GCSE MATHEMATICS

HIGHER TIER
COMPREHENSIVE LESSON NOTEBOOK
UNIT 4 | GRAPHS

TEACH-FROM-SCRATCH • PUBLICATION EDITION

10 detailed chapters • original diagrams • worked examples • guided practice • answers

INDEPENDENT PUBLICATION NOTICE

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COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Practice third.

FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each worked solution and reproduce it. 3 Complete practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final statement when interpretation is required.

Error loop

REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

Higher success requires connected graphical reasoning

OCR Higher graph questions connect algebra, transformations, rates and interpretation. Strong performance depends on identifying invariant features, reading scale accurately and justifying graphical conclusions.

COMPREHENSIVE LESSON NOTEBOOK

Unit 4 coverage map

OCR J560 Graphs of Equations and Functions content organised into a teachable sequence.

CH 01	OCR 7.01a–7.01c	Polynomial and reciprocal graphs
CH 02	OCR 7.01c; linked 6.01f	Quadratic graph features
CH 03	OCR 7.01d–7.01e	Exponential and trigonometric graphs
CH 04	OCR 7.01f; 7.02b	Circles and tangent equations
CH 05	OCR 7.02a–7.02b	Straight, parallel and perpendicular lines
CH 06	OCR 7.03a	Transformations of graphs
CH 07	OCR 6.03d; 7.01–7.03	Graphical equations and inequalities
CH 08	OCR 7.04a	Real-world and proportional graphs
CH 09	OCR 7.04b	Average and instantaneous rates
CH 10	OCR 7.04c	Areas under graphs

IMPORTANT TIER NOTE

This notebook covers OCR J560 section 7 and linked 6.03d at Higher Tier: coordinates, graph families, straight lines, graphical solutions and real-world graphs. Specification references are identifiers, not copied specification wording.

COMPREHENSIVE LESSON NOTEBOOK

Fast-reference method bank

Understand each relationship before memorising it.

Straight line	$y=mx+c$; $m=(y_2-y_1)/(x_2-x_1)$
Perpendicular gradients	$m_1m_2=-1$
Circle at origin	$x^2+y^2=r^2$
Tangent at (a,b)	$ax+by=r^2$
Quadratic vertex	$a(x-h)^2+k$ has vertex (h,k)
Graph translations	f(x)+a up a; f(x-a) right a
Reflections	-f(x) in x-axis; f(-x) in y-axis
Average rate	change in output ÷ change in input
Trapezium area	$\frac{1}{2}(a+b)h$
Rate accumulation	area under rate-time graph

CURRENT ASSESSMENT NOTE

OCR has announced Foundation and Higher formula sheets for GCSE Mathematics examinations in 2025, 2026 and 2027. Use the current official OCR sheet alongside this learning bank and check OCR for later exam-series arrangements.

COMPREHENSIVE LESSON NOTEBOOK

Polynomial and reciprocal graphs

Recognise, sketch and interpret linear, quadratic, cubic and reciprocal functions.

SPECIFICATION FOCUS

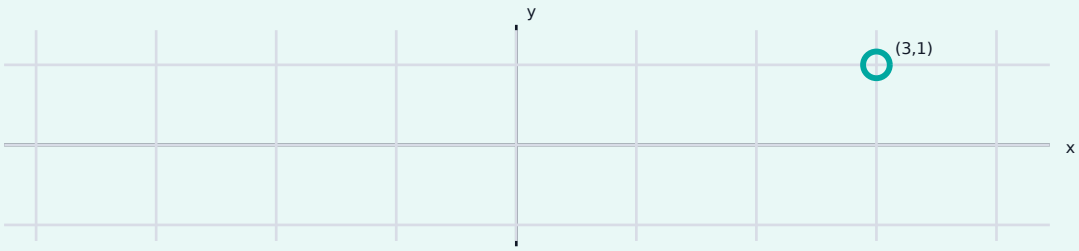
OCR reference OCR 7.01a–7.01c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A graph is a visual record of how outputs respond to inputs. Higher sketching uses algebraic structure—factors, powers, symmetry and undefined values—to predict features before plotting.

KEY VOCABULARY

function • root • intercept • asymptote • multiplicity



Why the method works

- Factors locate roots.
- Leading terms control end behaviour.
- Reciprocal denominators create vertical asymptotes.

CHECK BEFORE MOVING ON

Answer mentally: Sketch y=(x-2)(x+1), labelling roots and y-intercept. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Polynomial and reciprocal graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Use intercepts, symmetry and end behaviour before plotting extra points.
- A repeated root touches the x-axis; a simple root crosses it.
- Reciprocal graphs have asymptotes where values are undefined or approached.

WORKED EXAMPLE 1

Sketch $y=(x-2)(x+1)$, labelling roots and y-intercept.

SOLUTION

1. Factors give roots -1 and 2.
2. At $x=0$, $y=-2$.
3. Positive x^2 coefficient means upward opening.

ANSWER: roots -1,2; y-intercept -2; upward parabola

WORKED EXAMPLE 2

Describe the key features of $y=1/x$.

SOLUTION

1. Denominator forbids $x=0$.
2. As $|x|$ grows, y approaches 0.
3. x and y have the same sign.

ANSWER: asymptotes x=0,y=0; branches in quadrants I and III

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Polynomial and reciprocal graphs — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Polynomial and reciprocal models occur in optimisation and rates.

Common misconceptions

- Joining reciprocal branches across an asymptote.
- Missing repeated-root touching.
- Plotting without labelling scale.

GUIDED TRY

Find intercepts of $y=x^3-4x$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Find y-values of $y=2/x$ for $x=-4,-1,0.5,2$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Polynomial and reciprocal graphs

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Sketch y=(x-2)(x+1), labelling roots and y-intercept.** **[4]**

2. **Describe the key features of y=1/x.** **[4]**

3. **Find intercepts of $y=x^3-4x$.** **[4]**

4. **The graph $y=(x+3)^2(x-1)$ has which roots and crossing behaviour?** **[4]**

5. **State asymptotes and branch signs for y=6/(x-2)+1.** **[4]**

6. **A cubic has roots -2,1,4 and leading coefficient 1. Write an equation.** **[3]**

7. **Explain why $y=x^4+2$ has no x-intercepts.** **[3]**

8. **Find y-values of y=2/x for x=-4,-1,0.5,2.** **[3]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Quadratic graph features

Use completed-square form to identify vertices, axes, roots and ranges.

