GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 8 | PROBABILITY

TEACH-FROM-SCRATCH • PUBLICATION EDITION

10 detailed chapters • original diagrams • worked examples • guided practice • answers

INDEPENDENT PUBLICATION NOTICE

Independent GCSE Mathematics learning resource aligned to OCR J560. It is not produced, approved or endorsed by OCR or Cambridge University Press & Assessment. OCR identifies the qualification only. No official examination question, mark scheme, textbook page, logo, trade dress or third-party artwork is reproduced.

COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Practice third.

FIVE-PASS ROUTINE

1 Read the key idea without a calculator. 2 Cover each worked solution and reproduce it. 3 Complete practice independently. 4 Mark in a different colour and repair errors. 5 Retest after 2–7 days.

What a complete mathematics answer contains

- A clear model or calculation, not only a calculator display.
- Enough working for another person to follow the reasoning.
- Exact values until approximation is requested.
- Units and sensible rounding in context.
- A final statement when interpretation is required.

Error loop

REPAIR, DO NOT MERELY CORRECT

Label each error: concept, operation, arithmetic, calculator entry, accuracy, unit or interpretation. Write the corrected step and one sentence explaining the change.

Foundation does not mean superficial

OCR Foundation questions can combine familiar skills in unfamiliar contexts. Grade 4–5 success depends on selecting a method, connecting several steps and deciding whether the answer is reasonable.

COMPREHENSIVE LESSON NOTEBOOK

Unit 8 coverage map

OCR J560 Higher Probability content organised into a teachable sequence.

CH 01	OCR 11.01a; 11.02e	Probability models and algebraic totals
CH 02	OCR 11.02a-11.02b	Sample spaces and product-rule counting
CH 03	OCR 11.01b-11.01c	Relative frequency and frequency trees
CH 04	OCR 11.01d	Expected frequency and fairness
CH 05	OCR 11.02c	Venn diagrams and set reasoning
CH 06	OCR 11.02d-11.02f	Independent events and replacement trees
CH 07	OCR 11.02d-11.02f	Dependent events without replacement
CH 08	OCR 11.02f	Conditional probability from tables and trees
CH 09	OCR 11.02e-11.02f	Algebraic probability
CH 10	OCR 11.01-11.02 synthesis	Mixed Higher probability modelling

IMPORTANT TIER NOTE

This notebook covers OCR J560 section 11 at Higher Tier, including frequency trees, product-rule counting, conditional probability and independent/dependent event models. Specification references are identifiers, not copied specification wording.

COMPREHENSIVE LESSON NOTEBOOK

Fast-reference method bank

Understand each relationship before memorising it.

Probability bounds	$0 \le P(A) \le 1$
Complement	$P(\text{not } A)=1-P(A)$
Union	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
Conditional	$P(A\mid B)=P(A\cap B)/P(B)$
Product	$P(A\cap B)=P(A\mid B)P(B)$
Independence	$P(A\cap B)=P(A)P(B)$
Relative frequency	event frequency ÷ trials
Expected frequency	$n\times p$
Product rule	multiply available choices
Tree paths	multiply along; add disjoint paths

CURRENT ASSESSMENT NOTE

OCR has announced Foundation and Higher formula sheets for GCSE Mathematics examinations in 2025, 2026 and 2027. Use the current official OCR sheet alongside this learning bank and check OCR for later exam-series arrangements.

COMPREHENSIVE LESSON NOTEBOOK

Probability models and algebraic totals

Use complements, exhaustive outcomes and mutually exclusive events in numerical and algebraic models.

SPECIFICATION FOCUS

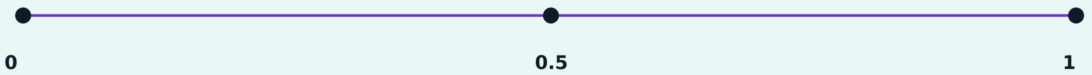
OCR reference OCR 11.01a; 11.02e. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A probability model assigns non-negative weights totalling 1 across an exhaustive set. Algebra expresses missing probabilities without concealing the model conditions.

KEY VOCABULARY

probability • complement • exhaustive • exclusive • algebraic model



Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: Events A, B and C are exhaustive and mutually exclusive. P(A)=0.27 and P(B)=0.41. Find P(C). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Probability models and algebraic totals — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Define the complete outcome set.
- Use total probability 1 only for exhaustive outcomes.
- Form and solve an equation, then validate every probability.

WORKED EXAMPLE 1

Events A, B and C are exhaustive and mutually exclusive. $P(A)=0.27$ and $P(B)=0.41$. Find $P(C)$.

SOLUTION

1. $P(C)=1-0.27-0.41=0.32$.

ANSWER: 0.32

WORKED EXAMPLE 2

A biased spinner gives red with probability x, blue with probability $2x$ and green with probability 0.28. Find x.

SOLUTION

1. $x+2x+0.28=1$.
2. $3x=0.72$, so $x=0.24$.

ANSWER: 0.24

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Probability models and algebraic totals — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Insurance and reliability models distribute total risk across outcomes.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

P(A)=0.64 and P(A or B)=0.81, where A and B are mutually exclusive. Find P(B). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

An outcome model uses probabilities x-0.1, x and 2x-0.2. Find x and state whether the model is valid. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Probability models and algebraic totals

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **Events A, B and C are exhaustive and mutually exclusive. P(A)=0.27 and P(B)=0.41. Find P(C).** [2]

2. **A biased spinner gives red with probability x, blue with probability 2x and green with probability 0.28. Find x.** [3]

3. **P(A)=0.64 and P(A or B)=0.81, where A and B are mutually exclusive. Find P(B).** [2]

4. **The probabilities of scores 1, 2, 3 and 4 are k, 2k, 3k and 0.4. Find k.** [3]

5. **P(not R)=3P(R). Find P(R).** [3]

6. **A and B are mutually exclusive. P(A)=2/7 and P(B)=3/8. Find P(neither A nor B).** [3]

7. **Explain why P(A)=0.7, P(B)=0.6 does not by itself make a probability model impossible.** [2]

8. **An outcome model uses probabilities x-0.1, x and 2x-0.2. Find x and state whether the model is valid.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Sample spaces and product-rule counting

Enumerate complex outcomes systematically and distinguish equally likely elementary outcomes from derived results.

