • common factor

x	x²	2y	total
-3	9	10	19

Why the method works

- Cancellation divides numerator and denominator by one common non-zero factor.
- A common denominator permits addition.
- Original restrictions remain after simplification.

CHECK BEFORE MOVING ON

Answer mentally: Simplify $(x^2-9)/(x^2+x-6)$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic fractions — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Factor before cancelling.
- Cancel common factors only, never separate terms.
- State excluded values inherited from every original denominator.

WORKED EXAMPLE 1

Simplify $(x^2-9)/(x^2+x-6)$.

SOLUTION

1. Factor numerator=$(x-3)(x+3)$.
2. Denominator=$(x+3)(x-2)$.
3. Cancel $x+3$ while retaining the original restrictions.

ANSWER: $(x-3)/(x-2)$, $x\neq-3,2$

WORKED EXAMPLE 2

Write $3/x + 2/(x+1)$ as one fraction.

SOLUTION

1. Common denominator $x(x+1)$.
2. Numerator=$3(x+1)+2x$.
3. Simplify; $x\neq0,-1$.

ANSWER: $(5x+3)/(x(x+1))$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic fractions — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Rational models describe rates and reciprocal relationships.

Common misconceptions

- Cancelling across sums.
- Ignoring zero denominators.
- Cross-multiplying without restrictions.

GUIDED TRY

Simplify $(4x^2-16)/(2x^2+12x+16)$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain why $(x^2-1)/(x-1)=x+1$ is not valid at $x=1$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Algebraic fractions

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Simplify $(x^2-9)/(x^2+x-6)$.** [4]

2. **Write $3/x + 2/(x+1)$ as one fraction.** [4]

3. **Simplify $(4x^2-16)/(2x^2+12x+16)$.** [4]

4. **Solve $2/(x-1)=5/(x+2)$.** [4]

5. **Simplify $[x/(x-2)] \div [3x/(x^2-4)]$.** [5]

SHOW ALL WORKING

6. **Solve $1/x + 1/(x+2)=3/4$.** [5]

SHOW ALL WORKING

7. **Write $1/(x-1)-1/(x+1)$ as one fraction.** [3]

8. **Explain why $(x^2-1)/(x-1)=x+1$ is not valid at $x=1$.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Formulae, rearrangement and kinematics

Substitute carefully and change subject when it appears twice, squared or reciprocal.

SPECIFICATION FOCUS

OCR reference OCR 6.02a–6.02e. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Changing the subject reorganises a relationship without changing it. Complex cases use factorisation, reciprocals, roots and domain restrictions.

KEY VOCABULARY

subject • inverse operation • formula • reciprocal • kinematics

$$y = 3x - 5$$

+5 both sides → y+5 = 3x → ÷3 → x=(y+5)/3

Why the method works

- Equality is preserved by applying the same valid operation to both sides.
- Factoring collects repeated subjects.
- Physical quantities select meaningful root signs.

CHECK BEFORE MOVING ON

Answer mentally: Make x the subject of y=(3x-5)/(x+2). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Formulae, rearrangement and kinematics — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Factor the required subject if it appears in multiple terms.
- Undo outer operations in reverse order.
- When reversing a square, consider sign and physical restrictions.

WORKED EXAMPLE 1

Make x the subject of $y=(3x-5)/(x+2)$.

SOLUTION

1. Multiply: $yx+2y=3x-5$.
2. Collect x terms: $x(y-3)=-5-2y$.
3. Divide and simplify.

ANSWER: $x=(5+2y)/(3-y)$

WORKED EXAMPLE 2

Make r the subject of $V=\pi r^2h$.

SOLUTION

1. Divide by πh.
2. Take the positive square root for a radius.

ANSWER: $r=\sqrt{V/(\pi h)}$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Formulae, rearrangement and kinematics — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Science and engineering formulas must be rearranged reliably.

Common misconceptions

- Moving terms without changing signs correctly.
- Taking a root without considering ±.
- Substituting negatives without brackets.

GUIDED TRY

Make x the subject of $p=1/(x+a)$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Make x the subject of $y=\sqrt{(x-2)/5}$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Formulae, rearrangement and kinematics

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Make x the subject of $y=(3x-5)/(x+2)$.** [5]

SHOW ALL WORKING

2. **Make r the subject of $V=\pi r^2h$.** [4]

3. **Make x the subject of $p=1/(x+a)$.** [3]

4. **Make t the subject of $v=u+at$.** [3]

5. **Make a the subject of $v^2=u^2+2as$.** [4]

6. **Make x the subject of $y=ax+bx+c$.** [4]

7. **A particle has $u=7$ m/s, $a=-1.5$ m/s², $t=4$ s. Use $s=ut+at^2/2$.** [4]

8. **Make x the subject of $y=\sqrt{(x-2)/5}$.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Linear and quadratic equations

Select factorisation, completing square or quadratic formula and interpret roots.