SPECIFICATION FOCUS

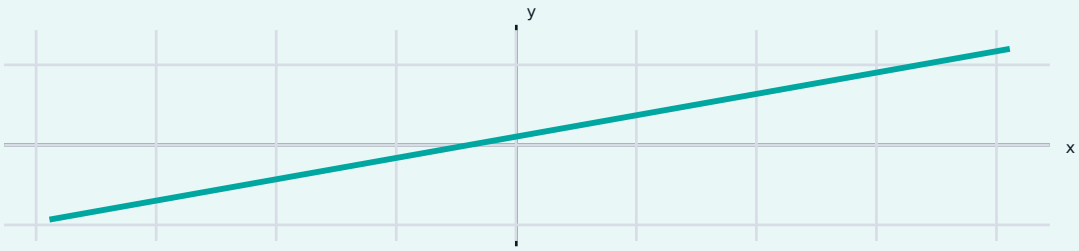
OCR reference OCR 7.01c; linked 6.01f. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A quadratic is controlled by its leading coefficient, vertex and roots. Completed-square form exposes translations and range; factor or formula form exposes intercepts.

KEY VOCABULARY

vertex • axis • discriminant • range • completed square



Why the method works

- Equivalent forms reveal different features.
- Symmetry passes through the vertex.
- Discriminant counts real crossings.

CHECK BEFORE MOVING ON

Answer mentally: Find turning point and axis of $y=x^2-8x+11$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Quadratic graph features — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Convert to $y=a(x-h)^2+k$ to read vertex (h,k).
- Use discriminant to determine how many x-intercepts exist.
- The sign of a determines minimum or maximum.

WORKED EXAMPLE 1

Find turning point and axis of $y=x^2-8x+11$.

SOLUTION

1. Complete square: $(x-4)^2-5$.
2. Read vertex and axis.

ANSWER: turning point (4,-5); axis x=4

WORKED EXAMPLE 2

Sketch $y=-2(x+1)^2+8$, labelling turning point and intercepts.

SOLUTION

1. Vertex is (-1,8).
2. Set y=0: $(x+1)^2=4$.
3. At x=0, y=6.

ANSWER: vertex (-1,8); x=-3,1; y=6

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Quadratic graph features — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Parabolic models describe trajectories and optimisation.

Common misconceptions

- Reading the sign of the vertex incorrectly.
- Forgetting the y-intercept.
- Assuming every quadratic crosses twice.

GUIDED TRY

Find range of $y=3(x-5)^2-7$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain how $y=x^2-6x+12$ shows the graph never crosses the x-axis. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Quadratic graph features

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Find turning point and axis of $y=x^2-8x+11$.** **[4]**

2. **Sketch $y=-2(x+1)^2+8$, labelling turning point and intercepts.** **[5]**

SHOW ALL WORKING

3. **Find range of $y=3(x-5)^2-7$.** **[3]**

4. **For $y=x^2+kx+9$, the axis is $x=2$. Find k.** **[3]**

5. **Determine number of real roots of $2x^2+3x+5=0$.** **[3]**

6. **A parabola has vertex $(3,-4)$ and passes through $(1,4)$. Find its equation.** **[5]**

SHOW ALL WORKING

7. **Find exact x-intercepts of $y=2x^2-4x-3$.** **[4]**

8. **Explain how $y=x^2-6x+12$ shows the graph never crosses the x-axis.** **[4]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Exponential and trigonometric graphs

Recognise exponential growth/decay and sketch sine, cosine and tangent graphs.

SPECIFICATION FOCUS

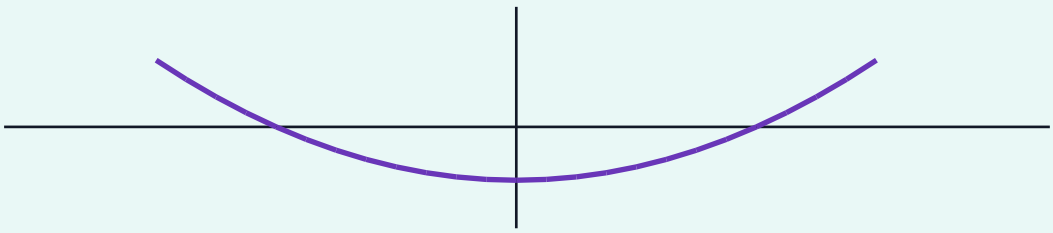
OCR reference OCR 7.01d–7.01e. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Exponential graphs model multiplicative change; trigonometric graphs model periodic behaviour. Their characteristic points and asymptotes permit accurate sketches without dense plotting.

KEY VOCABULARY

exponential • amplitude • period • asymptote • tangent



quadratic curves; straight lines keep a constant gradient

Why the method works

- Positive exponentials never reach zero.
- Sine and cosine repeat every 360°.
- Tangent repeats every 180° and has vertical asymptotes.

CHECK BEFORE MOVING ON

Answer mentally: Sketch y=2^x for -2≤x≤3, giving six key points. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Exponential and trigonometric graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Exponential y=k^x passes through (0,1) for k>0.
- Use amplitude, period and zeros to structure trig sketches.
- tan x is undefined at 90°+180°n.

WORKED EXAMPLE 1

Sketch y=2^x for -2≤x≤3, giving six key points.

SOLUTION

1. Evaluate integer powers.
2. Join with a smooth increasing curve.
3. State horizontal asymptote y=0.

ANSWER: (-2,1/4),(-1,1/2),(0,1),(1,2),(2,4),(3,8)

WORKED EXAMPLE 2

Solve 3^x=27 and interpret as a graph intersection.

SOLUTION

1. $27=3^3$.
2. It is where y=3^x meets y=27.

ANSWER: x=3

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Exponential and trigonometric graphs — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Growth, waves and rotations use these graph families.

Common misconceptions

- Drawing exponentials through the origin.
- Confusing sine and cosine starting values.
- Joining tangent through an asymptote.

GUIDED TRY

State amplitude and period of y=4sin x for degrees. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

How many solutions has 2^x=-3? Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Exponential and trigonometric graphs

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Sketch y=2^x for -2≤x≤3, giving six key points.** **[4]**

2. **Solve 3^x=27 and interpret as a graph intersection.** **[3]**

3. **State amplitude and period of y=4sin x for degrees.** **[3]**

4. **Give zeros of y=sin x for 0°≤x≤360°.** **[3]**

5. **Give maximum and minimum points of y=cos x for 0°≤x≤360°.** **[4]**

6. **State vertical asymptotes of y=tan x for -180°≤x≤180°.** **[3]**

7. **Describe y=(1/3)^x as x increases.** **[3]**

8. **How many solutions has 2^x=-3?** **[2]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Circles and tangent equations

Use $x^2+y^2=r^2$ and perpendicular gradients to form tangent equations.