SPECIFICATION FOCUS

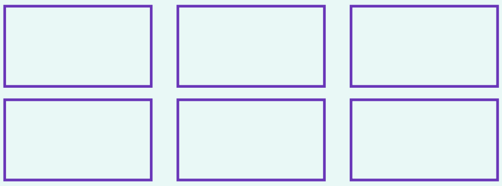
OCR reference OCR 11.02a-11.02b. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Counting rules describe the size of a sample space before probability is calculated. Restrictions change the number of choices at later stages.

KEY VOCABULARY

sample space • ordered outcome • product rule • permutation • restriction



Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A code contains one letter from A–E followed by two different digits from 0–9. How many codes are possible? Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Sample spaces and product-rule counting — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Identify the independent choices or positions.
- Multiply the number of choices when every choice can follow every previous one.
- For probability, count elementary outcomes with equal likelihood.

WORKED EXAMPLE 1

A code contains one letter from A–E followed by two different digits from 0–9. How many codes are possible?

SOLUTION

1. There are 5 choices, then 10, then 9.
2. $5\times10\times9=450$.

ANSWER: 450

WORKED EXAMPLE 2

Two fair dice are rolled. Find the probability that the product is a multiple of 6.

SOLUTION

1. Count ordered pairs with product divisible by 6: 15.
2. $15/36=5/12$.

ANSWER: 5/12

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Sample spaces and product-rule counting — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Codes, schedules and secure identifiers depend on systematic counting.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

Three different books are selected and arranged from 7 books. How many arrangements? First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A menu has 4 starters, 6 mains and 3 desserts. A customer chooses either starter+main or main+dessert. How many meals? Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Sample spaces and product-rule counting

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A code contains one letter from A–E followed by two different digits from 0–9. How many codes are possible?** [3]

2. **Two fair dice are rolled. Find the probability that the product is a multiple of 6.** [4]

3. **Three different books are selected and arranged from 7 books. How many arrangements?** [3]

4. **A fair 4-sector spinner labelled 1–4 and a fair die are used. Find P(spinner score × die score > 12).** [4]

5. **How many four-digit even numbers can be made from 1,2,3,4,5 without repetition?** [4]

6. **Two integers are chosen independently from {1,2,3,4,5}. Find P(their sum is prime).** [4]

7. **Explain why the totals from two dice are not equally likely.** [2]

8. **A menu has 4 starters, 6 mains and 3 desserts. A customer chooses either starter+main or main+dessert. How many meals?** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Relative frequency and frequency trees

Analyse repeated experiments, complete frequency trees and judge the strength of empirical evidence.

SPECIFICATION FOCUS

OCR reference OCR 11.01b-11.01c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A frequency tree preserves actual counts through successive classifications. Relative frequency estimates a probability, while branch proportions answer conditional questions.

KEY VOCABULARY

relative frequency • frequency tree • conditional split • sample size

observed proportion compared with expected proportion

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: In 600 trials an event occurs 174 times. Estimate its probability. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Relative frequency and frequency trees — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Convert between counts and conditional branch proportions.
- At each split, child frequencies total the parent frequency.
- Separate sampling variation from systematic bias.

WORKED EXAMPLE 1

In 600 trials an event occurs 174 times. Estimate its probability.

SOLUTION

1. 174/600=0.29.

ANSWER: 0.29

WORKED EXAMPLE 2

Of 480 orders, 300 are online. Of the online orders, 24 are returned; 9 shop orders are returned. Find P(return) for a randomly selected order.

SOLUTION

1. Total returns=24+9=33.
2. 33/480=11/160.

ANSWER: 11/160 or 0.06875

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Relative frequency and frequency trees — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Quality-control and customer data are analysed through classified counts.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

Using the data above, compare the conditional return rates. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A frequency tree has 900 people: 540 adults. Of all people, 252 travel by bus; 162 of the bus users are adults. Find P(bus | child). Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Relative frequency and frequency trees

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **In 600 trials an event occurs 174 times. Estimate its probability.** [2]

2. **Of 480 orders, 300 are online. Of the online orders, 24 are returned; 9 shop orders are returned. Find P(return) for a randomly selected order.** [4]

3. **Using the data above, compare the conditional return rates.** [4]

4. **A frequency tree begins with 750 items; 18% are faulty. Of the faulty items, 40% are repairable. Find the number faulty and repairable.** [3]

5. **A coin gives 286 heads in 500 tosses. State one reason this does not prove the coin is biased.** [2]

6. **After n trials an event has occurred 84 times and relative frequency 0.35. Find n.** [3]

7. **Two trials give estimates 0.31 from 100 trials and 0.294 from 5000 trials. Which is usually more reliable and why?** [3]

8. **A frequency tree has 900 people: 540 adults. Of all people, 252 travel by bus; 162 of the bus users are adults. Find P(bus | child).** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Expected frequency and fairness

Use probability to predict counts and evaluate whether observed evidence is consistent with a claimed model.

SPECIFICATION FOCUS

OCR reference OCR 11.01d. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Expected counts and values are long-run centres. A discrepancy is evidence to interpret, not automatic proof that a model is false.

KEY VOCABULARY

expected frequency • fairness • expected value • evidence • variation

observed proportion compared with expected proportion

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A component has failure probability 0.007. Find the expected failures among 12,000 components. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Expected frequency and fairness — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Expected frequency$=n\times p$.
- Compare observed and expected differences relative to sample size.
- State that expectation is a long-run benchmark, not a guarantee.

WORKED EXAMPLE 1

A component has failure probability 0.007. Find the expected failures among 12,000 components.

SOLUTION

1. $12000\times 0.007=84$.

ANSWER: 84

WORKED EXAMPLE 2

A spinner claims P(red)=0.35. In 800 spins red occurs 316 times. Compare observed and expected counts.

SOLUTION

1. $800\times 0.35=280$.