SPECIFICATION FOCUS

OCR reference OCR 6.03a–6.03b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Equations impose conditions. Higher solving selects an efficient exact method and checks any roots created by squaring or denominator clearing.

KEY VOCABULARY

root • discriminant • quadratic formula • extraneous solution

$$4x + 3 = 27$$

subtract 3 from both sides • divide both sides by 4

Why the method works

- Rearranging to zero enables factor methods.
- The discriminant classifies real roots.
- Checking protects against extraneous roots.

CHECK BEFORE MOVING ON

Answer mentally: Solve (3x-2)/5-(x+4)/3=2. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Linear and quadratic equations — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Clear fractions using a common non-zero multiplier.
- Rearrange a quadratic to zero before selecting a method.
- Check roots in the original equation and context.

WORKED EXAMPLE 1

Solve $(3x-2)/5-(x+4)/3=2$.

SOLUTION

1. Multiply by 15.
2. $3(3x-2)-5(x+4)=30$.
3. $4x-26=30$, so $x=14$.

ANSWER: $x=14$

WORKED EXAMPLE 2

Solve $3x^2-5x-4=0$. Give exact answers.

SOLUTION

1. Use quadratic formula.
2. Discriminant$=25+48=73$.
3. Divide by 6.

ANSWER: $x=(5\pm\sqrt{73})/6$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Linear and quadratic equations — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Design constraints and intersections often reduce to quadratics.

Common misconceptions

- Using formula before identifying a,b,c.
- Accepting impossible contextual roots.
- Dividing by an expression that could be zero.

GUIDED TRY

A rectangle has sides $x+2$ and $x-3$ and area 50. Find x. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

The roots of $x^2-7x+10=0$ are α,β. Find $\alpha^2+\beta^2$ without solving. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Linear and quadratic equations

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Solve $(3x-2)/5-(x+4)/3=2$.** [5]

SHOW ALL WORKING

2. **Solve $3x^2-5x-4=0$. Give exact answers.** [5]

SHOW ALL WORKING

3. **A rectangle has sides $x+2$ and $x-3$ and area 50. Find x.** [5]

SHOW ALL WORKING

4. **Solve $x^4-13x^2+36=0$.** [5]

SHOW ALL WORKING

5. **Solve $\sqrt{(2x+3)}=x$.** [5]

SHOW ALL WORKING

6. **Find k so $x^2-6x+k=0$ has one repeated root.** [4]

7. **Solve $2/(x-1)=x$, stating restrictions.** [5]

SHOW ALL WORKING

8. **The roots of $x^2-7x+10=0$ are α,β. Find $\alpha^2+\beta^2$ without solving.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Simultaneous equations: linear and quadratic

Find all intersections of two equations using substitution or elimination.

SPECIFICATION FOCUS

OCR reference OCR 6.03c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Simultaneous solutions satisfy both relationships and are graphically their intersection points. A line and curve can create a quadratic and multiple coordinate pairs.

KEY VOCABULARY

simultaneous • intersection • elimination • substitution • tangent

$$(x - 3)(x - 4) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0 \quad \rightarrow \quad x = 3 \text{ or } 4$$

Why the method works

- Elimination preserves the common solution set.
- Substitution enforces both equations.
- Each quadratic root needs back-substitution.

CHECK BEFORE MOVING ON

Answer mentally: Solve $3x+2y=19$ and $5x-y=23$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Simultaneous equations: linear and quadratic — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Use elimination for two linear equations.
- For a line and curve, substitute the linear expression into the quadratic.
- Back-substitute every valid root to form coordinate pairs.

WORKED EXAMPLE 1

Solve $3x+2y=19$ and $5x-y=23$.

SOLUTION

1. Double the second equation: $10x-2y=46$.
2. Add to the first: $13x=65$, so $x=5$.
3. Substitute to obtain $y=2$.

ANSWER: $x=5, y=2$

WORKED EXAMPLE 2

Solve $y=x+1$ and $y=x^2-5x+7$.