SPECIFICATION FOCUS

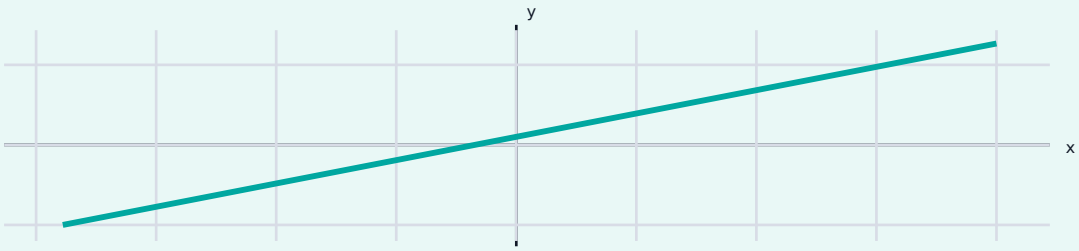
OCR reference OCR 7.01f; 7.02b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

The equation $x^2+y^2=r^2$ collects all points a fixed distance r from the origin. A tangent is perpendicular to the radius at the contact point.

KEY VOCABULARY

circle • radius • tangent • perpendicular • gradient



Why the method works

- Distance formula gives the circle equation.
- Perpendicular gradients multiply to -1.
- $ax+by=r^2$ describes the tangent at (a,b).

CHECK BEFORE MOVING ON

Answer mentally: Find radius and intercepts of $x^2+y^2=49$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Circles and tangent equations — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- For a centre-origin circle, radius is √(constant).
- A radius to (a,b) has gradient b/a when a≠0.
- Tangent gradient is the negative reciprocal of radius gradient.

WORKED EXAMPLE 1

Find radius and intercepts of $x^2+y^2=49$.

SOLUTION

1. Compare with $r^2=49$.
2. Set one coordinate to zero.

ANSWER: radius 7; intercepts (±7,0),(0,±7)

WORKED EXAMPLE 2

Show (3,4) lies on $x^2+y^2=25$ and find tangent gradient there.

SOLUTION

1. $3^2+4^2=25$.
2. Radius gradient=4/3.
3. Negative reciprocal=-3/4.

ANSWER: tangent gradient -3/4

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Circles and tangent equations — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Circular paths and contact geometry use tangent models.

Common misconceptions

- Using diameter instead of radius.
- Finding radius rather than tangent gradient.
- Ignoring vertical-line cases.

GUIDED TRY

Find tangent equation to $x^2+y^2=25$ at (3,4). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Find tangent at (-5,0) to $x^2+y^2=25$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Circles and tangent equations

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Find radius and intercepts of $x^2+y^2=49$.** **[3]**

2. **Show (3,4) lies on $x^2+y^2=25$ and find tangent gradient there.** **[4]**

3. **Find tangent equation to $x^2+y^2=25$ at (3,4).** **[5]**

SHOW ALL WORKING

4. **Find points on $x^2+y^2=100$ with $x=6$.** **[4]**

5. **A tangent at (a,b) to $x^2+y^2=R^2$ has equation $ax+by=R^2$. Verify for (3,4),R=5.** **[4]**

6. **Find tangent to $x^2+y^2=20$ at (2,4).** **[5]**

SHOW ALL WORKING

7. **Does line $y=x+6$ intersect $x^2+y^2=16$?** **[4]**

8. **Find tangent at (-5,0) to $x^2+y^2=25$.** **[3]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Straight, parallel and perpendicular lines

Find equations from points and gradients and analyse line relationships.

SPECIFICATION FOCUS

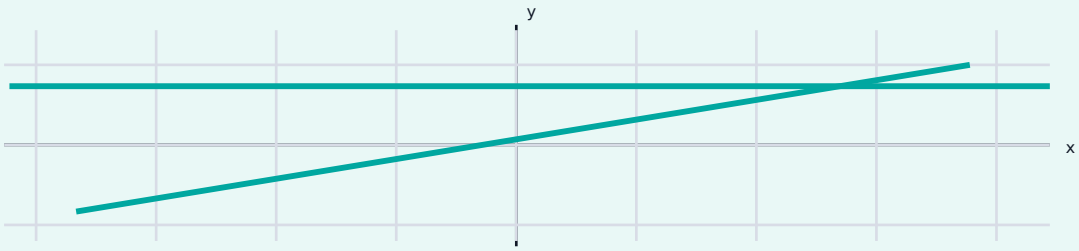
OCR reference OCR 7.02a–7.02b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A straight line is determined by gradient and one point. Parallelism preserves direction; perpendicularity turns direction through a right angle.

KEY VOCABULARY

gradient • intercept • parallel • perpendicular • point-gradient



Why the method works

- Gradient is a constant ratio.
- y-y1=m(x-x1) forces passage through a point.
- Negative reciprocal slopes form a right angle.

CHECK BEFORE MOVING ON

Answer mentally: Find equation through (2,-1) and (8,14). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Straight, parallel and perpendicular lines — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Gradient=(change in y)/(change in x).
- Parallel lines share gradient.
- Non-vertical perpendicular gradients multiply to -1.

WORKED EXAMPLE 1

Find equation through (2,-1) and (8,14).

SOLUTION

1. Gradient=15/6=5/2.
2. Use y=mx+c with point (2,-1).
3. c=-6.

ANSWER: y=(5/2)x-6

WORKED EXAMPLE 2

Find line parallel to 3x-2y=8 through (4,1).

SOLUTION

1. Rearrange gives gradient 3/2.
2. Keep coefficients 3x-2y.
3. Substitute point to find constant 10.

ANSWER: 3x-2y=10

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Straight, parallel and perpendicular lines — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Linear boundaries and trajectories use line equations.

Common misconceptions

- Reversing rise and run.
- Changing gradient for a parallel line.
- Using only reciprocal without negative sign.

GUIDED TRY

Find line perpendicular to y=-4x+7 through (8,-3). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

For what p are px+2y=7 and y=4x-1 perpendicular? Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Straight, parallel and perpendicular lines

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Find equation through (2,-1) and (8,14).** **[4]**

2. **Find line parallel to 3x-2y=8 through (4,1).** **[4]**

3. **Find line perpendicular to y=-4x+7 through (8,-3).** **[4]**

4. **Show A(-2,5),B(4,-1),C(7,2) form a right angle at B.** **[5]**

SHOW ALL WORKING

5. **Find intersection of y=3x-8 and y=-x+12.** **[3]**

6. **Line has x-intercept 6 and y-intercept -3. Find equation.** **[4]**

7. **For what k are y=(k-1)x+2 and y=3x-5 parallel?** **[3]**

8. **For what p are px+2y=7 and y=4x-1 perpendicular?** **[4]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Transformations of graphs

Translate and reflect graphs using changes inside and outside an equation.

