

GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 1: NUMBER

Mapped to the Eduqas GCSE Mathematics specification • HN1-HN16

16 chapters • concept teaching • worked methods • reasoning • practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

Independent publication notice

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How to learn from this notebook

Each chapter follows the same deliberate sequence: understand the mathematical object, see the method built line by line, apply it in unfamiliar settings, then complete a Higher-tier practice set.

1 • Understand

Read definitions aloud, annotate the diagram and answer the checkpoint without looking back.

2 • Build the method

Cover each next line of the example. Predict it, reveal it, and explain why it is permitted.

3 • Connect and reason

Use the misconception analysis and guided task to decide which method fits, not merely how to execute it.

4 • Practise at Higher demand

All assessed questions carry 4–6 marks. Show a mathematical setup, accurate progress and a supported conclusion.

5 • Diagnose and revisit

Mark from the worked answer. Record whether an error came from interpretation, structure, arithmetic, accuracy or communication.

Calculator discipline

Eduqas Number content may appear on either component. Keep exact values on non-calculator work; on calculator work, write the expression used, retain unrounded values and round once at the end.

Unit map • Eduqas Higher Number

The wording below is an independent teaching summary of the official HN1–HN16 content, not reproduced specification text.

Ch.	Ref.	Lesson
1	HN1	Ordering real numbers
2	HN2	Four operations and place value
3	HN3	Inverse operations and calculation priority
4	HN4	Prime structure, HCF and LCM
5	HN5	Systematic listing and product rule
6	HN6	Powers, roots and estimation
7	HN7	Integer and fractional indices
8	HN8	Exact fractions, surds and multiples of π
9	HN9	Standard form
10	HN10	Terminating and recurring decimals
11	HN11	Fractions in ratio problems
12	HN12	Fractions and percentages as operators
13	HN13	Measures and compound measures
14	HN14	Estimation and checking
15	HN15	Rounding and error intervals
16	HN16	Limits of accuracy and bounds

Assessment design: questions deliberately connect Number with algebra, ratio, geometry and contexts. Marks shown are independent teaching allocations and are not official Eduqas marks.

Higher Number method bank

Exact before approximate

Keep fractions, surds and pi exact until a decimal is explicitly requested.

Operation structure

Use brackets and fraction bars to make the intended hierarchy visible.

Multiplicative thinking

Translate percentages, ratios and compound units into multipliers or formulae.

Proof and justification

State the general form, derive the result and close with the required conclusion.

Error intervals

Write the permitted interval before choosing extreme values.

Reasonableness

Estimate, calculate, compare and explain any difference.

Units and accuracy

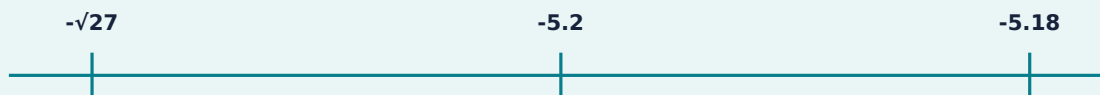
Convert first, carry units, retain precision, then round once.

Exam communication

Show the decisive setup; a bare calculator answer rarely demonstrates a multi-mark method.

Chapter 1 • Ordering real numbers

HN1 • Compare exact and approximate forms on one number line.



Understand the idea

A real number can be represented as a fraction, decimal, percentage, root or signed quantity. Ordering is reliable only after the forms have been made comparable; negative values reverse the usual magnitude intuition.

Key vocabulary
integer; rational; irrational; inequality; modulus; reciprocal

Why the method works

Choose a common exact form where possible. For close surds, compare squares of positive magnitudes; when values are negative, reverse the final order.

Prior-knowledge check
Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint
Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 1 • Build the method

Ordering real numbers

Reliable routine

Choose a common exact form where possible. For close surds, compare squares of positive magnitudes; when values are negative, reverse the final order.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Order $-\sqrt{27}$, -5.18 and $-26/5$.

Worked reasoning

Since $(26/5)^2 = 27.04$ and $5.18^2 = 26.8324$, $26/5 > \sqrt{27} > 5.18$. Therefore $-26/5 < -\sqrt{27} < -5.18$.

Worked example 2

Question. Solve $x^2 < 30$ and $|x-2| > 6$ for negative integer x .

Worked reasoning

The first condition gives $-\sqrt{30} < x < \sqrt{30}$. The second gives $x < -4$ or $x > 8$. Their negative-integer intersection is $x = -5$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 1 • Apply and reason

Ordering real numbers

Common misconception

Converting a recurring decimal to a short rounded decimal can destroy an exact comparison.

Guided problem

4 marks. A bank balance is -£146.80. A payment of £235.50 arrives, three equal bills of £41.75 leave, and a £9.20 fee is charged. Find the balance and interpret its sign.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 1 • Independent practice

HN1 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A bank balance is -£146.80. A payment of £235.50 arrives, three equal bills of £41.75 leave, and a £9.20 fee is charged. Find the balance and interpret its sign.

Q2 | 6 MARKS

A game score begins at 40. Correct answers add 7, incorrect answers subtract 4 and omitted answers subtract 1. After 18 questions, 10 were correct and the final score was 91. Find the numbers incorrect and omitted.

Q3 | 5 MARKS

Evaluate $[(-4)^3 - (-2)^5] \div [-3^2 + 5]$. Explain the difference between -3^2 and $(-3)^2$.

Q4 | 5 MARKS

Show that $0.58 < \sqrt{1/3} < 0.59$ is false as written, then repair it using two-decimal bounds.

Q5 | 6 MARKS

The four values p , -9 , 14 and $2p$ have range 41. Find every possible value of p .

Q6 | 4 MARKS

Evaluate $(-1)^{101} + (-1)^{202} - (-2)^4 + -2^4$. Explain each sign before combining.

Q7 | 5 MARKS

A number x is negative and satisfies $x^2 < 30$ and $|x-2| > 6$. Find all possible integer values.