SOLUTION

1. Set expressions equal.
2. $x^2-6x+6=0$.
3. Use the quadratic formula: $x=3\pm\sqrt{3}$.
4. Then $y=x+1=4\pm\sqrt{3}$.

ANSWER: $(3-\sqrt{3}, 4-\sqrt{3})$ and $(3+\sqrt{3}, 4+\sqrt{3})$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Simultaneous equations: linear and quadratic — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Linked price, geometry and motion models use shared unknowns.

Common misconceptions

- Finding x values but no y values.
- Discarding a second intersection.
- Combining unlike equations incorrectly.

GUIDED TRY

Solve $x+y=7$ and $xy=10$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain geometrically why a line and parabola can have 0, 1 or 2 real simultaneous solutions. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Simultaneous equations: linear and quadratic

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Solve $3x+2y=19$ and $5x-y=23$.** [4]

2. **Solve $y=x+1$ and $y=x^2-5x+7$.** [5]
SHOW ALL WORKING

3. **Solve $x+y=7$ and $xy=10$.** [4]

4. **Solve $y=2x-3$ and $x^2+y^2=25$.** [5]
SHOW ALL WORKING

5. **Solve $x^2+y^2=13$ and $y=x-1$.** [5]
SHOW ALL WORKING

6. **A line $y=kx$ intersects $y=x^2$ at exactly one point. What can be said?** [4]

7. **Solve $2x+3y=7$ and $4x-3y=11$.** [3]

8. **Explain geometrically why a line and parabola can have 0, 1 or 2 real simultaneous solutions.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Approximation and iteration

Locate roots using sign changes and generate convergent iterative estimates.

SPECIFICATION FOCUS

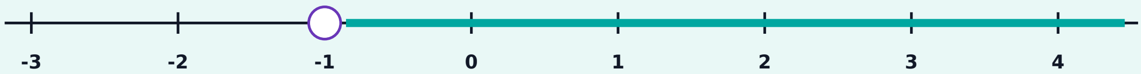
OCR reference OCR 6.03d–6.03e. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Approximation is necessary when exact algebra is unavailable. Sign changes locate roots; iteration repeatedly applies a rearranged rule and must be checked for stability.

KEY VOCABULARY

iteration • sign change • interval • convergence • accuracy



x > -1: open boundary, values to the right

Why the method works

- Continuity plus a sign change guarantees a crossing.
- More iterations can stabilise digits.
- Different rearrangements have different convergence behaviour.

CHECK BEFORE MOVING ON

Answer mentally: Show $x^3-x-1=0$ has a root between 1.3 and 1.4. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Approximation and iteration — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- A sign change across a continuous interval traps at least one root.
- Round only after producing sufficient iterations.
- Check convergence: successive values should stabilise.

WORKED EXAMPLE 1

Show $x^3-x-1=0$ has a root between 1.3 and 1.4.

SOLUTION

1. $f(1.3)=2.197-1.3-1=-0.103$.
2. $f(1.4)=2.744-1.4-1=0.344$.
3. Sign change implies a root.

ANSWER: $f(1.3)<0$ and $f(1.4)>0$

WORKED EXAMPLE 2

Use iteration $x_{n+1}=\sqrt[3]{x_n+1}$, $x_1=1.3$, to find x_4.

SOLUTION

1. $x_2=\sqrt[3]{2.3}\approx1.320006$.
2. $x_3\approx\sqrt[3]{2.320006}\approx1.323822$.
3. $x_4\approx\sqrt[3]{2.323822}\approx1.324548$.

ANSWER: about 1.3245

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Approximation and iteration — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Numerical methods power scientific and computer solutions.

Common misconceptions

- Claiming a root from same-sign endpoints.
- Rounding each iterate.
- Assuming every iteration converges.

GUIDED TRY

Find an interval width 0.1 containing a root of $x^2-3x-1=0$ near 3. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A graph of $y=f(x)$ crosses the x-axis at $x\approx-1.6$ and 2.4. State approximate solutions of $f(x)=0$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Approximation and iteration

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Show $x^3-x-1=0$ has a root between 1.3 and 1.4.** [4]

2. **Use iteration $x_{n+1}=\sqrt[3]{x_n+1}$, $x_1=1.3$, to find x_4.** [4]

3. **Find an interval width 0.1 containing a root of $x^2-3x-1=0$ near 3.** [4]

4. **Given $x_{n+1}=\sqrt{6+x_n}$, $x_1=2.5$, calculate x_3.** [3]

5. **An iteration alternates 1,4,1,4,... Does it converge?** [2]

6. **Explain why rounding every iterate to 1 decimal place may give a wrong final root.** [3]

7. **Use decimal search to solve $x^3=20$ to 2 decimal places.** [5]

SHOW ALL WORKING

8. **A graph of $y=f(x)$ crosses the x-axis at $x\approx-1.6$ and 2.4. State approximate solutions of $f(x)=0$.** [2]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Linear, quadratic and graphical inequalities

Solve ranges and represent intersections on number lines or coordinate planes.