SPECIFICATION FOCUS

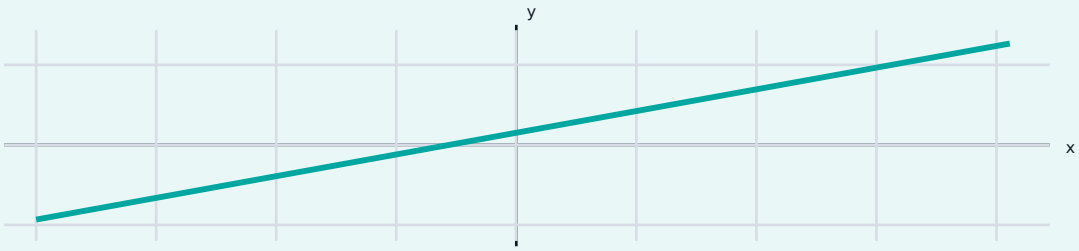
OCR reference OCR 7.03a. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Graph transformations change inputs or outputs systematically. Outside changes act vertically; inside changes act horizontally in the opposite apparent direction.

KEY VOCABULARY

translation • reflection • input • output • invariant



Why the method works

- Adding to outputs moves every y-value.
- Replacing x by x-a requires an input a larger.
- Negating outputs reflects vertically.

CHECK BEFORE MOVING ON

Answer mentally: Describe transformation y=f(x)+5. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Transformations of graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- y=f(x)+a moves vertically by a.
- y=f(x-a) moves right by a.
- y=-f(x) reflects in x-axis; y=f(-x) reflects in y-axis.

WORKED EXAMPLE 1

Describe transformation y=f(x)+5.

SOLUTION

1. Outside addition changes outputs.

ANSWER: translation 5 units up

WORKED EXAMPLE 2

Describe y=f(x-3).

SOLUTION

1. Inside subtraction changes inputs oppositely.

ANSWER: translation 3 units right

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Transformations of graphs — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Transformations compare related models efficiently.

Common misconceptions

- Moving f(x-a) left.
- Confusing f(-x) and -f(x).
- Transforming only intercepts.

GUIDED TRY

Point (2,-1) lies on y=f(x). Where on y=f(x)+4? First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Graph y=f(x) has roots -2 and 5. Find roots of y=f(x-3). Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Transformations of graphs

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Describe transformation y=f(x)+5.** **[2]**

2. **Describe y=f(x-3).** **[2]**

3. **Point (2,-1) lies on y=f(x). Where on y=f(x)+4?** **[3]**

4. **Point (-3,5) lies on y=f(x). Where on y=f(-x)?** **[3]**

5. **Describe y=-f(x)+2.** **[3]**

6. **Sketch $y=(x-4)^2-3$ from $y=x^2$ and state vertex.** **[3]**

7. **How does y=2^x change to y=2^(x+1)?** **[3]**

8. **Graph y=f(x) has roots -2 and 5. Find roots of y=f(x-3).** **[4]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Graphical equations and inequalities

Use roots and intersections to solve equations and graphical regions.

SPECIFICATION FOCUS

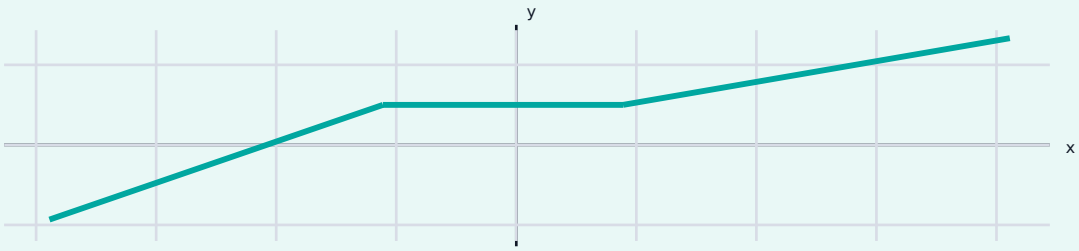
OCR reference OCR 6.03d; 7.01–7.03. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Equations become graphical conditions: roots are axis crossings and simultaneous solutions are intersections. Inequalities select whole regions.

KEY VOCABULARY

intersection • root • region • boundary • tangent



Why the method works

- Equal outputs locate intersections.
- Quadratic sign follows position relative to axis.
- Tangency corresponds to a repeated root.

CHECK BEFORE MOVING ON

Answer mentally: Graphs $y=x^2$ and $y=2x+3$ intersect. Find exact intersection x-values. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Graphical equations and inequalities — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- f(x)=0 corresponds to x-axis intersections.
- f(x)=g(x) corresponds to graph intersections.
- Solid boundaries include equality; dashed boundaries exclude it.

WORKED EXAMPLE 1

Graphs $y=x^2$ and $y=2x+3$ intersect. Find exact intersection x-values.

SOLUTION

1. Set $x^2=2x+3$.
2. $x^2-2x-3=0$.
3. Factor.

ANSWER: x=-1,3

WORKED EXAMPLE 2

Use $y=x^3-x$ and $y=4$ to describe solutions of $x^3-x=4$.

SOLUTION

1. Plot or use the two graphs.
2. Read crossing x-values.

ANSWER: x-coordinates of intersections

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Graphical equations and inequalities — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Graphical solutions handle equations without simple exact methods.

Common misconceptions

- Reading y-coordinate as a solution for x.
- Using a solid line for strict inequality.
- Omitting a second intersection.

GUIDED TRY

Solve $x^2-4x-5\leq 0$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Find k so $y=x^2$ and $y=4x+k$ are tangent. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Graphical equations and inequalities

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Graphs $y=x^2$ and $y=2x+3$ intersect. Find exact intersection x-values.** **[4]**

2. **Use $y=x^3-x$ and $y=4$ to describe solutions of $x^3-x=4$.** **[3]**

3. **Solve $x^2-4x-5\leq 0$.** **[4]**

4. **What region represents $y>2x-1$?** **[3]**

5. **Solve simultaneously $y=x^2-1$ and $y=3x+3$.** **[5]**

SHOW ALL WORKING

6. **A graph of $y=f(x)$ has roots 1.2 and 4.7. Give solutions of $f(x)>0$ if it opens upward.** **[3]**

7. **Explain why a tangent intersection gives a repeated quadratic root.** **[4]**

8. **Find k so $y=x^2$ and $y=4x+k$ are tangent.** **[5]**

SHOW ALL WORKING

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Real-world and proportional graphs

Interpret rates, intercepts, direct/inverse proportion and piecewise change.