Q8 | 4 MARKS

Three consecutive integers have mean -7 . Their squares are then given signs $+$, $-$, $+$ in order. Evaluate the resulting signed sum.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 2 • Four operations and place value

HN2 • Calculate fluently with signed numbers, decimals and fractions.

exact

operate

check

Understand the idea

Place value determines the size of every digit; operation signs determine direction. Fractions retain exactness, while formal decimal algorithms require aligned place-value columns.

Key vocabulary

place value; dividend; divisor; mixed number; improper fraction; reciprocal

Why the method works

Estimate first, preserve brackets, convert mixed numbers before multiplication or division, and align decimal points for column work.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 2 • Build the method

Four operations and place value

Reliable routine

Estimate first, preserve brackets, convert mixed numbers before multiplication or division, and align decimal points for column work.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Evaluate $(7/12 - 5/18) \div (11/24 + 1/9)$.

Worked reasoning

The numerator is $11/36$ and the denominator is $41/72$. Multiply by the reciprocal: $11/36 \times 72/41 = 22/41$.

Worked example 2

Question. A tank is $5/6$ full. One quarter of its water is removed, then 18 L makes it $3/4$ full. Find capacity.

Worked reasoning

After removal it is $5/8$ full. The increase from $5/8$ to $3/4$ is $1/8$, so $1/8$ is 18 L and capacity is 144 L.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 2 • Apply and reason

Four operations and place value

Common misconception

Cancellation is legal between factors, not across terms joined by addition or subtraction.

Guided problem

4 marks. Calculate $\frac{4}{7} + \frac{4}{7}$ and $\frac{4}{7} \times \frac{4}{7}$ in simplest form. Then calculate $\frac{4}{7}$ divided by $\frac{4}{7}$ and explain the reciprocal step.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 2 • Independent practice

HN2 • Questions 1–8 • 4–6 marks each

Q1 | 4 MARKS

Calculate $\frac{4}{7} + \frac{4}{7}$ and $\frac{4}{7} \times \frac{4}{7}$ in simplest form. Then calculate $\frac{4}{7}$ divided by $\frac{4}{7}$ and explain the reciprocal step.

Q2 | 4 MARKS

Calculate $\frac{1}{4} + \frac{6}{8}$ and $\frac{1}{4} \times \frac{6}{8}$ in simplest form. Then calculate $\frac{1}{4}$ divided by $\frac{6}{8}$ and explain the reciprocal step.

Q3 | 4 MARKS

Calculate $\frac{4}{7} + \frac{1}{8}$ and $\frac{4}{7} \times \frac{1}{8}$ in simplest form. Then calculate $\frac{4}{7}$ divided by $\frac{1}{8}$ and explain the reciprocal step.

Q4 | 4 MARKS

Calculate $\frac{1}{4} + \frac{1}{9}$ and $\frac{1}{4} \times \frac{1}{9}$ in simplest form. Then calculate $\frac{1}{4}$ divided by $\frac{1}{9}$ and explain the reciprocal step.

Q5 | 4 MARKS

Calculate $\frac{4}{7} + \frac{2}{9}$ and $\frac{4}{7} \times \frac{2}{9}$ in simplest form. Then calculate $\frac{4}{7}$ divided by $\frac{2}{9}$ and explain the reciprocal step.

Q6 | 4 MARKS

Calculate $\frac{1}{4} + \frac{1}{10}$ and $\frac{1}{4} \times \frac{1}{10}$ in simplest form. Then calculate $\frac{1}{4}$ divided by $\frac{1}{10}$ and explain the reciprocal step.

Q7 | 4 MARKS

Calculate $\frac{4}{7} + \frac{1}{10}$ and $\frac{4}{7} \times \frac{1}{10}$ in simplest form. Then calculate $\frac{4}{7}$ divided by $\frac{1}{10}$ and explain the reciprocal step.

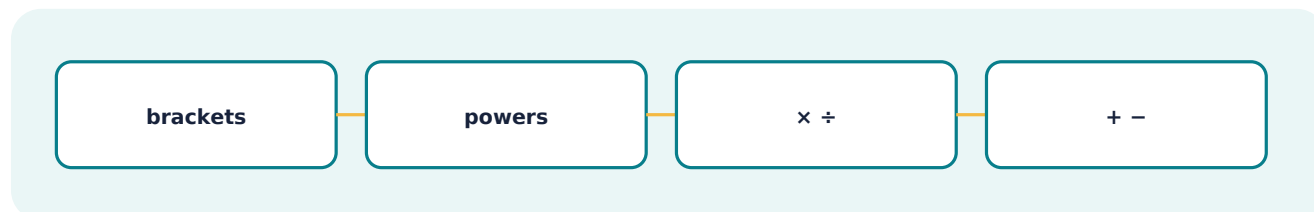
Q8 | 4 MARKS

Calculate $\frac{1}{4} + \frac{9}{11}$ and $\frac{1}{4} \times \frac{9}{11}$ in simplest form. Then calculate $\frac{1}{4}$ divided by $\frac{9}{11}$ and explain the reciprocal step.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 3 • Inverse operations and calculation priority

HN3 • Use operation structure, brackets, powers and reciprocals.



Understand the idea

An expression is a hierarchy, not a left-to-right sentence. Brackets and fraction bars create groups; powers act on their stated bases; multiplication and division then proceed left to right.

Key vocabulary

inverse; operation; exponent; base; reciprocal; hierarchy

Why the method works

Mark the deepest grouping, evaluate powers, then complete multiplication/division and addition/subtraction at equal priority from left to right.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 3 • Build the method

Inverse operations and calculation priority

Reliable routine

Mark the deepest grouping, evaluate powers, then complete multiplication/division and addition/subtraction at equal priority from left to right.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Evaluate $84 - 3[7 - 2(5 - 9)] + 6^2 \div 4$.

Worked reasoning

$5 - 9 = -4$, so $7 - 2(-4) = 15$. Also $6^2 \div 4 = 9$. Hence $84 - 45 + 9 = 48$.

Worked example 2

Question. Explain -3^2 versus $(-3)^2$.