SPECIFICATION FOCUS

OCR reference OCR 6.04a–6.04b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

An inequality describes a set. Quadratic signs change at roots; graphical inequalities divide the plane into half-planes whose intersection is the feasible region.

KEY VOCABULARY

inequality • interval • boundary • region • critical value

4	7	10	13	...

common difference +3 → nth term 3n + 1

Why the method works

- Multiplying by a negative reverses order.
- Test intervals determine quadratic sign.
- Included equality uses a solid boundary.

CHECK BEFORE MOVING ON

Answer mentally: Solve 5-3x≤17 and give interval notation. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Linear, quadratic and graphical inequalities — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Reverse the inequality when multiplying or dividing by a negative.
- For a quadratic, find critical roots and test intervals.
- For two variables, test a point to select the correct half-plane.

WORKED EXAMPLE 1

Solve $5-3x\leq17$ and give interval notation.

SOLUTION

1. Subtract 5: $-3x\leq12$.
2. Divide by -3 and reverse sign.

ANSWER: $x\geq-4$, $[-4,\infty)$

WORKED EXAMPLE 2

Solve $x^2-5x+6<0$.

SOLUTION

1. Factor $(x-2)(x-3)$.
2. Upward parabola is negative between roots.

ANSWER: $2<x<3$

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Linear, quadratic and graphical inequalities — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Feasible regions express budgets, capacities and safety limits.

Common misconceptions

- Treating an inequality as one value.
- Forgetting to reverse a sign.
- Shading without testing a point.

GUIDED TRY

Solve $2x^2+x-6\geq 0$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Solve $(x-1)/(x+2)>0$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Linear, quadratic and graphical inequalities

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Solve $5-3x\leq 17$ and give interval notation.** [3]

2. **Solve $x^2-5x+6<0$.** [4]

3. **Solve $2x^2+x-6\geq 0$.** [5]

SHOW ALL WORKING

4. **Solve $-2<3x+4\leq 13$.** [4]

5. **Find integer solutions of $x^2\leq 20$.** [3]

6. **Does (2,3) satisfy $y>2x-2$ and $x+y\leq 6$?** [3]

7. **Describe the boundary and shading for $y\leq -x+4$.** [3]

8. **Solve $(x-1)/(x+2)>0$.** [5]

SHOW ALL WORKING

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Functions and advanced sequences

Use inverse/composite processes and derive quadratic or geometric sequence rules.

SPECIFICATION FOCUS

OCR reference OCR 6.05a–6.06b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Functions are processes with defined inputs and outputs. Sequences are functions on positive integer positions; differences, ratios and recurrences reveal their structure.

KEY VOCABULARY

function • input • output • inverse • composite • recurrence

model	simplify	solve	check	interpret

a complete algebraic argument has a logical chain

Why the method works

- Inverse processes undo in reverse order.
- Composition order matters.
- Constant second difference identifies a quadratic sequence.

CHECK BEFORE MOVING ON

Answer mentally: Function f multiplies by 3 then subtracts 5. Find f(7) and the inverse process. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Functions and advanced sequences — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- For composite processes, apply functions in the stated order.
- Reverse an operation chain in reverse order.
- For quadratic sequences, constant second difference gives coefficient a=second difference/2.

WORKED EXAMPLE 1

Function f multiplies by 3 then subtracts 5. Find f(7) and the inverse process.

SOLUTION

1. $f(7)=21-5=16$.
2. Undo subtraction, then multiplication.

ANSWER: 16; add 5 then divide by 3

WORKED EXAMPLE 2

$f(x)=2x+1$ and $g(x)=x^2$. Find $g(f(3))$ and $f(g(3))$.

SOLUTION

1. $f(3)=7$, then $g(7)=49$.
2. $g(3)=9$, then $f(9)=19$.

ANSWER: 49 and 19

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Functions and advanced sequences — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Functions organise transformations, algorithms and repeated models.

Common misconceptions

- Assuming $f(g(x))=g(f(x))$.
- Using term-to-term pattern as nth term.
- Forgetting $n-1$ in a geometric rule.