ANSWER: Expected 280; observed is 36 higher

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Expected frequency and fairness — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Games and production processes are evaluated against claimed rates.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

A game costs £3. The prize is £12 with probability 0.18 and £5 with probability 0.22. Find the expected net gain to the player. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

State two limitations of using past relative frequency to forecast future demand. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Expected frequency and fairness

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A component has failure probability 0.007. Find the expected failures among 12,000 components.** [2]

2. **A spinner claims P(red)=0.35. In 800 spins red occurs 316 times. Compare observed and expected counts.** [3]

3. **A game costs £3. The prize is £12 with probability 0.18 and £5 with probability 0.22. Find the expected net gain to the player.** [4]

4. **A fair die is rolled 1500 times. Estimate the number of results greater than 4.** [2]

5. **A machine claims defect probability 0.02. A sample of 50 has 3 defects. Explain why this is weak evidence of a changed rate.** [3]

6. **A prize game pays £20 with probability 0.08 and £4 with probability 0.30. What entry fee gives a fair game?** [3]

7. **In 2500 independent trials, the expected number of failures is 65. Find the modelled failure probability.** [2]

8. **State two limitations of using past relative frequency to forecast future demand.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Venn diagrams and set reasoning

Construct two-set models and calculate unconditional and conditional probabilities from regions.

SPECIFICATION FOCUS

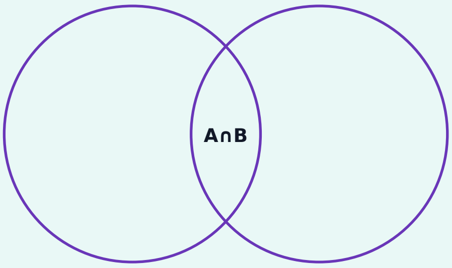
OCR reference OCR 11.02c. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

A Venn diagram partitions a population into disjoint regions. Conditional probability changes the denominator to the region known to contain the outcome.

KEY VOCABULARY

set • overlap • union • intersection • complement • conditional



Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: In a group of 80, 47 study French, 39 study Spanish and 18 study both. Find the number studying neither. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Venn diagrams and set reasoning — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Enter the intersection first.
- Recover only-regions by subtraction.
- For a conditional probability, restrict the denominator to the stated set.

WORKED EXAMPLE 1

In a group of 80, 47 study French, 39 study Spanish and 18 study both. Find the number studying neither.

SOLUTION

1. At least one=47+39-18=68.
2. Neither=80-68=12.

ANSWER: 12

WORKED EXAMPLE 2

Using the same data, find P(French | Spanish).

SOLUTION

1. 18 of the 39 Spanish students study French.
2. 18/39=6/13.

ANSWER: 6/13

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Venn diagrams and set reasoning — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Medical, subject-choice and market segments often overlap.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

In a universal set of 120, $n(A)=72$, $n(B)=55$ and $n(\text{neither})=18$. Find $n(A\cap B)$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain why the denominator changes from the whole sample to $n(B)$ in $P(A \mid B)$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Venn diagrams and set reasoning

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **In a group of 80, 47 study French, 39 study Spanish and 18 study both. Find the number studying neither.** [3]

2. **Using the same data, find P(French | Spanish).** [3]

3. **In a universal set of 120, $n(A)=72$, $n(B)=55$ and n(neither)=18. Find $n(A \cap B)$.** [3]

4. **For the data above, find P(A only).** [2]

5. **A number is selected from 1–30. A is “multiple of 3”, B is “factor of 30”. Find $n(A \cap B)$.** [3]

6. **$P(A)=0.58$, $P(B)=0.46$ and $P(A \cap B)=0.21$. Find $P(A \cup B)$.** [2]

7. **For the previous probabilities, find $P(A \mid B)$.** [3]

8. **Explain why the denominator changes from the whole sample to $n(B)$ in $P(A \mid B)$.** [2]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Independent events and replacement trees

Calculate multi-stage probabilities and test independence using conditional and intersection probabilities.

SPECIFICATION FOCUS

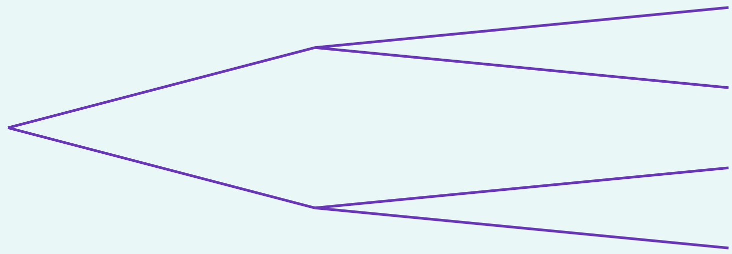
OCR reference OCR 11.02d-11.02f. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Independence means learning one event occurred does not change the probability of the other. Repeated identical trials permit compact complement and binomial-style reasoning.

KEY VOCABULARY

independent • replacement • intersection • complement • repeated trial



multiply along a path • add alternative paths

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: $P(A)=0.65$ and $P(B)=0.4$. A and B are independent. Find $P(A\cap B)$. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Independent events and replacement trees — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Multiply along a path and add disjoint paths.
- With replacement, restore the original composition.
- Test independence using $P(A\cap B)=P(A)P(B)$ or $P(A|B)=P(A)$.

WORKED EXAMPLE 1

$P(A)=0.65$ and $P(B)=0.4$. A and B are independent. Find $P(A\cap B)$.

SOLUTION

1. $0.65\times 0.4=0.26$.

ANSWER: 0.26

WORKED EXAMPLE 2

A biased coin has $P(H)=0.7$ and is tossed three times. Find P(exactly two heads).

SOLUTION

1. There are three HHT-type paths.
2. $3\times 0.7^2\times 0.3=0.441$.

ANSWER: 0.441

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Independent events and replacement trees — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Repeated tests and component systems use independent-event models.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

For the same coin, find P(at least one tail). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain why identical marginal probabilities do not prove two events are independent. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Independent events and replacement trees

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **P(A)=0.65 and P(B)=0.4. A and B are independent. Find P(A∩B).** [2]

2. **A biased coin has P(H)=0.7 and is tossed three times. Find P(exactly two heads).** [4]

3. **For the same coin, find P(at least one tail).** [3]

4. **A bag contains 5 red and 7 blue counters. A counter is replaced after each of three draws. Find P(all the same colour).** [4]

5. **Events C and D satisfy P(C)=0.3, P(D)=0.5 and P(C∩D)=0.15. Are they independent?** [3]

6. **Independent events have P(A)=x, P(B)=0.4 and P(A∩B)=0.28. Find x.** [3]

7. **A test has sensitivity 0.92. Two independent tests are used on an affected item. Find P(at least one positive).** [3]

8. **Explain why identical marginal probabilities do not prove two events are independent.** [3]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Dependent events without replacement

Update conditional branches and solve multi-stage selection problems without replacement.