Worked reasoning

Without brackets the exponent acts on 3, giving $-(3^2) = -9$. With brackets the base is -3, so $(-3)^2 = 9$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 3 • Apply and reason

Inverse operations and calculation priority

Common misconception

A calculator entry is not evidence of the intended grouping; use extra brackets whenever the structure could be read two ways.

Guided problem

5 marks. Solve $(3/5)x - 7/10 = 11/6$ exactly. Verify by substitution.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 3 • Independent practice

HN3 • Questions 1-8 • 4-6 marks each

Q1 | 5 MARKS

Solve $(\frac{3}{5})x - \frac{7}{10} = \frac{11}{6}$ exactly. Verify by substitution.

Q2 | 5 MARKS

Find the exact value of $(0.6 \text{ recurring} + 0.25) / (1.2 - \frac{5}{8})$. Convert recurring notation before calculating.

Q3 | 5 MARKS

A journey is split into $\frac{2}{5}$ at 48 km/h and the remaining $\frac{3}{5}$ at 72 km/h. For a total distance of 180 km, find the exact total travel time.

Q4 | 5 MARKS

Evaluate $(\frac{3}{7} \text{ of } \frac{14}{15}) \div (\frac{5}{9} \text{ of } \frac{27}{28})$. Show the meaning of “of” before division.

Q5 | 5 MARKS

Evaluate exactly: $(\frac{11}{12} + \frac{3}{8}) \div (\frac{11}{12} - \frac{3}{8})$. Include a check that the divisor is non-zero.

Q6 | 5 MARKS

Evaluate exactly: $(\frac{9}{10} + \frac{7}{25}) \div (\frac{9}{10} - \frac{7}{25})$. Include a check that the divisor is non-zero.

Q7 | 5 MARKS

Evaluate exactly: $(\frac{7}{16} + \frac{3}{20}) \div (\frac{7}{16} - \frac{3}{20})$. Include a check that the divisor is non-zero.

Q8 | 5 MARKS

Solve $5/(x-1) = 7/3$. State the excluded value and keep the solution exact.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 4 • Prime structure, HCF and LCM

HN4 • Use unique prime factorisation in calculation and proof.

prime powers

minimum → HCF

maximum → LCM

Understand the idea

Every integer greater than 1 has one prime factorisation apart from factor order. Minimum exponents give an HCF; maximum exponents give an LCM.

Key vocabulary

prime; factor; multiple; index; HCF; LCM; divisor

Why the method works

Write each number as a prime product in aligned columns. Select minimum exponents for common divisors and maximum exponents for a common multiple.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 4 • Build the method

Prime structure, HCF and LCM

Reliable routine

Write each number as a prime product in aligned columns. Select minimum exponents for common divisors and maximum exponents for a common multiple.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Find the least square multiple of 18 and 30.

Worked reasoning

$\text{LCM} = 2 \times 3^2 \times 5 = 90$. A square needs even exponents, so multiply by 2×5 . The least square is $2^2 \times 3^2 \times 5^2 = 900$.

Worked example 2

Question. $\text{HCF} = 12$, $\text{LCM} = 420$ and one number is 60. Find the other.

Worked reasoning

For positive integers, product of the numbers equals $\text{HCF} \times \text{LCM}$. The other is $12 \times 420 \div 60 = 84$; prime factors verify both conditions.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 4 • Apply and reason

Prime structure, HCF and LCM

Common misconception

Multiplying the two original numbers is usually not the LCM because common prime factors are counted twice.

Guided problem

6 marks. Find the smallest positive integer that leaves remainder 2 when divided by 5, remainder 3 when divided by 7, and is divisible by 4.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 4 • Independent practice

HN4 • Questions 1-8 • 4-6 marks each

Q1 | 6 MARKS

Find the smallest positive integer that leaves remainder 2 when divided by 5, remainder 3 when divided by 7, and is divisible by 4.

Q2 | 5 MARKS

A number is written as $2^4 \times 3^3 \times 5$. How many of its positive factors are multiples of 60?

Q3 | 5 MARKS

The LCM of $2^a \times 3^2$ and $2^3 \times 3^b$ is $2^5 \times 3^4$, while their HCF is $2^3 \times 3^2$. Find a and b .

Q4 | 6 MARKS

The sum of two positive integers is 84 and their HCF is 12. Find all unordered pairs.

Q5 | 6 MARKS

A positive integer has exactly three positive factors. Prove that it must be the square of a prime.

Q6 | 6 MARKS

Show that the product of four consecutive integers is always divisible by 24.

Q7 | 6 MARKS

Find all primes p for which $p+10$ and $p+14$ are also prime.

Q8 | 6 MARKS

Two positive integers have product 7560 and HCF 6. Find the pair with the smallest possible difference.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 5 • Systematic listing and product rule

HN5 • Count complete outcomes without omission or duplication.



Understand the idea

The product rule multiplies the number of available choices at successive stages. Restrictions may change later branch counts, so a diagram or cases are often essential.

Key vocabulary

outcome; branch; product rule; permutation; combination; restriction

Why the method works

Define the stages, count choices on each branch, split into cases if branch sizes differ, then add mutually exclusive case totals.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 5 • Build the method

Systematic listing and product rule

Reliable routine

Define the stages, count choices on each branch, split into cases if branch sizes differ, then add mutually exclusive case totals.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Form four-digit even numbers from 0–5 without repetition.

Worked reasoning

End 0: $5 \times 4 \times 3 = 60$. End 2 or 4: $2 \times 4 \times 4 \times 3 = 96$. Total 156.

Worked example 2

Question. Ten teams each play every other team once.

Worked reasoning

10×9 counts each pairing twice, once in each order. Divide by 2 to obtain 45 matches.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 5 • Apply and reason

Systematic listing and product rule

Common misconception

A rectangular product is wrong when a restriction means not every branch has the same number of choices.

Guided problem

6 marks. Use digits 1-7 once each to form a seven-digit number. How many arrangements have 1 before 2 before 3, not necessarily adjacent?