GUIDED TRY

Find the nth term of 5,12,23,38,57,... First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain why sequence n^2+n is always even. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Functions and advanced sequences

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Function f multiplies by 3 then subtracts 5. Find f(7) and the inverse process.** [3]

2. **$f(x)=2x+1$ and $g(x)=x^2$. Find $g(f(3))$ and $f(g(3))$.** [4]

3. **Find the nth term of 5,12,23,38,57,...** [5]

SHOW ALL WORKING

4. **Find the next two terms and nth term of 3,6,12,24,...** [3]

5. **Which term of $4n^2-3$ is 897?** [4]

6. **A quadratic sequence has nth term an^2+bn+c and begins 6,15,28. Find a,b,c.** [5]

SHOW ALL WORKING

7. **The recurrence $u_{(n+1)}=0.5u_n+6$, $u_1=20$. Find u_4 and limiting value.** [5]

SHOW ALL WORKING

8. **Explain why sequence n^2+n is always even.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Algebraic structure, indices and proof

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Simplify $18a^5b^{-2} \div 6a^{-1}b^3$. Give positive indices.

ANSWER: $3a^6/b^5$

Method cue: Divide coefficients. Check sign, size, accuracy and units.

2. Prove that the product of two consecutive integers is even.

ANSWER: $n(n+1)$ is even

Method cue: Let the integers be n and $n+1$. Check sign, size, accuracy and units.

3. Show that $(3x-2)^2-(3x+2)^2$ simplifies to $-24x$.

ANSWER: $-24x$

Method cue: Expand both squares. Check sign, size, accuracy and units.

4. Disprove: "n^2+n+41 is prime for every positive integer n."

ANSWER: $n=41$ gives 41×43

Method cue: A single counterexample is sufficient. Check sign, size, accuracy and units.

5. Simplify $(12x^3y^2)^2/(18x^4y^5)$.

ANSWER: $8x^2/(y)$

Method cue: Square numerator: $144x^6y^4$. Check sign, size, accuracy and units.

6. Prove the difference of the squares of consecutive integers is odd.

ANSWER: $(n+1)^2-n^2=2n+1$

Method cue: Represent consecutive integers $n,n+1$. Check sign, size, accuracy and units.

7. If $x+y$ is odd, prove x^2+y^2 is odd for integer x,y.

ANSWER: One integer is even and the other odd

Method cue: Opposite parity is necessary for an odd sum. Check sign, size, accuracy and units.

8. State whether $4x(x-3)=4x^2-12x$ is an equation or identity, and justify.

ANSWER: Identity

Method cue: Expanding the left gives the right for every x. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Expansion and factorisation

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Expand and simplify $(2x-3)(x+5)(x-1)$.

ANSWER: $2x^3+5x^2-22x+15$

Method cue: First expand $(x+5)(x-1)=x^2+4x-5$. Check sign, size, accuracy and units.

2. Factorise completely $6x^2+x-2$.

ANSWER: $(3x+2)(2x-1)$

Method cue: $ac=-12$; use 4 and -3. Check sign, size, accuracy and units.

3. Factorise $12x^3y-27xy^3$ completely.

ANSWER: $3xy(2x-3y)(2x+3y)$

Method cue: Take common factor $3xy$. Check sign, size, accuracy and units.

4. Find k if $(x+3)(2x+k)$ has x-coefficient 11.

ANSWER: $k=5$

Method cue: Expand: $2x^2+(k+6)x+3k$. Check sign, size, accuracy and units.

5. Factorise $8x^2-2$.

ANSWER: $2(2x-1)(2x+1)$

Method cue: Take factor 2. Check sign, size, accuracy and units.

6. Expand $(x-2)^3$.

ANSWER: $x^3-6x^2+12x-8$

Method cue: Use $(x-2)^2=x^2-4x+4$. Check sign, size, accuracy and units.

7. Solve by factorising $10x^2-17x+6=0$.

ANSWER: $x=6/5$ or $1/2$

Method cue: Find factors of 60 with sum -17: -12 and -5. Check sign, size, accuracy and units.

8. Show $x^2+6x+11$ cannot be factorised into real linear factors.

ANSWER: Discriminant is $-8<0$

Method cue: Calculate $b^2-4ac=36-44=-8$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Completing the square and quadratic form

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Write $x^2-10x+7$ as $(x-a)^2+b$.

ANSWER: $(x-5)^2-18$

Method cue: Half -10 to get -5. Check sign, size, accuracy and units.

2. Write $3x^2+12x-5$ in completed-square form.

ANSWER: $3(x+2)^2-17$

Method cue: Factor 3 from x terms. Check sign, size, accuracy and units.