SPECIFICATION FOCUS

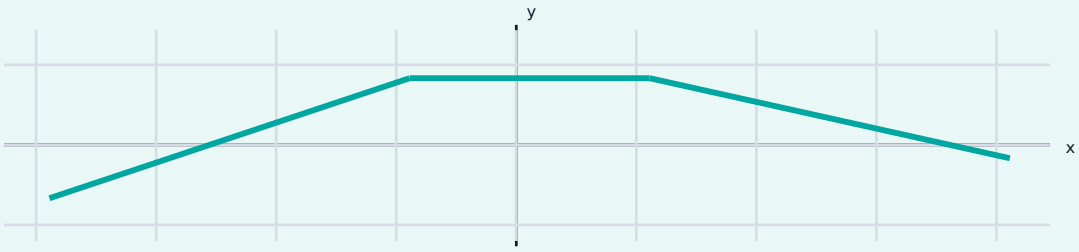
OCR reference OCR 7.04a. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A contextual graph compresses a changing process. Meaning comes from axes, units, intercept, slope and shape—not from visual appearance alone.

KEY VOCABULARY

context • unit • direct • inverse • piecewise



Why the method works

- Gradient inherits output/input units.
- Horizontal segments mean zero output change.
- Direct proportion passes through the origin.

CHECK BEFORE MOVING ON

Answer mentally: A distance-time graph rises 150 m in 12 s. Find speed. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Real-world and proportional graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Read axes and units before calculating.
- Gradient gives rate only when axes support that interpretation.
- Curves indicate changing rate; horizontal sections indicate zero rate.

WORKED EXAMPLE 1

A distance-time graph rises 150 m in 12 s. Find speed.

SOLUTION

1. Gradient=150/12.
2. Include m/s.

ANSWER: 12.5 m/s

WORKED EXAMPLE 2

A taxi cost graph has y-intercept £3.50 and gradient £1.80/km. Write equation.

SOLUTION

1. Intercept is fixed charge.
2. Gradient is price per km.

ANSWER: C=1.8d+3.5

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Real-world and proportional graphs — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Travel, finance, conversion and filling processes use graphs.

Common misconceptions

- Ignoring units.
- Calling any curve inverse proportion.
- Interpreting steepness without axis scale.

GUIDED TRY

Points on a conversion graph include (8,11.2). It is direct proportion. Find equation. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Why can a currency conversion graph change gradient over different dates? Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Real-world and proportional graphs

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A distance-time graph rises 150 m in 12 s. Find speed.** **[3]**

2. **A taxi cost graph has y-intercept £3.50 and gradient £1.80/km. Write equation.** **[3]**

3. **Points on a conversion graph include (8,11.2). It is direct proportion. Find equation.** **[3]**

4. **For inverse graph y=k/x through (5,18), find y at x=12.** **[4]**

5. **A distance-time graph is horizontal from 14:20 to 14:35. Interpret.** **[2]**

6. **A tank-volume graph steepens. What does this say about inflow rate?** **[3]**

7. **Two travel graphs intersect at (40,28). Interpret.** **[3]**

8. **Why can a currency conversion graph change gradient over different dates?** **[3]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Average and instantaneous rates

Calculate chord gradients and estimate tangent gradients on curves.

SPECIFICATION FOCUS

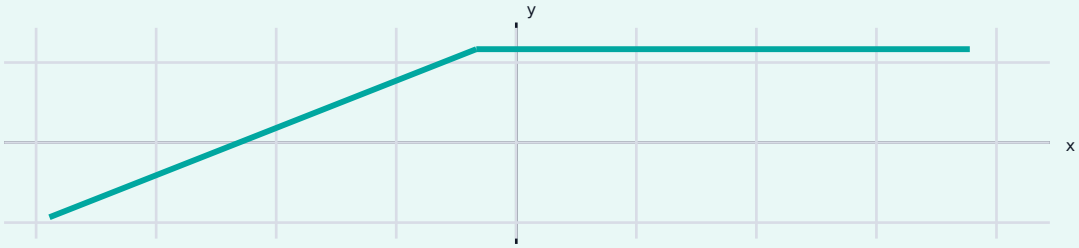
OCR reference OCR 7.04b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Average rate measures change across an interval; instantaneous rate is the limiting local slope represented by a tangent.

KEY VOCABULARY

chord • tangent • average rate • instantaneous rate • stationary



Why the method works

- Chord gradient uses two curve points.
- Tangent gradient uses two convenient tangent-line points.
- Horizontal tangent has zero instantaneous rate.

CHECK BEFORE MOVING ON

Answer mentally: For $s=t^3-2t$, find average rate from $t=1$ to $t=3$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Average and instantaneous rates — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Average rate uses a chord between two points.
- Instantaneous rate uses the tangent at one point.
- Choose widely separated readable tangent points, not necessarily curve data points.

WORKED EXAMPLE 1

For $s=t^3-2t$, find average rate from $t=1$ to $t=3$.

SOLUTION

1. $s(1)=-1$ and $s(3)=21$.
2. Change in s is 22 while change in t is 2.
3. Average rate$=22\div2=11$.

ANSWER: 11 units/time

WORKED EXAMPLE 2

A tangent passes through (2,7) and (8,31). Estimate gradient.

SOLUTION

1. Gradient$=(31-7)/(8-2)=24/6$.

ANSWER: 4

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Average and instantaneous rates — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Science and economics use local and interval rates.

Common misconceptions

- Using curve points off the tangent.
- Confusing value with rate.
- Ignoring scale units.

GUIDED TRY

Explain difference between average and instantaneous speed. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Why should tangent points be far apart when reading a drawn graph? Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Average and instantaneous rates

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **For $s=t^3-2t$, find average rate from t=1 to t=3.** [4]

2. **A tangent passes through (2,7) and (8,31). Estimate gradient.** [3]

3. **Explain difference between average and instantaneous speed.** [3]

4. **A curve is increasing but flattening. Describe gradient.** [2]

5. **Estimate derivative-like rate of $y=x^2$ at x=3 using chord from 2.9 to 3.1.** [4]

6. **A population graph tangent has gradient 450 people/year. Interpret.** [2]

7. **When is instantaneous rate zero on a smooth graph?** [3]

8. **Why should tangent points be far apart when reading a drawn graph?** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Areas under graphs

Calculate or estimate accumulated quantities from rates and velocity-time graphs.