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 5 • Independent practice

HN5 • Questions 1-8 • 4-6 marks each

Q1 | 6 MARKS

Use digits 1-7 once each to form a seven-digit number. How many arrangements have 1 before 2 before 3, not necessarily adjacent?

Q2 | 6 MARKS

Find the number of positive integer solutions to $a+b+c=12$ where $a,b,c \geq 1$. Give a systematic counting argument.

Q3 | 4 MARKS

A two-digit number is formed from 1,2,3,4,5 without repetition. List systematically those divisible by 3 and count them.

Q4 | 6 MARKS

A delivery driver visits four different towns and returns home. Travel must begin at home; reversing a tour counts as the same route. Find the number of distinct tours.

Q5 | 6 MARKS

A staircase has 8 steps. A person may climb 1 or 2 steps at a time. Count the possible step sequences systematically.

Q6 | 4 MARKS

A code uses one of 4 letters followed by two digits chosen from 5 available digits, with repetition allowed. Exactly 3 specified digit-pairs are forbidden. Find the number of valid codes.

Q7 | 4 MARKS

A code uses one of 3 letters followed by two digits chosen from 8 available digits, with repetition allowed. Exactly 5 specified digit-pairs are forbidden. Find the number of valid codes.

Q8 | 4 MARKS

A journey has 4 choices for stage 1, 2 for stage 2 and 6 for stage 3. One complete combination is unavailable for each stage-1 choice. Find the number of valid journeys.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 6 • Powers, roots and estimation

HN6 • Recognise power structure and locate roots between known powers.



Understand the idea

A root reverses a power. Exact neighbouring squares or cubes give a rigorous interval; a midpoint comparison decides the nearest integer or requested rounding.

Key vocabulary
square; cube; root; perfect power; interval; midpoint

Why the method works

Bracket the target between consecutive powers. For nearest-integer decisions, compare the target with the power of the numerical midpoint.

Prior-knowledge check
Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint
Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 6 • Build the method

Powers, roots and estimation

Reliable routine

Bracket the target between consecutive powers. For nearest-integer decisions, compare the target with the power of the numerical midpoint.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Estimate $\sqrt{73.5}$ to 1 decimal place.

Worked reasoning

$8.5^2 = 72.25$ and $8.6^2 = 73.96$. Since $8.55^2 = 73.1025 < 73.5$, the root exceeds 8.55 and rounds to 8.6.

Worked example 2

Question. Find the least k for which $3^k > 20000$.

Worked reasoning

$3^9 = 19683 < 20000$ and $3^{10} = 59049 > 20000$, so $k = 10$. Both inequalities prove minimality.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 6 • Apply and reason

Powers, roots and estimation

Common misconception

Being between 8 and 9 does not by itself prove which whole number is nearer.

Guided problem

5 marks. Find all integer x satisfying $2^{(x+1)} \leq 64 < 2^{(x+3)}$.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 6 • Independent practice

HN6 • Questions 1-8 • 4-6 marks each

Q1 | 5 MARKS

Find all integer x satisfying $2^{x+1} \leq 64 < 2^{x+3}$.

Q2 | 4 MARKS

A student concludes that $\sqrt[3]{-64}$ is undefined because $\sqrt{-64}$ is not real. Correct the argument.

Q3 | 5 MARKS

Simplify $(x^6 y^4)^2 \div (x^5 y^{-1})$. Give a single expression with positive indices and state restrictions.

Q4 | 5 MARKS

Simplify $(x^7 y^3)^2 \div (x^6 y^1)$. Give a single expression with positive indices and state restrictions.

Q5 | 5 MARKS

Simplify $(x^3 y^6)^2 \div (x^1 y^5)$. Give a single expression with positive indices and state restrictions.

Q6 | 4 MARKS

Solve $2^{3x+5} = 2^{13}$, then verify by substitution.

Q7 | 4 MARKS

Solve $4^{2x+2} = 4^{12}$, then verify by substitution.

Q8 | 5 MARKS

Prove that $\sqrt{190}$ lies between 13 and 14, then determine the nearer integer without finding the decimal root.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 7 • Integer and fractional indices

HN7 • Interpret negative and fractional powers before applying laws.

root

power

reciprocal

Understand the idea

In $a^{(m/n)}$, the denominator names the root and the numerator names the power. A negative exponent takes a reciprocal; it does not make the value negative.

Key vocabulary

index; base; reciprocal; fractional index; principal root; restriction

Why the method works

Convert to a root when evaluating numbers. When simplifying symbols, apply the index laws and then state restrictions created by roots or denominators.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 7 • Build the method

Integer and fractional indices

Reliable routine

Convert to a root when evaluating numbers. When simplifying symbols, apply the index laws and then state restrictions created by roots or denominators.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Evaluate $81^{-3/4}$.

Worked reasoning

The fourth root of 81 is 3; cubing gives 27; the negative exponent gives the reciprocal $1/27$.

Worked example 2

Question. Solve $27^{(x/3)}=9$.

Worked reasoning

Write $27=3^3$ and $9=3^2$. Then $(3^3)^{(x/3)}=3^x=3^2$, so $x=2$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 7 • Apply and reason

Integer and fractional indices

Common misconception

The denominator of a fractional index is the root; swapping numerator and denominator changes the value.

Guided problem

4 marks. Evaluate $4^{(5/2)}$ and $4^{(-5/2)}$ exactly. Verify by identifying the 2th root first.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 7 • Independent practice

HN7 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Evaluate $4^{(5/2)}$ and $4^{(-5/2)}$ exactly. Verify by identifying the 2th root first.

Q2 | 4 MARKS

Evaluate $25^{(3/2)}$ and $25^{(-3/2)}$ exactly. Verify by identifying the 2th root first.

Q3 | 4 MARKS

Evaluate $64^{(3/2)}$ and $64^{(-3/2)}$ exactly. Verify by identifying the 2th root first.

Q4 | 4 MARKS

Evaluate $1331^{(2/3)}$ and $1331^{(-2/3)}$ exactly. Verify by identifying the 3th root first.