SPECIFICATION FOCUS

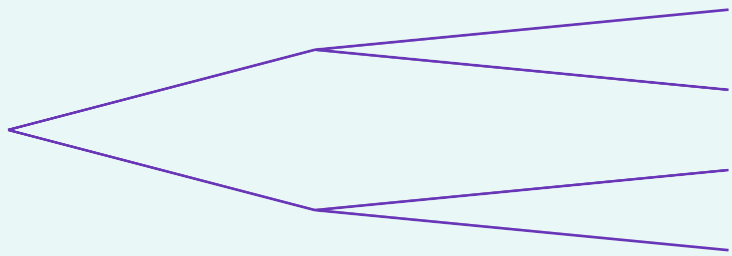
OCR reference OCR 11.02d-11.02f. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Sampling without replacement changes the composition. Every later branch is conditioned on the path already followed.

KEY VOCABULARY

dependent • without replacement • conditional branch • remaining



multiply along a path • add alternative paths

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A bag has 6 red, 5 blue and 4 green counters. Two are drawn without replacement. Find P(both red). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Dependent events without replacement — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- After a draw, reduce the total and the selected category.
- Write each path as a product of changing fractions.
- Combine all disjoint orders that satisfy the event.

WORKED EXAMPLE 1

A bag has 6 red, 5 blue and 4 green counters. Two are drawn without replacement. Find P(both red).

SOLUTION

1. $6/15 \times 5/14 = 30/210 = 1/7$.

ANSWER: 1/7

WORKED EXAMPLE 2

For the same bag, find P(one red and one blue).

SOLUTION

1. $RB + BR = 6/15 \times 5/14 + 5/15 \times 6/14 = 60/210 = 2/7$.

ANSWER: 2/7

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Dependent events without replacement — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Committee selection and batch inspection involve dependent choices.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

Five cards numbered 1–5 are shuffled. Two are drawn. Find P(their sum is 6). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain precisely why the second draw is dependent on the first. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Dependent events without replacement

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A bag has 6 red, 5 blue and 4 green counters. Two are drawn without replacement. Find P(both red).** **[3]**

2. **For the same bag, find P(one red and one blue).** **[4]**

3. **Five cards numbered 1–5 are shuffled. Two are drawn. Find P(their sum is 6).** **[4]**

4. **A box contains x white and 3 black balls. Two are drawn without replacement. P(both black)=1/10. Find x.** **[5]**
SHOW ALL WORKING

5. **A committee of 2 is chosen from 4 women and 3 men. Find P(one of each).** **[4]**

6. **From 8 bulbs, 2 are faulty. Three are chosen without replacement. Find P(no faulty bulb).** **[3]**

7. **For the same bulbs, find P(at least one faulty).** **[3]**

8. **Explain precisely why the second draw is dependent on the first.** **[2]**

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Conditional probability from tables and trees

Restrict a sample space after information is known and reverse conditional information when appropriate.

SPECIFICATION FOCUS

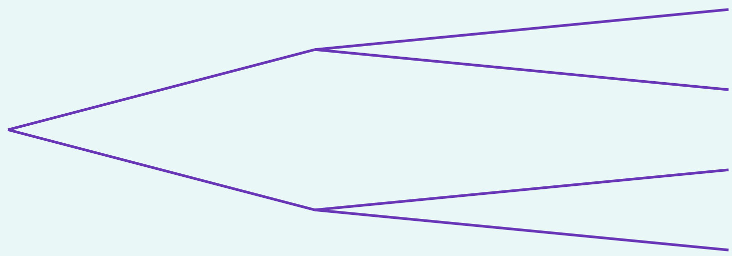
OCR reference OCR 11.02f. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Conditional probability focuses on outcomes compatible with known information. Reversing the condition usually gives a different probability.

KEY VOCABULARY

given • restricted sample space • reverse conditional • base rate



multiply along a path • add alternative paths

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A table records 90 students: 54 study science; 30 of those study music. Find P(science | music) if 42 study music. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Conditional probability from tables and trees — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Read “given B” as restrict to B.
- Use $P(A|B)=P(A\cap B)/P(B)$.
- Do not reverse a conditional statement without calculation.

WORKED EXAMPLE 1

A table records 90 students: 54 study science; 30 of those study music. Find P(science | music) if 42 study music.

SOLUTION

1. $30/42=5/7$.

ANSWER: 5/7

WORKED EXAMPLE 2

Using the same data, find P(music | science).

SOLUTION

1. $30/54=5/9$.

ANSWER: 5/9

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Conditional probability from tables and trees — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Screening tests illustrate why base rates matter.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

$P(A \cap B)=0.18$ and $P(B)=0.45$. Find $P(A|B)$. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Explain the error in assuming $P(A|B)=P(B|A)$. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Conditional probability from tables and trees

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A table records 90 students: 54 study science; 30 of those study music. Find P(science | music) if 42 study music.** [3]

2. **Using the same data, find P(music | science).** [2]

3. **$P(A \cap B)=0.18$ and $P(B)=0.45$. Find $P(A|B)$.** [2]

4. **$P(A)=0.6$ and $P(B|A)=0.35$. Find $P(A \cap B)$.** [3]

5. **P(rain)=0.3, P(late|rain)=0.4 and P(late|no rain)=0.1. Find P(late).** [4]

6. **Using the previous model, find P(rain | late).** [4]

7. **A disease affects 2% of people. A test is positive for 95% of affected people and 6% of unaffected people. Find P(affected | positive).** [5]
SHOW ALL WORKING

8. **Explain the error in assuming $P(A|B)=P(B|A)$.** [2]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic probability

Translate probability conditions into equations and reject roots that do not fit the model.