SPECIFICATION FOCUS

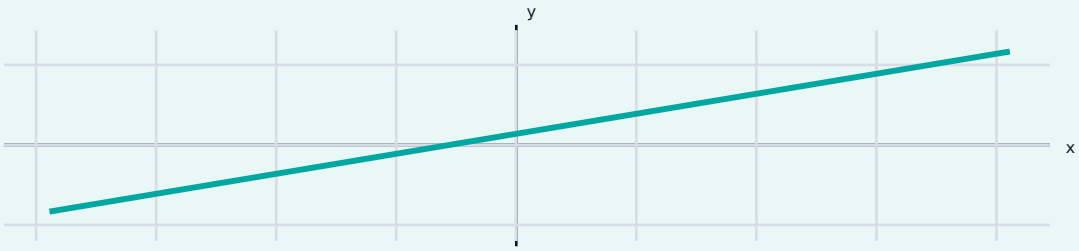
OCR reference OCR 7.04c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Area beneath a rate-time graph accumulates the underlying quantity. Shape decomposition or numerical strips converts a graph into a total.

KEY VOCABULARY

area • accumulation • velocity • displacement • trapezium



Why the method works

- Rate×time has units of quantity.
- Below-axis velocity contributes negative displacement.
- Trapezia improve curved-area estimates.

CHECK BEFORE MOVING ON

Answer mentally: Velocity rises uniformly from 0 to 18 m/s in 6 s. Find distance. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Areas under graphs — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Area under a rate-time graph equals accumulated quantity.
- Split polygonal regions into rectangles, triangles and trapezia.
- Signed area below the time axis represents negative displacement.

WORKED EXAMPLE 1

Velocity rises uniformly from 0 to 18 m/s in 6 s. Find distance.

SOLUTION

1. Area is triangle.
2. $1/2 \times 6 \times 18 = 54$.

ANSWER: 54 m

WORKED EXAMPLE 2

A car travels at 12 m/s for 8 s then slows uniformly to 0 over 4 s. Find distance.

SOLUTION

1. Rectangle area $= 12 \times 8 = 96$.
2. Triangle $= 1/2 \times 4 \times 12 = 24$.
3. Total $= 120$.

ANSWER: 120 m

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Areas under graphs — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Motion, flow and financial rates accumulate through area.

Common misconceptions

- Using perimeter.
- Ignoring signed area.
- Applying area interpretation to incompatible axes.

GUIDED TRY

Velocity is -5 m/s for 3 s. Find displacement and distance. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain why area under a distance-time graph is not normally distance travelled. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Areas under graphs

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Velocity rises uniformly from 0 to 18 m/s in 6 s. Find distance.** **[3]**

2. **A car travels at 12 m/s for 8 s then slows uniformly to 0 over 4 s. Find distance.** **[4]**

3. **Velocity is -5 m/s for 3 s. Find displacement and distance.** **[3]**

4. **A rate graph forms a trapezium with parallel heights 6 and 14 litres/min over 5 min. Find volume.** **[3]**

5. **Estimate area under a curve using four strips of width 2 with heights 3,5,8,10,11 using trapezium rule.** **[5]**

SHOW ALL WORKING

6. **A velocity-time graph has area 80 above and 25 below axis. Find displacement and distance.** **[4]**

7. **Acceleration is gradient of a velocity-time graph. Velocity rises 7 to 19 m/s in 4 s. Find acceleration.** **[3]**

8. **Explain why area under a distance-time graph is not normally distance travelled.** **[3]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Polynomial and reciprocal graphs

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Sketch y=(x-2)(x+1), labelling roots and y-intercept.

ANSWER: roots -1,2; y-intercept -2; upward parabola

Method cue: Factors give roots -1 and 2. Check sign, size, accuracy and units.

2. Describe the key features of y=1/x.

ANSWER: asymptotes x=0,y=0; branches in quadrants I and III

Method cue: Denominator forbids x=0. Check sign, size, accuracy and units.

3. Find intercepts of $y=x^3-4x$.

ANSWER: x-intercepts -2,0,2; y-intercept 0

Method cue: Factor $x(x^2-4)=x(x-2)(x+2)$. Check sign, size, accuracy and units.

4. The graph $y=(x+3)^2(x-1)$ has which roots and crossing behaviour?

ANSWER: x=-3 touches; x=1 crosses

Method cue: Roots come from factors. Check sign, size, accuracy and units.

5. State asymptotes and branch signs for y=6/(x-2)+1.

ANSWER: x=2,y=1; above-right and below-left relative to (2,1)

Method cue: Denominator zero gives vertical asymptote x=2. Check sign, size, accuracy and units.

6. A cubic has roots -2,1,4 and leading coefficient 1. Write an equation.

ANSWER: y=(x+2)(x-1)(x-4)

Method cue: Each root r gives factor x-r. Check sign, size, accuracy and units.

7. Explain why $y=x^4+2$ has no x-intercepts.

ANSWER: $x^4\geq 0$, so $y\geq 2$

Method cue: Even power is non-negative. Check sign, size, accuracy and units.

8. Find y-values of y=2/x for x=-4,-1,0.5,2.

ANSWER: -0.5,-2,4,1

Method cue: Substitute each non-zero x. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Quadratic graph features

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Find turning point and axis of $y=x^2-8x+11$.

ANSWER: turning point (4,-5); axis x=4

Method cue: Complete square: $(x-4)^2-5$. Check sign, size, accuracy and units.

2. Sketch $y=-2(x+1)^2+8$, labelling turning point and intercepts.

ANSWER: vertex (-1,8); x=-3,1; y=6

Method cue: Vertex is (-1,8). Check sign, size, accuracy and units.

3. Find range of $y=3(x-5)^2-7$.

ANSWER: $y\geq-7$

Method cue: Square is non-negative. Check sign, size, accuracy and units.

4. For $y=x^2+kx+9$, the axis is x=2. Find k.

ANSWER: k=-4

Method cue: Axis $x=-k/2$. Check sign, size, accuracy and units.

5. Determine number of real roots of $2x^2+3x+5=0$.

ANSWER: none

Method cue: Discriminant=9-40=-31. Check sign, size, accuracy and units.

6. A parabola has vertex (3,-4) and passes through (1,4). Find its equation.

ANSWER: $y=2(x-3)^2-4$

Method cue: Use $y=a(x-3)^2-4$. Check sign, size, accuracy and units.