Q5 | 4 MARKS

Evaluate $125^{(4/3)}$ and $125^{(-4/3)}$ exactly. Verify by identifying the 3th root first.

Q6 | 5 MARKS

Simplify $z^{(7/4)} \times z^{(3/4)} \div z^{(1/2)}$ as one power. Show the exact fraction arithmetic and state restrictions.

Q7 | 5 MARKS

Simplify $z^{(9/5)} \times z^{(-4/5)} \div z^{(2/5)}$ as one power. Show the exact fraction arithmetic and state restrictions.

Q8 | 5 MARKS

Simplify $z^{(15/10)} \times z^{(-3/10)} \div z^{(1/5)}$ as one power. Show the exact fraction arithmetic and state restrictions.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 8 • Exact fractions, surds and multiples of pi

HN8 • Manipulate exact values without premature approximation.

simplify

collect

conjugate

Understand the idea

Surds and multiples of pi preserve exact information. Like surds collect like algebraic terms; conjugates remove a two-term surd denominator through a difference of squares.

Key vocabulary

surd; irrational; exact; rationalise; conjugate; like terms

Why the method works

Extract square factors, simplify before collecting, expand every product, and multiply a binomial denominator by its conjugate.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 8 • Build the method

Exact fractions, surds and multiples of pi

Reliable routine

Extract square factors, simplify before collecting, expand every product, and multiply a binomial denominator by its conjugate.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Simplify $\sqrt{72} - \sqrt{18} + \sqrt{8}$.

Worked reasoning

The terms are $6\sqrt{2}$, $3\sqrt{2}$ and $2\sqrt{2}$, so the result is $5\sqrt{2}$.

Worked example 2

Question. Rationalise $6/(3+\sqrt{2})$.

Worked reasoning

Multiply by $(3-\sqrt{2})/(3-\sqrt{2})$. The denominator is $9-2=7$, giving $(18-6\sqrt{2})/7$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 8 • Apply and reason

Exact fractions, surds and multiples of pi

Common misconception

$\sqrt{a+b}$ is not generally $\sqrt{a} + \sqrt{b}$; test a simple counterexample such as $1+4$.

Guided problem

4 marks. Without a calculator, determine which is greater: $5\sqrt{3}$ or $3\sqrt{8}$. Justify a comparison of squares.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 8 • Independent practice

HN8 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Without a calculator, determine which is greater: $5\sqrt{3}$ or $3\sqrt{8}$. Justify a comparison of squares.

Q2 | 4 MARKS

A rectangle has length $4+\sqrt{3}$ cm and width $4-\sqrt{3}$ cm. Find its exact area and explain why it is rational.

Q3 | 4 MARKS

Simplify $4\sqrt{20}-2\sqrt{5}$ completely, showing the square factor used.

Q4 | 4 MARKS

Simplify $5\sqrt{63}-3\sqrt{7}$ completely, showing the square factor used.

Q5 | 4 MARKS

Simplify $4\sqrt{75}-1\sqrt{3}$ completely, showing the square factor used.

Q6 | 5 MARKS

Expand and simplify $(2+\sqrt{3})(5-\sqrt{3})$. Give an exact answer.

Q7 | 5 MARKS

Expand and simplify $(5+\sqrt{6})(4-\sqrt{6})$. Give an exact answer.

Q8 | 5 MARKS

Expand and simplify $(7+\sqrt{2})(5-\sqrt{2})$. Give an exact answer.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 9 • Standard form

HN9 • Represent very large and small values as $A \times 10^n$.

coefficient

$\times 10^n$

normalise

Understand the idea

Standard form separates size into a coefficient $1 \leq A < 10$ and an integer power of ten. Arithmetic acts separately on coefficients and powers before renormalising.

Key vocabulary

coefficient; exponent; standard form; order of magnitude; normalise

Why the method works

Multiply or divide coefficients, add or subtract powers, then shift the decimal so the coefficient returns to the required interval.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 9 • Build the method

Standard form

Reliable routine

Multiply or divide coefficients, add or subtract powers, then shift the decimal so the coefficient returns to the required interval.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Calculate $(3.6 \times 10^7)(5 \times 10^{-3})$.

Worked reasoning

Coefficients give 18 and powers give 10^4 , so $18 \times 10^4 = 1.8 \times 10^5$.

Worked example 2

Question. Calculate $(8.4 \times 10^{-6}) / (2.1 \times 10^3)$.

Worked reasoning

$8.4 \div 2.1 = 4$ and $-6 - 3 = -9$, giving 4×10^{-9} .

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 9 • Apply and reason

Standard form

Common misconception

An answer such as 18×10^4 is equivalent but not in standard form because the coefficient is not below 10.

Guided problem

4 marks. Write $1.9\text{e}+06$ in standard form. Calculate $(1.9 \times 10^6) \times (2 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 9 • Independent practice

HN9 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Write $1.9\text{e}+06$ in standard form. Calculate $(1.9 \times 10^6) \times (2 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q2 | 4 MARKS

Write 1400 in standard form. Calculate $(1.4 \times 10^3) \times (5 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q3 | 4 MARKS

Write $1.7\text{e}+06$ in standard form. Calculate $(1.7 \times 10^6) \times (3 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q4 | 4 MARKS

Write 1200 in standard form. Calculate $(1.2 \times 10^3) \times (6 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q5 | 4 MARKS

Write $1.5\text{e}+06$ in standard form. Calculate $(1.5 \times 10^6) \times (4 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q6 | 4 MARKS

Write 1800 in standard form. Calculate $(1.8 \times 10^3) \times (2 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q7 | 4 MARKS

Write $1.3\text{e}+06$ in standard form. Calculate $(1.3 \times 10^6) \times (5 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

Q8 | 4 MARKS

Write 1600 in standard form. Calculate $(1.6 \times 10^3) \times (3 \times 10^1)$ and give the answer in standard form, explaining how the coefficient is normalised.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 10 • Terminating and recurring decimals

HN10 • Convert recurring decimals to exact fractions by algebra.