SPECIFICATION FOCUS

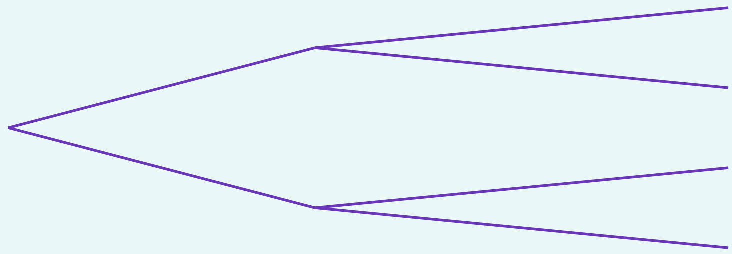
OCR reference OCR 11.02e-11.02f. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Algebraic probability combines structural equations with domain checks. A solved root must still represent a possible probability or whole-number count.

KEY VOCABULARY

variable • equation • valid root • count constraint • probability constraint



multiply along a path • add alternative paths

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A bag contains x red and 5 blue counters. P(red)=3/8. Find x. Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic probability — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Express every branch or region in terms of the variable.
- Form an equation from a stated combined probability.
- Solve and reject values that give negative counts or probabilities outside [0,1].

WORKED EXAMPLE 1

A bag contains x red and 5 blue counters. P(red)=3/8. Find x.

SOLUTION

1. $x/(x+5)=3/8$; $8x=3x+15$.

ANSWER: 3

WORKED EXAMPLE 2

A spinner has P(A)=x, P(B)=x+0.2 and P(C)=2x-0.1. Find x.

SOLUTION

1. $4x+0.1=1$, hence $x=0.225$.

ANSWER: 0.225

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Algebraic probability — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Unknown compositions and biased devices produce probability equations.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

Independent events have $P(A)=x$ and $P(B)=x+0.1$. If $P(A\cap B)=0.30$, find x. First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

A box contains n items, 4 defective. Two are selected without replacement and $P(\text{both defective})=1/15$. Find n. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Algebraic probability

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A bag contains x red and 5 blue counters. P(red)=3/8. Find x.** [3]

2. **A spinner has P(A)=x, P(B)=x+0.2 and P(C)=2x-0.1. Find x.** [3]

3. **Independent events have P(A)=x and P(B)=x+0.1. If P(A∩B)=0.30, find x.** [4]

4. **A bag has x red and x+2 blue counters. Two are drawn without replacement. P(both red)=1/6. Find x.** [5]

SHOW ALL WORKING

5. **P(A)=x, P(B)=0.7 and P(A∪B)=0.82. If P(A∩B)=0.18, find x.** [3]

6. **A biased coin has P(H)=p. It is tossed twice and P(exactly one head)=0.48. Find p.** [5]

SHOW ALL WORKING

7. **Why are both roots valid in the previous question?** [2]

8. **A box contains n items, 4 defective. Two are selected without replacement and P(both defective)=1/15. Find n.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Mixed Higher probability modelling

Select representations for unstructured problems and justify assumptions, calculations and conclusions.

SPECIFICATION FOCUS

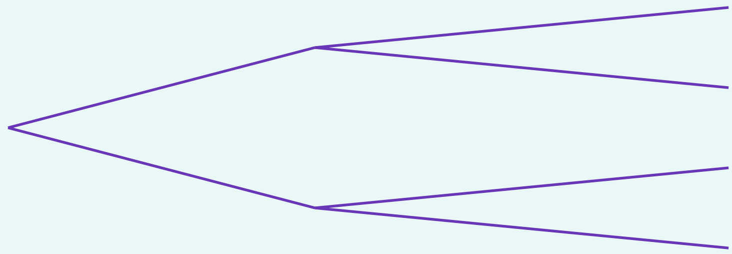
OCR reference OCR 11.01-11.02 synthesis. This lesson develops representation, accurate calculation, reasoning and interpretation within Higher Tier.

Understand the idea

Higher problems often withhold the diagram. The solver must choose a representation, justify assumptions and communicate limitations.

KEY VOCABULARY

model choice • assumption • validation • unstructured problem • inference



multiply along a path • add alternative paths

Why the method works

- A complete outcome model prevents omitted or duplicated cases.
- Multiplication follows one conditional route; addition combines disjoint routes.
- Complements, totals and alternative representations provide checks.

CHECK BEFORE MOVING ON

Answer mentally: A factory uses machines A and B for 65% and 35% of output. Their defect rates are 1.5% and 2.4%. Find P(a random item is defective). Then define one key word from this page in your own words.

COMPREHENSIVE LESSON NOTEBOOK

Mixed Higher probability modelling — method and examples

Follow the reasoning; do not copy steps mechanically.

Reliable method

- Identify what is known, conditioned upon and required.
- Choose a table, Venn diagram, frequency tree or probability tree.
- Validate totals, dependence, exactness and context.

WORKED EXAMPLE 1

A factory uses machines A and B for 65% and 35% of output. Their defect rates are 1.5% and 2.4%. Find P(a random item is defective).

SOLUTION

1. $0.65\times0.015+0.35\times0.024=0.01815$.

ANSWER: 0.01815

WORKED EXAMPLE 2

Given an item is defective in the previous model, find P(it came from B).

SOLUTION

1. B and defective$=0.35\times0.024=0.0084$.
2. $0.0084/0.01815=56/121$.

ANSWER: 56/121 or about 0.463

SELF-CHECK QUESTION

Which step carries the mathematical reason, and which step carries only arithmetic? Being able to distinguish them makes a solution easier to repair.

COMPREHENSIVE LESSON NOTEBOOK

Mixed Higher probability modelling — apply and reason

Connect the method to decisions and common errors.

Where this mathematics appears

Risk, diagnosis and industrial decisions combine several probability models.

Common misconceptions

- Treating derived results as equally likely without checking elementary outcomes.
- Using an unconditional denominator after information has restricted the sample space.
- Assuming independence, or failing to update a without-replacement branch.

GUIDED TRY

Three independent alarms detect a fault with probabilities 0.8, 0.7 and 0.6. Find P(exactly two detect it). First predict the form or approximate size of the answer. Then show the complete method.

REASONING PROMPT

Give four checks for a completed Higher probability solution. Explain why each condition or operation you use is valid.

EXAM COMMUNICATION

Use one mathematical statement per line. Label units and restrictions. If the result is approximate, show the unrounded value before the final rounded answer.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

Mixed Higher probability modelling

Complete without notes. Show all working, then use the answer section to diagnose and repair errors.