7. Find exact x-intercepts of $y=2x^2-4x-3$.

ANSWER: $x=1\pm\sqrt{10}/2$

Method cue: Use quadratic formula. Check sign, size, accuracy and units.

8. Explain how $y=x^2-6x+12$ shows the graph never crosses the x-axis.

ANSWER: $y=(x-3)^2+3>0$

Method cue: Complete the square. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Exponential and trigonometric graphs

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Sketch y=2^x for -2≤x≤3, giving six key points.

ANSWER: (-2,1/4),(-1,1/2),(0,1),(1,2),(2,4),(3,8)

Method cue: Evaluate integer powers. Check sign, size, accuracy and units.

2. Solve 3^x=27 and interpret as a graph intersection.

ANSWER: x=3

Method cue: 27=3³. Check sign, size, accuracy and units.

3. State amplitude and period of y=4sin x for degrees.

ANSWER: amplitude 4; period 360°

Method cue: Coefficient gives amplitude. Check sign, size, accuracy and units.

4. Give zeros of y=sin x for 0°≤x≤360°.

ANSWER: 0°,180°,360°

Method cue: Sine is zero on the horizontal axis every 180°. Check sign, size, accuracy and units.

5. Give maximum and minimum points of y=cos x for 0°≤x≤360°.

ANSWER: max (0°,1),(360°,1); min (180°,-1)

Method cue: Use standard cosine key values. Check sign, size, accuracy and units.

6. State vertical asymptotes of y=tan x for -180°≤x≤180°.

ANSWER: x=-90°,90°

Method cue: Tangent is undefined where cos x=0. Check sign, size, accuracy and units.

7. Describe y=(1/3)^x as x increases.

ANSWER: positive decreasing curve through (0,1), approaching 0

Method cue: Base lies between 0 and 1. Check sign, size, accuracy and units.

8. How many solutions has 2^x=-3?

ANSWER: none

Method cue: 2^x is always positive. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Circles and tangent equations

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Find radius and intercepts of $x^2+y^2=49$.

ANSWER: radius 7; intercepts $(\pm7,0),(0,\pm7)$

Method cue: Compare with $r^2=49$. Check sign, size, accuracy and units.

2. Show (3,4) lies on $x^2+y^2=25$ and find tangent gradient there.

ANSWER: tangent gradient -3/4

Method cue: $3^2+4^2=25$. Check sign, size, accuracy and units.

3. Find tangent equation to $x^2+y^2=25$ at (3,4).

ANSWER: $y-4=-(3/4)(x-3)$

Method cue: Use tangent gradient -3/4. Check sign, size, accuracy and units.

4. Find points on $x^2+y^2=100$ with $x=6$.

ANSWER: (6,8),(6,-8)

Method cue: $36+y^2=100$. Check sign, size, accuracy and units.

5. A tangent at (a,b) to $x^2+y^2=R^2$ has equation $ax+by=R^2$. Verify for (3,4),R=5.

ANSWER: $3x+4y=25$

Method cue: Substitute values. Check sign, size, accuracy and units.

6. Find tangent to $x^2+y^2=20$ at (2,4).

ANSWER: $x+2y=10$

Method cue: Use $ax+by=R^2$: $2x+4y=20$. Check sign, size, accuracy and units.

7. Does line $y=x+6$ intersect $x^2+y^2=16$?

ANSWER: No

Method cue: Substitute to get $x^2+(x+6)^2=16$. Check sign, size, accuracy and units.

8. Find tangent at (-5,0) to $x^2+y^2=25$.

ANSWER: $x=-5$

Method cue: Radius is horizontal. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Straight, parallel and perpendicular lines

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Find equation through (2,-1) and (8,14).

ANSWER: y=(5/2)x-6

Method cue: Gradient=15/6=5/2. Check sign, size, accuracy and units.

2. Find line parallel to 3x-2y=8 through (4,1).

ANSWER: 3x-2y=10

Method cue: Rearrange gives gradient 3/2. Check sign, size, accuracy and units.

3. Find line perpendicular to y=-4x+7 through (8,-3).

ANSWER: y=x/4-5

Method cue: Perpendicular gradient=1/4. Check sign, size, accuracy and units.

4. Show A(-2,5),B(4,-1),C(7,2) form a right angle at B.

ANSWER: AB gradient -1; BC gradient 1; product -1

Method cue: Calculate both gradients. Check sign, size, accuracy and units.

5. Find intersection of y=3x-8 and y=-x+12.

ANSWER: (5,7)

Method cue: Set equations equal. Check sign, size, accuracy and units.

6. Line has x-intercept 6 and y-intercept -3. Find equation.

ANSWER: y=x/2-3

Method cue: Use points (6,0),(0,-3). Check sign, size, accuracy and units.

7. For what k are y=(k-1)x+2 and y=3x-5 parallel?

ANSWER: k=4

Method cue: Equal gradients: k-1=3. Check sign, size, accuracy and units.

8. For what p are px+2y=7 and y=4x-1 perpendicular?

ANSWER: p=1/2

Method cue: First gradient=-p/2. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Transformations of graphs

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Describe transformation y=f(x)+5.

ANSWER: translation 5 units up

Method cue: Outside addition changes outputs. Check sign, size, accuracy and units.

2. Describe y=f(x-3).

ANSWER: translation 3 units right

Method cue: Inside subtraction changes inputs oppositely. Check sign, size, accuracy and units.

3. Point (2,-1) lies on y=f(x). Where on y=f(x)+4?

ANSWER: (2,3)

Method cue: x unchanged. Check sign, size, accuracy and units.

4. Point (-3,5) lies on y=f(x). Where on y=f(-x)?

ANSWER: (3,5)

Method cue: Reflect in y-axis. Check sign, size, accuracy and units.

5. Describe y=-f(x)+2.

ANSWER: reflect in x-axis, then translate 2 up

Method cue: Negate outputs. Check sign, size, accuracy and units.

6. Sketch $y=(x-4)^2-3$ from $y=x^2$ and state vertex.

ANSWER: right 4, down 3; vertex (4,-3)

Method cue: Recognise inside and outside shifts. Check sign, size, accuracy and units.

7. How does y=2^x change to y=2^(x+1)?

ANSWER: translation 1 left

Method cue: x+1 inside means left 1. Check sign, size, accuracy and units.

8. Graph y=f(x) has roots -2 and 5. Find roots of y=f(x-3).

ANSWER: 1 and 8

Method cue: Translation is 3 right. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Graphical equations and inequalities

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Graphs $y=x^2$ and $y=2x+3$ intersect. Find exact intersection x-values.