Understand the idea

A terminating decimal is a rational number with a finite place-value expansion. For a recurring decimal, multiplying by a suitable power of ten aligns the repeating blocks so subtraction removes them.

Key vocabulary
terminating; recurring; period; rational; aligned subtraction

Why the method works

Let x equal the decimal, create two equations whose recurring tails align, subtract, solve, and reduce the fraction. Check by division.

Prior-knowledge check
Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint
Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 10 • Build the method

Terminating and recurring decimals

Reliable routine

Let x equal the decimal, create two equations whose recurring tails align, subtract, solve, and reduce the fraction. Check by division.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
--	--	--	---

Worked example 1

Question. Convert $0.272727\ldots$ to a fraction.

Worked reasoning

Let $x=0.272727\ldots$ Then $100x=27.272727\ldots$ Subtract: $99x=27$, so $x=27/99=3/11$.

Worked example 2

Question. Convert $0.1\overline{6}$ recurring (only 6 repeats).

Worked reasoning

Let $x=0.1666\ldots$ Then $10x=1.666\ldots$ and $100x=16.666\ldots$ Subtract: $90x=15$, so $x=1/6$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 10 • Apply and reason

Terminating and recurring decimals

Common misconception

The multiplier must match the length and starting position of the repeating block; subtracting misaligned tails does not cancel.

Guided problem

4 marks. Let $x=0.8444\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 10 • Independent practice

HN10 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Let $x=0.8444\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q2 | 4 MARKS

Let $x=0.3333\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q3 | 4 MARKS

Let $x=0.6222\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q4 | 4 MARKS

Let $x=0.1888\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q5 | 4 MARKS

Let $x=0.4777\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q6 | 4 MARKS

Let $x=0.7666\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q7 | 4 MARKS

Let $x=0.2555\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

Q8 | 4 MARKS

Let $x=0.5444\ldots$. Use an algebraic subtraction method to write x as a fraction, then verify the decimal pattern.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 11 • Fractions in ratio problems

HN11 • Connect fractions of wholes to multiplicative ratios.

same whole

ratio parts

link

Understand the idea

A fraction statement identifies a part relative to a specified whole. Ratio parts must refer to the same whole before they can be compared or linked.

Key vocabulary

part; whole; ratio; scale factor; common whole; remainder

Why the method works

Name the whole for each fraction, convert each relationship into ratio parts, link shared quantities using a common multiple, then answer in context.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 11 • Build the method

Fractions in ratio problems

Reliable routine

Name the whole for each fraction, convert each relationship into ratio parts, link shared quantities using a common multiple, then answer in context.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
--	--	--	---

Worked example 1

Question. $\frac{3}{5}$ of a group are adults; $\frac{2}{3}$ of the adults are members. What fraction are adult members?

Worked reasoning

The operators compose: $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$ of the whole group.

Worked example 2

Question. Boys:girls=4:5 and $\frac{3}{8}$ of boys cycle. What fraction of all pupils are boys who cycle?

Worked reasoning

Boys are $\frac{4}{9}$ of all pupils. Cycling boys are $\frac{3}{8} \times \frac{4}{9} = \frac{1}{6}$ of all pupils.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 11 • Apply and reason

Fractions in ratio problems

Common misconception

Adding denominators or multiplying unrelated fractions usually signals that the named wholes have not been tracked.

Guided problem

4 marks. $\frac{4}{6}$ of a fund of GBP 480 is reserved. The remainder is shared in the ratio 2:6. Find all three amounts and express the second share as a fraction of the whole fund.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 11 • Independent practice

HN11 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

$\frac{4}{6}$ of a fund of GBP 480 is reserved. The remainder is shared in the ratio 2:6. Find all three amounts and express the second share as a fraction of the whole fund.

Q2 | 4 MARKS

$\frac{5}{9}$ of a fund of GBP 720 is reserved. The remainder is shared in the ratio 5:3. Find all three amounts and express the second share as a fraction of the whole fund.

Q3 | 4 MARKS

$\frac{1}{5}$ of a fund of GBP 450 is reserved. The remainder is shared in the ratio 3:6. Find all three amounts and express the second share as a fraction of the whole fund.

Q4 | 4 MARKS

$\frac{1}{8}$ of a fund of GBP 720 is reserved. The remainder is shared in the ratio 6:3. Find all three amounts and express the second share as a fraction of the whole fund.

Q5 | 4 MARKS

$\frac{1}{11}$ of a fund of GBP 1100 is reserved. The remainder is shared in the ratio 4:6. Find all three amounts and express the second share as a fraction of the whole fund.

Q6 | 4 MARKS

$\frac{1}{7}$ of a fund of GBP 350 is reserved. The remainder is shared in the ratio 2:3. Find all three amounts and express the second share as a fraction of the whole fund.

Q7 | 4 MARKS

$\frac{2}{10}$ of a fund of GBP 1100 is reserved. The remainder is shared in the ratio 5:6. Find all three amounts and express the second share as a fraction of the whole fund.

Q8 | 4 MARKS

$\frac{1}{6}$ of a fund of GBP 360 is reserved. The remainder is shared in the ratio 3:3. Find all three amounts and express the second share as a fraction of the whole fund.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 12 • Fractions and percentages as operators

HN12 • Model forward, reverse and repeated percentage change.



Understand the idea

A percentage operator is a multiplier. Repeated changes multiply; they do not add. Reverse percentage divides by the original multiplier because the final amount represents a known fraction of the start.

Key vocabulary

multiplier; original; final; reverse percentage; compound change

Why the method works

Translate each change into a multiplier, write the complete equation, then multiply forward or divide backward. Retain full calculator precision until the end.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 12 • Build the method

Fractions and percentages as operators

Reliable routine

Translate each change into a multiplier, write the complete equation, then multiply forward or divide backward. Retain full calculator precision until the end.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. A value rises 18% then falls 12%. Find the net percentage change.