1. **A factory uses machines A and B for 65% and 35% of output. Their defect rates are 1.5% and 2.4%. Find P(a random item is defective).** [4]

2. **Given an item is defective in the previous model, find P(it came from B).** [4]

3. **Three independent alarms detect a fault with probabilities 0.8, 0.7 and 0.6. Find P(exactly two detect it).** [5]

SHOW ALL WORKING

4. **A class has 18 girls and 12 boys. Two pupils are chosen without replacement. Find P(at least one boy).** [3]

5. **P(A)=0.5, P(B)=0.4 and P(A∪B)=0.7. Determine whether A and B are independent.** [4]

6. **A trial is repeated until the first success, with independent success probability 0.3. Find P(first success occurs on trial 4).** [3]

7. **A survey estimate is based on volunteers from one website. Explain two threats to the probability model.** [3]

8. **Give four checks for a completed Higher probability solution.** [4]

REFLECTION

Circle one: secure / almost / reteach. Record the question number of your most useful error and the rule that repairs it.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Probability models and algebraic totals

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. Events A, B and C are exhaustive and mutually exclusive. P(A)=0.27 and P(B)=0.41. Find P...

ANSWER: 0.32

Method cue: P(C)=1-0.27-0.41=0.32. Check sign, size, accuracy and units.

2. A biased spinner gives red with probability x, blue with probability 2x and green with p...

ANSWER: 0.24

Method cue: x+2x+0.28=1. Check sign, size, accuracy and units.

3. P(A)=0.64 and P(A or B)=0.81, where A and B are mutually exclusive. Find P(B).

ANSWER: 0.17

Method cue: P(B)=0.81-0.64. Check sign, size, accuracy and units.

4. The probabilities of scores 1, 2, 3 and 4 are k, 2k, 3k and 0.4. Find k.

ANSWER: 0.10

Method cue: 6k+0.4=1, hence k=0.1. Check sign, size, accuracy and units.

5. P(not R)=3P(R). Find P(R).

ANSWER: 1/4

Method cue: Let P(R)=r. Then 3r+r=1. Check sign, size, accuracy and units.

6. A and B are mutually exclusive. P(A)=2/7 and P(B)=3/8. Find P(neither A nor B).

ANSWER: 19/56

Method cue: 1-2/7-3/8=56/56-16/56-21/56. Check sign, size, accuracy and units.

7. Explain why P(A)=0.7, P(B)=0.6 does not by itself make a probability model impossible.

ANSWER: The events may overlap

Method cue: Only if A and B are mutually exclusive would their sum exceeding 1 be impossible. Check sign, size, accuracy and units.

8. An outcome model uses probabilities x-0.1, x and 2x-0.2. Find x and state whether the mo...

ANSWER: x=0.325; valid

Method cue: 4x-0.3=1, so x=0.325. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Sample spaces and product-rule counting

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A code contains one letter from A–E followed by two different digits from 0–9. How many ...

ANSWER: 450

Method cue: There are 5 choices, then 10, then 9. Check sign, size, accuracy and units.

2. Two fair dice are rolled. Find the probability that the product is a multiple of 6.

ANSWER: 5/12

Method cue: Count ordered pairs with product divisible by 6: 15. Check sign, size, accuracy and units.

3. Three different books are selected and arranged from 7 books. How many arrangements?

ANSWER: 210

Method cue: $7\times6\times5=210$. Check sign, size, accuracy and units.

4. A fair 4-sector spinner labelled 1–4 and a fair die are used. Find P(spinner score × die...

ANSWER: 5/24

Method cue: There are 24 equally likely pairs. Check sign, size, accuracy and units.

5. How many four-digit even numbers can be made from 1,2,3,4,5 without repetition?

ANSWER: 48

Method cue: Final digit: 2 choices (2 or 4). Check sign, size, accuracy and units.

6. Two integers are chosen independently from {1,2,3,4,5}. Find P(their sum is prime).

ANSWER: 11/25

Method cue: Prime sums are 2,3,5,7. Check sign, size, accuracy and units.

7. Explain why the totals from two dice are not equally likely.

ANSWER: They have unequal numbers of elementary ordered pairs

Method cue: For example, total 2 has one pair while total 7 has six. Check sign, size, accuracy and units.

8. A menu has 4 starters, 6 mains and 3 desserts. A customer chooses either starter+main or...

ANSWER: 42

Method cue: Starter+main: $4\times6=24$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Relative frequency and frequency trees

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. In 600 trials an event occurs 174 times. Estimate its probability.

ANSWER: 0.29

Method cue: 174/600=0.29. Check sign, size, accuracy and units.

2. Of 480 orders, 300 are online. Of the online orders, 24 are returned; 9 shop orders are ...

ANSWER: 11/160 or 0.06875

Method cue: Total returns=24+9=33. Check sign, size, accuracy and units.

3. Using the data above, compare the conditional return rates.

ANSWER: Online 0.08; shop 0.05; online is higher by 0.03

Method cue: Online:24/300. Check sign, size, accuracy and units.

4. A frequency tree begins with 750 items; 18% are faulty. Of the faulty items, 40% are rep...

ANSWER: 54

Method cue: 750×0.18×0.40=54. Check sign, size, accuracy and units.

5. A coin gives 286 heads in 500 tosses. State one reason this does not prove the coin is b...

ANSWER: Random variation can produce a departure from 250

Method cue: A larger or repeated investigation is needed. Check sign, size, accuracy and units.

6. After n trials an event has occurred 84 times and relative frequency 0.35. Find n.

ANSWER: 240

Method cue: 84/n=0.35, so n=240. Check sign, size, accuracy and units.

7. Two trials give estimates 0.31 from 100 trials and 0.294 from 5000 trials. Which is usua...

ANSWER: 0.294 from 5000 trials

Method cue: A larger sample normally has less random fluctuation, assuming comparable sampling. Check sign, size, accuracy and units.

8. A frequency tree has 900 people: 540 adults. Of all people, 252 travel by bus; 162 of th...

ANSWER: 1/4

Method cue: Children=360. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Expected frequency and fairness

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A component has failure probability 0.007. Find the expected failures among 12,000 compo...

ANSWER: 84

Method cue: 12000×0.007=84. Check sign, size, accuracy and units.