3. Find the minimum of $2x^2-8x+11$ and the x-value where it occurs.

ANSWER: minimum 3 at $x=2$

Method cue: Complete square: $2(x-2)^2+3$. Check sign, size, accuracy and units.

4. Solve $x^2+6x-4=0$ by completing the square.

ANSWER: $x=-3\pm\sqrt{13}$

Method cue: $(x+3)^2=13$. Check sign, size, accuracy and units.

5. Find the range of $y=-(x-4)^2+9$.

ANSWER: $y\leq9$

Method cue: The square is non-negative. Check sign, size, accuracy and units.

6. The graph $y=x^2+px+12$ has turning point $x=3$. Find p.

ANSWER: $p=-6$

Method cue: Axis of symmetry is $x=-p/2$. Check sign, size, accuracy and units.

7. Show x^2+4x+8 is always positive.

ANSWER: $(x+2)^2+4>0$

Method cue: Complete the square. Check sign, size, accuracy and units.

8. Find exact roots of $2x^2-12x+7=0$ using completed square.

ANSWER: $x=3\pm\sqrt{22}/2$

Method cue: Divide by 2: $x^2-6x+7/2=0$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Algebraic fractions

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Simplify $(x^2-9)/(x^2+x-6)$.

ANSWER: $(x-3)/(x-2)$, $x\neq-3,2$

Method cue: Factor numerator=$(x-3)(x+3)$. Check sign, size, accuracy and units.

2. Write $3/x + 2/(x+1)$ as one fraction.

ANSWER: $(5x+3)/(x(x+1))$

Method cue: Common denominator $x(x+1)$. Check sign, size, accuracy and units.

3. Simplify $(4x^2-16)/(2x^2+12x+16)$.

ANSWER: $2(x-2)/(x+4)$

Method cue: Factor $4(x-2)(x+2)$ and $2(x+2)(x+4)$. Check sign, size, accuracy and units.

4. Solve $2/(x-1)=5/(x+2)$.

ANSWER: $x=3$

Method cue: Cross-multiply $2(x+2)=5(x-1)$. Check sign, size, accuracy and units.

5. Simplify $[x/(x-2)] \div [3x/(x^2-4)]$.

ANSWER: $(x+2)/3$, where $x\neq-2,0,2$

Method cue: Multiply by reciprocal. Check sign, size, accuracy and units.

6. Solve $1/x + 1/(x+2)=3/4$.

ANSWER: $x=2$ or $x=-4/3$

Method cue: Multiply by $4x(x+2)$. Check sign, size, accuracy and units.

7. Write $1/(x-1)-1/(x+1)$ as one fraction.

ANSWER: $2/(x^2-1)$

Method cue: Common denominator $(x-1)(x+1)$. Check sign, size, accuracy and units.

8. Explain why $(x^2-1)/(x-1)=x+1$ is not valid at $x=1$.

ANSWER: Original denominator is zero

Method cue: Factor and cancel for $x\neq1$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Formulae, rearrangement and kinematics

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Make x the subject of $y=(3x-5)/(x+2)$.

ANSWER: $x=(5+2y)/(3-y)$

Method cue: Multiply: $yx+2y=3x-5$. Check sign, size, accuracy and units.

2. Make r the subject of $V=\pi r^2h$.

ANSWER: $r=\sqrt{(V/(\pi h))}$

Method cue: Divide by πh. Check sign, size, accuracy and units.

3. Make x the subject of $p=1/(x+a)$.

ANSWER: $x=1/p-a$

Method cue: Take reciprocals: $x+a=1/p$. Check sign, size, accuracy and units.

4. Make t the subject of $v=u+at$.

ANSWER: $t=(v-u)/a$

Method cue: Subtract u. Check sign, size, accuracy and units.

5. Make a the subject of $v^2=u^2+2as$.

ANSWER: $a=(v^2-u^2)/(2s)$

Method cue: Subtract u^2. Check sign, size, accuracy and units.

6. Make x the subject of $y=ax+bx+c$.

ANSWER: $x=(y-c)/(a+b)$

Method cue: Factor x: $y=x(a+b)+c$. Check sign, size, accuracy and units.

7. A particle has $u=7$ m/s, $a=-1.5$ m/s², $t=4$ s. Use $s=ut+at^2/2$.

ANSWER: 16 m

Method cue: Substitute with brackets. Check sign, size, accuracy and units.