Worked reasoning

Multiplier = $1.18 \times 0.88 = 1.0384$, so the net change is a 3.84% increase.

Worked example 2

Question. After a 15% reduction, a price is £238. Find the original.

Worked reasoning

£238 is 85% of the original. Divide by 0.85 to get £280.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 12 • Apply and reason

Fractions and percentages as operators

Common misconception

Equal percentage increases and decreases do not cancel because the second percentage acts on a different amount.

Guided problem

4 marks. Find 40% of GBP 540. Increase GBP 540 by 40%, then reverse your increase to recover the original amount. Explain why subtracting 40% of the new amount would not reverse it exactly.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 12 • Independent practice

HN12 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Find 40% of GBP 540. Increase GBP 540 by 40%, then reverse your increase to recover the original amount. Explain why subtracting 40% of the new amount would not reverse it exactly.

Q2 | 4 MARKS

Find 15% of GBP 600. Increase GBP 600 by 15%, then reverse your increase to recover the original amount. Explain why subtracting 15% of the new amount would not reverse it exactly.

Q3 | 4 MARKS

Find 30% of GBP 660. Increase GBP 660 by 30%, then reverse your increase to recover the original amount. Explain why subtracting 30% of the new amount would not reverse it exactly.

Q4 | 4 MARKS

Find 5% of GBP 720. Increase GBP 720 by 5%, then reverse your increase to recover the original amount. Explain why subtracting 5% of the new amount would not reverse it exactly.

Q5 | 4 MARKS

Find 20% of GBP 780. Increase GBP 780 by 20%, then reverse your increase to recover the original amount. Explain why subtracting 20% of the new amount would not reverse it exactly.

Q6 | 4 MARKS

Find 35% of GBP 840. Increase GBP 840 by 35%, then reverse your increase to recover the original amount. Explain why subtracting 35% of the new amount would not reverse it exactly.

Q7 | 4 MARKS

Find 10% of GBP 900. Increase GBP 900 by 10%, then reverse your increase to recover the original amount. Explain why subtracting 10% of the new amount would not reverse it exactly.

Q8 | 4 MARKS

Find 25% of GBP 960. Increase GBP 960 by 25%, then reverse your increase to recover the original amount. Explain why subtracting 25% of the new amount would not reverse it exactly.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 13 • Measures and compound measures

HN13 • Use units consistently in multi-stage rate problems.



Understand the idea

Compound units combine measures through multiplication or division. A numerical formula is trustworthy only when every input unit is compatible with the requested output.

Key vocabulary
unit; conversion; speed; density; pressure; compound measure

Why the method works

Write the governing relationship, convert before substitution, carry units through cancellation, and test whether the final unit and size are sensible.

Prior-knowledge check
Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint
Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 13 • Build the method

Measures and compound measures

Reliable routine

Write the governing relationship, convert before substitution, carry units through cancellation, and test whether the final unit and size are sensible.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. A car travels 150 km in 2 h 24 min. Find average speed.

Worked reasoning

2 h 24 min = 2.4 h, so speed = $150/2.4 = 62.5$ km/h.

Worked example 2

Question. Mass is 3.36 kg and volume 420 cm^3 . Find density in g/cm^3 .

Worked reasoning

3.36 kg = 3360 g. Density = $3360/420 = 8$ g/cm^3 .

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 13 • Apply and reason

Measures and compound measures

Common misconception

Using 2.24 hours for 2 hours 24 minutes is a place-value error; minutes are sixtieths of an hour.

Guided problem

4 marks. A component has mass 2700 g and volume 75 cm³. Find its density. A force of 600 N acts over 40 cm²; convert the area to m² and find the pressure in Pa.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 13 • Independent practice

HN13 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A component has mass 2700 g and volume 75 cm³. Find its density. A force of 600 N acts over 40 cm²; convert the area to m² and find the pressure in Pa.

Q2 | 4 MARKS

A component has mass 3000 g and volume 50 cm³. Find its density. A force of 660 N acts over 25 cm²; convert the area to m² and find the pressure in Pa.

Q3 | 4 MARKS

A component has mass 3300 g and volume 65 cm³. Find its density. A force of 720 N acts over 40 cm²; convert the area to m² and find the pressure in Pa.

Q4 | 4 MARKS

A component has mass 3600 g and volume 40 cm³. Find its density. A force of 780 N acts over 25 cm²; convert the area to m² and find the pressure in Pa.

Q5 | 4 MARKS

A component has mass 3900 g and volume 55 cm³. Find its density. A force of 840 N acts over 40 cm²; convert the area to m² and find the pressure in Pa.

Q6 | 4 MARKS

A component has mass 4200 g and volume 70 cm³. Find its density. A force of 900 N acts over 25 cm²; convert the area to m² and find the pressure in Pa.

Q7 | 4 MARKS

A component has mass 4500 g and volume 45 cm³. Find its density. A force of 960 N acts over 40 cm²; convert the area to m² and find the pressure in Pa.

Q8 | 4 MARKS

A component has mass 4800 g and volume 60 cm³. Find its density. A force of 1020 N acts over 25 cm²; convert the area to m² and find the pressure in Pa.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 14 • Estimation and checking

HN14 • Use approximations to control and validate calculation.

rough value

exact value

compare

Understand the idea

An estimate is a purposeful nearby calculation, not a random rounding exercise. It predicts order of magnitude and exposes keying, unit or operation errors.

Key vocabulary

estimate; approximation; significant figure; order of magnitude; reasonableness

Why the method works

Round inputs to friendly values, preserve the operation structure, calculate mentally, then compare the actual answer with the expected scale and sign.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 14 • Build the method

Estimation and checking

Reliable routine

Round inputs to friendly values, preserve the operation structure, calculate mentally, then compare the actual answer with the expected scale and sign.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. Estimate $(19.8 \times 0.503) / 0.098$.

Worked reasoning

Use $20 \times 0.5 \div 0.1 = 100$. An accurate answer should therefore be close to 100.

Worked example 2

Question. A calculator shows $0.00482 \times 6020 = 29016.4$. Assess it.