2. A spinner claims P(red)=0.35. In 800 spins red occurs 316 times. Compare observed and ex...

ANSWER: Expected 280; observed is 36 higher

Method cue: 800×0.35=280. Check sign, size, accuracy and units.

3. A game costs £3. The prize is £12 with probability 0.18 and £5 with probability 0.22. Fi...

ANSWER: £0.26 gain

Method cue: Expected prize=12(0.18)+5(0.22)=£3.26. Check sign, size, accuracy and units.

4. A fair die is rolled 1500 times. Estimate the number of results greater than 4.

ANSWER: 500

Method cue: P(5 or 6)=2/6=1/3; 1500/3=500. Check sign, size, accuracy and units.

5. A machine claims defect probability 0.02. A sample of 50 has 3 defects. Explain why this...

ANSWER: The sample is small and random variation is substantial

Method cue: The expected count is 1, but one small sample cannot establish a new long-run rate. Check sign, size, accuracy and units.

6. A prize game pays £20 with probability 0.08 and £4 with probability 0.30. What entry fee...

ANSWER: £2.80

Method cue: Fair fee=expected payout=20(0.08)+4(0.30)=2.80. Check sign, size, accuracy and units.

7. In 2500 independent trials, the expected number of failures is 65. Find the modelled fai...

ANSWER: 0.026

Method cue: 65/2500=0.026. Check sign, size, accuracy and units.

8. State two limitations of using past relative frequency to forecast future demand.

ANSWER: The underlying conditions may change; the sample may be unrepresentative

Method cue: Random variation also remains, so the prediction is not exact. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Venn diagrams and set reasoning

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. In a group of 80, 47 study French, 39 study Spanish and 18 study both. Find the number s...

ANSWER: 12

Method cue: At least one=47+39-18=68. Check sign, size, accuracy and units.

2. Using the same data, find P(French | Spanish).

ANSWER: 6/13

Method cue: 18 of the 39 Spanish students study French. Check sign, size, accuracy and units.

3. In a universal set of 120, n(A)=72, n(B)=55 and n(neither)=18. Find n(A∩B).

ANSWER: 25

Method cue: n(A∪B)=102. Check sign, size, accuracy and units.

4. For the data above, find P(A only).

ANSWER: 47/120

Method cue: A only=72-25=47. Check sign, size, accuracy and units.

5. A number is selected from 1–30. A is "multiple of 3", B is "factor of 30". Find n(A∩B).

ANSWER: 4

Method cue: Common members are 3,6,15,30. Check sign, size, accuracy and units.

6. P(A)=0.58, P(B)=0.46 and P(A∩B)=0.21. Find P(A∪B).

ANSWER: 0.83

Method cue: 0.58+0.46-0.21=0.83. Check sign, size, accuracy and units.

7. For the previous probabilities, find P(A | B).

ANSWER: 21/46

Method cue: P(A|B)=P(A∩B)/P(B)=0.21/0.46. Check sign, size, accuracy and units.

8. Explain why the denominator changes from the whole sample to n(B) in P(A | B).

ANSWER: The condition tells us B has occurred

Method cue: Only outcomes inside B remain possible, so B becomes the restricted sample space. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Independent events and replacement trees

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. P(A)=0.65 and P(B)=0.4. A and B are independent. Find P(A∩B).

ANSWER: 0.26

Method cue: 0.65×0.4=0.26. Check sign, size, accuracy and units.

2. A biased coin has P(H)=0.7 and is tossed three times. Find P(exactly two heads).

ANSWER: 0.441

Method cue: There are three HHT-type paths. Check sign, size, accuracy and units.

3. For the same coin, find P(at least one tail).

ANSWER: 0.657

Method cue: Use the complement: 1-P(HHH). Check sign, size, accuracy and units.

4. A bag contains 5 red and 7 blue counters. A counter is replaced after each of three draw...

ANSWER: 13/48

Method cue: P(RRR)+P(BBB)=(5/12)^3+(7/12)^3. Check sign, size, accuracy and units.

5. Events C and D satisfy P(C)=0.3, P(D)=0.5 and P(C∩D)=0.15. Are they independent?

ANSWER: Yes

Method cue: P(C)P(D)=0.15, equal to P(C∩D). Check sign, size, accuracy and units.

6. Independent events have P(A)=x, P(B)=0.4 and P(A∩B)=0.28. Find x.

ANSWER: 0.7

Method cue: 0.4x=0.28. Check sign, size, accuracy and units.

7. A test has sensitivity 0.92. Two independent tests are used on an affected item. Find P(...

ANSWER: 0.9936

Method cue: 1-P(both negative)=1-0.08^2. Check sign, size, accuracy and units.

8. Explain why identical marginal probabilities do not prove two events are independent.

ANSWER: Independence concerns whether one event changes the probability of the other

Method cue: You must compare a conditional probability or the intersection product. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Dependent events without replacement

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A bag has 6 red, 5 blue and 4 green counters. Two are drawn without replacement. Find P(...

ANSWER: 1/7

Method cue: 6/15×5/14=30/210=1/7. Check sign, size, accuracy and units.

2. For the same bag, find P(one red and one blue).

ANSWER: 2/7

Method cue: RB+BR=6/15×5/14+5/15×6/14=60/210=2/7. Check sign, size, accuracy and units.

3. Five cards numbered 1–5 are shuffled. Two are drawn. Find P(their sum is 6).

ANSWER: 1/5

Method cue: Favourable ordered pairs: (1,5),(5,1),(2,4),(4,2). Check sign, size, accuracy and units.

4. A box contains x white and 3 black balls. Two are drawn without replacement. P(both blac...

ANSWER: x=6

Method cue: 3/(x+3)×2/(x+2)=1/10. Check sign, size, accuracy and units.

5. A committee of 2 is chosen from 4 women and 3 men. Find P(one of each).

ANSWER: 4/7

Method cue: WM+MW=4/7×3/6+3/7×4/6=4/7. Check sign, size, accuracy and units.

6. From 8 bulbs, 2 are faulty. Three are chosen without replacement. Find P(no faulty bulb)...

ANSWER: 5/14

Method cue: 6/8×5/7×4/6=5/14. Check sign, size, accuracy and units.

7. For the same bulbs, find P(at least one faulty).

ANSWER: 9/14

Method cue: Use complement:1-5/14=9/14. Check sign, size, accuracy and units.