8. Make x the subject of $y=\sqrt{((x-2)/5)}$.

ANSWER: $x=5y^2+2$

Method cue: Square both sides. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Linear and quadratic equations

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Solve $(3x-2)/5-(x+4)/3=2$.

ANSWER: $x=14$

Method cue: Multiply by 15. Check sign, size, accuracy and units.

2. Solve $3x^2-5x-4=0$. Give exact answers.

ANSWER: $x=(5\pm\sqrt{73})/6$

Method cue: Use quadratic formula. Check sign, size, accuracy and units.

3. A rectangle has sides $x+2$ and $x-3$ and area 50. Find x.

ANSWER: $x=8$

Method cue: Form $(x+2)(x-3)=50$. Check sign, size, accuracy and units.

4. Solve $x^4-13x^2+36=0$.

ANSWER: $x=\pm2,\pm3$

Method cue: Let $y=x^2$. Check sign, size, accuracy and units.

5. Solve $\sqrt{(2x+3)}=x$.

ANSWER: $x=3$

Method cue: Require $x\geq0$. Check sign, size, accuracy and units.

6. Find k so $x^2-6x+k=0$ has one repeated root.

ANSWER: $k=9$

Method cue: Repeated root means discriminant zero. Check sign, size, accuracy and units.

7. Solve $2/(x-1)=x$, stating restrictions.

ANSWER: $x=2$ or $x=-1$

Method cue: $x\neq1$. Check sign, size, accuracy and units.

8. The roots of $x^2-7x+10=0$ are α,β. Find $\alpha^2+\beta^2$ without solving.

ANSWER: 29

Method cue: $\alpha+\beta=7$ and $\alpha\beta=10$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Simultaneous equations: linear and quadratic

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Solve $3x+2y=19$ and $5x-y=23$.

ANSWER: $x=5, y=2$

Method cue: Double the second equation: $10x-2y=46$. Check sign, size, accuracy and units.

2. Solve $y=x+1$ and $y=x^2-5x+7$.

ANSWER: $(3-\sqrt{3}, 4-\sqrt{3})$ and $(3+\sqrt{3}, 4+\sqrt{3})$

Method cue: Set expressions equal. Check sign, size, accuracy and units.

3. Solve $x+y=7$ and $xy=10$.

ANSWER: $(2,5),(5,2)$

Method cue: Use $y=7-x$. Check sign, size, accuracy and units.

4. Solve $y=2x-3$ and $x^2+y^2=25$.

ANSWER: $((6\pm2\sqrt{29})/5, (-3\pm4\sqrt{29})/5)$

Method cue: Substitute: $x^2+(2x-3)^2=25$. Check sign, size, accuracy and units.

5. Solve $x^2+y^2=13$ and $y=x-1$.

ANSWER: $(3,2),(-2,-3)$

Method cue: Substitute $x^2+(x-1)^2=13$. Check sign, size, accuracy and units.

6. A line $y=kx$ intersects $y=x^2$ at exactly one point. What can be said?

ANSWER: They always meet at $x=0$; exactly one distinct point only if $k=0$

Method cue: Set $x^2=kx$: $x(x-k)=0$. Check sign, size, accuracy and units.

7. Solve $2x+3y=7$ and $4x-3y=11$.

ANSWER: $x=3, y=1/3$

Method cue: Add equations: $6x=18$. Check sign, size, accuracy and units.

8. Explain geometrically why a line and parabola can have 0, 1 or 2 real simultaneous solut...

ANSWER: Solutions are intersection points

Method cue: No crossing gives none. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Approximation and iteration

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Show $x^3-x-1=0$ has a root between 1.3 and 1.4.

ANSWER: $f(1.3)<0$ and $f(1.4)>0$

Method cue: $f(1.3)=2.197-1.3-1=-0.103$. Check sign, size, accuracy and units.

2. Use iteration $x_{n+1}=\sqrt[3]{x_n+1}$, $x_1=1.3$, to find x_4.

ANSWER: about 1.3245

Method cue: $x_2=\sqrt[3]{2.3}\approx 1.320006$. Check sign, size, accuracy and units.

3. Find an interval width 0.1 containing a root of $x^2-3x-1=0$ near 3.

ANSWER: $3.3<x<3.4$

Method cue: $f(3.3)=10.89-9.9-1=-0.01$. Check sign, size, accuracy and units.

4. Given $x_{n+1}=\sqrt{6+x_n}$, $x_1=2.5$, calculate x_3.

ANSWER: about 2.986

Method cue: $x_2=\sqrt{8.5}\approx 2.9155$. Check sign, size, accuracy and units.