ANSWER: $x=-1,3$

Method cue: Set $x^2=2x+3$. Check sign, size, accuracy and units.

2. Use $y=x^3-x$ and $y=4$ to describe solutions of $x^3-x=4$.

ANSWER: x-coordinates of intersections

Method cue: Plot or use the two graphs. Check sign, size, accuracy and units.

3. Solve $x^2-4x-5\leq 0$.

ANSWER: $-1\leq x\leq 5$

Method cue: Roots -1,5. Check sign, size, accuracy and units.

4. What region represents $y>2x-1$?

ANSWER: above dashed line $y=2x-1$

Method cue: Strict inequality gives dashed boundary. Check sign, size, accuracy and units.

5. Solve simultaneously $y=x^2-1$ and $y=3x+3$.

ANSWER: (-1,0),(4,15)

Method cue: Set equal: $x^2-3x-4=0$. Check sign, size, accuracy and units.

6. A graph of $y=f(x)$ has roots 1.2 and 4.7. Give solutions of $f(x)>0$ if it opens upward.

ANSWER: $x<1.2$ or $x>4.7$

Method cue: Upward quadratic is positive outside roots. Check sign, size, accuracy and units.

7. Explain why a tangent intersection gives a repeated quadratic root.

ANSWER: The line and curve meet once without crossing

Method cue: Algebraically discriminant is zero. Check sign, size, accuracy and units.

8. Find k so $y=x^2$ and $y=4x+k$ are tangent.

ANSWER: $k=-4$

Method cue: Intersection: $x^2-4x-k=0$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Real-world and proportional graphs

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A distance-time graph rises 150 m in 12 s. Find speed.

ANSWER: 12.5 m/s

Method cue: Gradient=150/12. Check sign, size, accuracy and units.

2. A taxi cost graph has y-intercept £3.50 and gradient £1.80/km. Write equation.

ANSWER: C=1.8d+3.5

Method cue: Intercept is fixed charge. Check sign, size, accuracy and units.

3. Points on a conversion graph include (8,11.2). It is direct proportion. Find equation.

ANSWER: y=1.4x

Method cue: k=11.2/8=1.4. Check sign, size, accuracy and units.

4. For inverse graph y=k/x through (5,18), find y at x=12.

ANSWER: 7.5

Method cue: k=xy=90. Check sign, size, accuracy and units.

5. A distance-time graph is horizontal from 14:20 to 14:35. Interpret.

ANSWER: stationary for 15 minutes

Method cue: Distance does not change. Check sign, size, accuracy and units.

6. A tank-volume graph steepens. What does this say about inflow rate?

ANSWER: inflow rate is increasing

Method cue: Gradient is volume change per time. Check sign, size, accuracy and units.

7. Two travel graphs intersect at (40,28). Interpret.

ANSWER: At time 40, both are at distance/position 28 in the stated units

Method cue: Intersection means equal coordinates. Check sign, size, accuracy and units.

8. Why can a currency conversion graph change gradient over different dates?

ANSWER: exchange rate changes

Method cue: Gradient represents units of one currency per unit of another. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Average and instantaneous rates

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. For $s=t^3-2t$, find average rate from t=1 to t=3.

ANSWER: 11 units/time

Method cue: s(1)=-1 and s(3)=21. Check sign, size, accuracy and units.

2. A tangent passes through (2,7) and (8,31). Estimate gradient.

ANSWER: 4

Method cue: Gradient=(31-7)/(8-2)=24/6. Check sign, size, accuracy and units.

3. Explain difference between average and instantaneous speed.

ANSWER: average uses interval; instantaneous uses tangent at an instant

Method cue: Chord spans two times. Check sign, size, accuracy and units.

4. A curve is increasing but flattening. Describe gradient.

ANSWER: positive but decreasing toward zero

Method cue: Increasing gives positive slope. Check sign, size, accuracy and units.

5. Estimate derivative-like rate of $y=x^2$ at x=3 using chord from 2.9 to 3.1.

ANSWER: 6

Method cue: Values 8.41 and 9.61. Check sign, size, accuracy and units.

6. A population graph tangent has gradient 450 people/year. Interpret.

ANSWER: at that instant population is increasing about 450 per year

Method cue: Positive gradient means growth. Check sign, size, accuracy and units.

7. When is instantaneous rate zero on a smooth graph?

ANSWER: at a horizontal tangent, often a stationary point

Method cue: Zero gradient means no first-order change. Check sign, size, accuracy and units.

8. Why should tangent points be far apart when reading a drawn graph?

ANSWER: reduces percentage reading error

Method cue: A larger run and rise make small coordinate-reading errors less influential. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Areas under graphs

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Velocity rises uniformly from 0 to 18 m/s in 6 s. Find distance.

ANSWER: 54 m

Method cue: Area is triangle. Check sign, size, accuracy and units.

2. A car travels at 12 m/s for 8 s then slows uniformly to 0 over 4 s. Find distance.

ANSWER: 120 m

Method cue: Rectangle area=12×8=96. Check sign, size, accuracy and units.

3. Velocity is -5 m/s for 3 s. Find displacement and distance.

ANSWER: displacement -15 m; distance 15 m

Method cue: Signed area=-5×3. Check sign, size, accuracy and units.

4. A rate graph forms a trapezium with parallel heights 6 and 14 litres/min over 5 min. Fin...

ANSWER: 50 litres

Method cue: Area=1/2(6+14)×5. Check sign, size, accuracy and units.

5. Estimate area under a curve using four strips of width 2 with heights 3,5,8,10,11 using ...

ANSWER: 60 square units

Method cue: Use width/2[first+last+2(sum interior)]. Check sign, size, accuracy and units.

6. A velocity-time graph has area 80 above and 25 below axis. Find displacement and distanc...

ANSWER: 55 m; 105 m

Method cue: Signed displacement=80-25. Check sign, size, accuracy and units.

7. Acceleration is gradient of a velocity-time graph. Velocity rises 7 to 19 m/s in 4 s. Fi...

ANSWER: 3 m/s²

Method cue: Gradient=(19-7)/4. Check sign, size, accuracy and units.

8. Explain why area under a distance-time graph is not normally distance travelled.

ANSWER: distance is vertical coordinate; its integral has units distance×time

Method cue: Check dimensions. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Independent authorship and release check

OCR Higher Mathematics • Unit 4 Graphs

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- Original vector diagrams and two worked examples appear in every chapter.
- Applications, misconceptions, guided reasoning and independent practice are included.
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