8. Explain precisely why the second draw is dependent on the first.

ANSWER: The first draw changes the composition of the remaining set

Method cue: Therefore the conditional probability on the second draw changes. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Conditional probability from tables and trees

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A table records 90 students: 54 study science; 30 of those study music. Find P(science |...

ANSWER: 5/7

Method cue: 30/42=5/7. Check sign, size, accuracy and units.

2. Using the same data, find P(music | science).

ANSWER: 5/9

Method cue: 30/54=5/9. Check sign, size, accuracy and units.

3. P(A∩B)=0.18 and P(B)=0.45. Find P(A|B).

ANSWER: 0.4

Method cue: 0.18/0.45=0.4. Check sign, size, accuracy and units.

4. P(A)=0.6 and P(B|A)=0.35. Find P(A∩B).

ANSWER: 0.21

Method cue: P(A∩B)=P(B|A)P(A)=0.35×0.6. Check sign, size, accuracy and units.

5. P(rain)=0.3, P(late|rain)=0.4 and P(late|no rain)=0.1. Find P(late).

ANSWER: 0.19

Method cue: Rain-late=0.3×0.4=0.12. Check sign, size, accuracy and units.

6. Using the previous model, find P(rain | late).

ANSWER: 12/19

Method cue: P(rain and late)/P(late)=0.12/0.19=12/19. Check sign, size, accuracy and units.

7. A disease affects 2% of people. A test is positive for 95% of affected people and 6% of ...

ANSWER: 95/389

Method cue: Affected-positive=0.02×0.95=0.019. Check sign, size, accuracy and units.

8. Explain the error in assuming P(A|B)=P(B|A).

ANSWER: The restricted sample spaces and denominators are different

Method cue: The intersection is common, but it is divided by P(B) in one direction and P(A) in the other. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Algebraic probability

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A bag contains x red and 5 blue counters. P(red)=3/8. Find x.

ANSWER: 3

Method cue: $x/(x+5)=3/8$; $8x=3x+15$. Check sign, size, accuracy and units.

2. A spinner has P(A)=x, P(B)=x+0.2 and P(C)=2x-0.1. Find x.

ANSWER: 0.225

Method cue: $4x+0.1=1$, hence $x=0.225$. Check sign, size, accuracy and units.

3. Independent events have P(A)=x and P(B)=x+0.1. If P(A∩B)=0.30, find x.

ANSWER: 0.5

Method cue: $x(x+0.1)=0.30$. Check sign, size, accuracy and units.

4. A bag has x red and x+2 blue counters. Two are drawn without replacement. P(both red)=1/...

ANSWER: No valid integer x

Method cue: The equation is $6x(x-1)=(2x+2)(2x+1)$. Check sign, size, accuracy and units.

5. P(A)=x, P(B)=0.7 and P(A∪B)=0.82. If P(A∩B)=0.18, find x.

ANSWER: 0.30

Method cue: $x+0.7-0.18=0.82$. Check sign, size, accuracy and units.

6. A biased coin has P(H)=p. It is tossed twice and P(exactly one head)=0.48. Find p.

ANSWER: 0.4 or 0.6

Method cue: $2p(1-p)=0.48$. Check sign, size, accuracy and units.

7. Why are both roots valid in the previous question?

ANSWER: Interchanging head and tail probabilities leaves “exactly one head” unchanged

Method cue: Both roots lie between 0 and 1. Check sign, size, accuracy and units.

8. A box contains n items, 4 defective. Two are selected without replacement and P(both def...

ANSWER: 15

Method cue: $4/n\times3/(n-1)=1/15$. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Practice answers and repair notes

Mixed Higher probability modelling

Use only after a genuine attempt. Rework incorrect questions from a blank line rather than copying the final value.

1. A factory uses machines A and B for 65% and 35% of output. Their defect rates are 1.5% a...

ANSWER: 0.01815

Method cue: $0.65\times0.015+0.35\times0.024=0.01815$. Check sign, size, accuracy and units.

2. Given an item is defective in the previous model, find P(it came from B).

ANSWER: 56/121 or about 0.463

Method cue: B and defective$=0.35\times0.024=0.0084$. Check sign, size, accuracy and units.

3. Three independent alarms detect a fault with probabilities 0.8, 0.7 and 0.6. Find P(exac...

ANSWER: 0.452

Method cue: $0.8\times0.7\times0.4 + 0.8\times0.3\times0.6 + 0.2\times0.7\times0.6$. Check sign, size, accuracy and units.

4. A class has 18 girls and 12 boys. Two pupils are chosen without replacement. Find P(at l...

ANSWER: 94/145

Method cue: Use the complement. Check sign, size, accuracy and units.

5. $P(A)=0.5$, $P(B)=0.4$ and $P(A\cup B)=0.7$. Determine whether A and B are independent.

ANSWER: Yes

Method cue: $P(A\cap B)=0.5+0.4-0.7=0.2$. Check sign, size, accuracy and units.

6. A trial is repeated until the first success, with independent success probability 0.3. F...

ANSWER: 0.1029

Method cue: Fail three times then succeed:$0.7^3\times0.3$. Check sign, size, accuracy and units.

7. A survey estimate is based on volunteers from one website. Explain two threats to the pr...

ANSWER: Self-selection bias and lack of population coverage

Method cue: The sample may not represent the target population; repeated responses or wording effects may also distort it. Check sign, size, accuracy and units.

8. Give four checks for a completed Higher probability solution.

ANSWER: Probabilities in [0,1]; exhaustive totals equal 1; conditional denominator is correct; dependence/equal-likelihood assumptions are justified

Method cue: Also check exact arithmetic and whether all disjoint routes were included. Check sign, size, accuracy and units.

COMPREHENSIVE LESSON NOTEBOOK

Independent authorship and release check

OCR Higher Mathematics • Unit 8 Probability

ORIGINAL-CONTENT STATEMENT

Release checklist

- 10 publisher-organised chapters mapped to OCR J560 section 11.
- Every chapter begins on a new page and contains conceptual explanation.
- Original vector diagrams and two worked examples appear in every chapter.
- Applications, misconceptions, guided reasoning and independent practice are included.
- Answers are collected at the end with method cues.
- Copyright, trademark and non-endorsement wording included.

USE OF OFFICIAL MATERIALS