5. An iteration alternates 1,4,1,4,... Does it converge?

ANSWER: No

Method cue: Terms do not approach one limiting value. Check sign, size, accuracy and units.

6. Explain why rounding every iterate to 1 decimal place may give a wrong final root.

ANSWER: Rounding error feeds into later iterates

Method cue: Each new value depends on the previous one. Check sign, size, accuracy and units.

7. Use decimal search to solve $x^3=20$ to 2 decimal places.

ANSWER: $x\approx 2.71$

Method cue: $2.71^3\approx 19.9025$ and $2.72^3\approx 20.1236$. Check sign, size, accuracy and units.

8. A graph of $y=f(x)$ crosses the x-axis at $x\approx -1.6$ and 2.4. State approximate solutions of f...

ANSWER: $x\approx -1.6$ and $x\approx 2.4$

Method cue: Roots are x-coordinates where $y=0$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Linear, quadratic and graphical inequalities

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Solve $5-3x\le17$ and give interval notation.

ANSWER: $x\ge-4$, $[-4,\infty)$

Method cue: Subtract 5: $-3x\le12$. Check sign, size, accuracy and units.

2. Solve $x^2-5x+6<0$.

ANSWER: $2<x<3$

Method cue: Factor $(x-2)(x-3)$. Check sign, size, accuracy and units.

3. Solve $2x^2+x-6\ge0$.

ANSWER: $x\le-2$ or $x\ge3/2$

Method cue: Factor $(2x-3)(x+2)$. Check sign, size, accuracy and units.

4. Solve $-2<3x+4\le13$.

ANSWER: $-2<x\le3$

Method cue: Subtract 4: $-6<3x\le9$. Check sign, size, accuracy and units.

5. Find integer solutions of $x^2\le20$.

ANSWER: -4,-3,-2,-1,0,1,2,3,4

Method cue: $|x|\le\sqrt{20}\approx4.47$. Check sign, size, accuracy and units.

6. Does (2,3) satisfy $y>2x-2$ and $x+y\le6$?

ANSWER: Yes

Method cue: $3>2$ and $5\le6$. Check sign, size, accuracy and units.

7. Describe the boundary and shading for $y\le-x+4$.

ANSWER: Solid line; shade on or below

Method cue: Equality is included, so boundary solid. Check sign, size, accuracy and units.

8. Solve $(x-1)/(x+2)>0$.

ANSWER: $x<-2$ or $x>1$

Method cue: Critical values -2 (excluded) and 1. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Functions and advanced sequences

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Function f multiplies by 3 then subtracts 5. Find f(7) and the inverse process.

ANSWER: 16; add 5 then divide by 3

Method cue: $f(7)=21-5=16$. Check sign, size, accuracy and units.

2. $f(x)=2x+1$ and $g(x)=x^2$. Find $g(f(3))$ and $f(g(3))$.

ANSWER: 49 and 19

Method cue: $f(3)=7$, then $g(7)=49$. Check sign, size, accuracy and units.

3. Find the nth term of 5,12,23,38,57,...

ANSWER: $2n^2+n+2$

Method cue: First differences 7,11,15,19; second difference 4, so $a=2$. Check sign, size, accuracy and units.

4. Find the next two terms and nth term of 3,6,12,24,...

ANSWER: 48,96; $3\times2^{(n-1)}$

Method cue: Common ratio 2. Check sign, size, accuracy and units.

5. Which term of $4n^2-3$ is 897?

ANSWER: 15th

Method cue: Solve $4n^2-3=897$. Check sign, size, accuracy and units.

6. A quadratic sequence has nth term an^2+bn+c and begins 6,15,28. Find a,b,c.

ANSWER: $2n^2+3n+1$

Method cue: Second difference is 4, so $a=2$. Check sign, size, accuracy and units.

7. The recurrence $u_{(n+1)}=0.5u_n+6$, $u_1=20$. Find u_4 and limiting value.

ANSWER: $u_4=13$; limit 12

Method cue: $u_2=16, u_3=14, u_4=13$. Check sign, size, accuracy and units.

8. Explain why sequence n^2+n is always even.

ANSWER: $n(n+1)$ is product of consecutive integers

Method cue: Factor $n^2+n=n(n+1)$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

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- 10 publisher-organised chapters mapped to OCR J560 section 6.
- Every chapter begins on a new page and contains conceptual explanation.
- Original vector diagrams and two worked examples appear in every chapter.
- Applications, misconceptions, guided reasoning and independent practice are included.
- Answers are collected at the end with method cues.
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