Worked reasoning

Estimate $0.005 \times 6000 = 30$. The display is about 1000 times too large; the likely error is a misplaced decimal.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 14 • Apply and reason

Estimation and checking

Common misconception

Rounding every value in the same direction does not automatically create a bound; it creates only an estimate.

Guided problem

4 marks. Round 233.106 to 1 decimal place(s) and 3 significant figures. Estimate $(233.1 \times 20)/4$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 14 • Independent practice

HN14 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Round 233.106 to 1 decimal place(s) and 3 significant figures. Estimate $(233.1 \times 20)/4$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q2 | 4 MARKS

Round 255.036 to 1 decimal place(s) and 2 significant figures. Estimate $(255.0 \times 23)/7$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q3 | 4 MARKS

Round 276.966 to 1 decimal place(s) and 3 significant figures. Estimate $(277.0 \times 19)/5$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q4 | 4 MARKS

Round 298.896 to 1 decimal place(s) and 2 significant figures. Estimate $(298.9 \times 22)/8$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q5 | 4 MARKS

Round 320.826 to 1 decimal place(s) and 3 significant figures. Estimate $(320.8 \times 25)/6$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q6 | 4 MARKS

Round 342.756 to 1 decimal place(s) and 2 significant figures. Estimate $(342.8 \times 21)/4$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q7 | 4 MARKS

Round 364.686 to 1 decimal place(s) and 3 significant figures. Estimate $(364.7 \times 24)/7$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

Q8 | 4 MARKS

Round 386.616 to 1 decimal place(s) and 2 significant figures. Estimate $(386.6 \times 20)/5$ using one significant figure values. Write the error interval for your decimal-place rounded value. Before calculating exactly, produce a one-significant-figure estimate and use it to assess whether your final answer is reasonable.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 15 • Rounding and error intervals

HN15 • Translate rounded or truncated values into inequalities.



Understand the idea

A rounded value represents an interval centred on it, with half a rounding unit on each side. The lower endpoint is included; the upper endpoint is excluded.

Key vocabulary

degree of accuracy; rounding unit; lower bound; upper bound; truncation

Why the method works

Identify the rounding step, halve it, subtract and add that half-step, then write $\text{lower} \leq x < \text{upper}$. For truncation, the interval starts at the displayed value.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 15 • Build the method

Rounding and error intervals

Reliable routine

Identify the rounding step, halve it, subtract and add that half-step, then write $\text{lower} \leq x < \text{upper}$. For truncation, the interval starts at the displayed value.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. A length is 7.4 cm to nearest 0.1 cm.

Worked reasoning

The half-step is 0.05 cm, so $7.35 \leq L < 7.45$.

Worked example 2

Question. A positive value is truncated to 3.27 at 2 d.p.

Worked reasoning

The truncation step is 0.01, so $3.27 \leq x < 3.28$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 15 • Apply and reason

Rounding and error intervals

Common misconception

Writing $\leq$ at the upper endpoint includes a value that would round or truncate to the next displayed number.

Guided problem

4 marks. Round 328.136 to 2 decimal place(s) and 2 significant figures. Estimate $(328.1 \times 19)/7$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 15 • Independent practice

HN15 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Round 328.136 to 2 decimal place(s) and 2 significant figures. Estimate $(328.1 \times 19)/7$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q2 | 4 MARKS

Round 350.066 to 2 decimal place(s) and 3 significant figures. Estimate $(350.1 \times 22)/5$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q3 | 4 MARKS

Round 371.996 to 2 decimal place(s) and 2 significant figures. Estimate $(372.0 \times 25)/8$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q4 | 4 MARKS

Round 393.926 to 2 decimal place(s) and 3 significant figures. Estimate $(393.9 \times 21)/6$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q5 | 5 MARKS

Round 415.856 to 2 decimal place(s) and 2 significant figures. Estimate $(415.9 \times 24)/4$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q6 | 5 MARKS

Round 437.786 to 2 decimal place(s) and 3 significant figures. Estimate $(437.8 \times 20)/7$ using one significant figure values. Write the error interval for your decimal-place rounded value. State the relevant rounding or truncation interval using inequalities before completing the task.

Q7 | 4 MARKS

Round 125.5 to the nearest whole number. State the relevant rounding or truncation interval using inequalities before completing the task.

Q8 | 4 MARKS

Round 131.8 to the nearest whole number. State the relevant rounding or truncation interval using inequalities before completing the task.

For every question worth 5 marks or more, show the setup and a final check or justification.

Chapter 16 • Limits of accuracy and bounds

HN16 • Choose extreme inputs to bound a complete calculation.

input intervals

extreme values

result bound

Understand the idea

A bound is guaranteed by the permitted intervals. For positive products choose like extremes; for quotients, the greatest quotient uses the greatest numerator and smallest denominator.

Key vocabulary

upper bound; lower bound; interval; extreme value; guaranteed accuracy

Why the method works

Write every input interval first. Decide how increasing each input affects the result, substitute the correct extremes, then round only after the bound is calculated.

Prior-knowledge check

Can you identify the operation structure, preserve exact values and explain how sign, size or units constrain the answer? Repair any weakness before attempting the Higher extension.

Checkpoint

Explain the diagram in one complete sentence. Then identify the controlling decision, give a simple example, and construct one non-example that breaks a required condition.

Chapter 16 • Build the method

Limits of accuracy and bounds

Reliable routine

Write every input interval first. Decide how increasing each input affects the result, substitute the correct extremes, then round only after the bound is calculated.

1 • Interpret Name the target and conditions.	2 • Represent Write an exact structure.	3 • Execute Show valid transformations.	4 • Verify Test size, sign, unit and accuracy.
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Worked example 1

Question. $L=8.2$ cm and $W=3.6$ cm, each nearest 0.1 cm. Bound the area.

Worked reasoning

$8.15 \leq L < 8.25$ and $3.55 \leq W < 3.65$. Thus $28.9325 \leq A < 30.1125 \text{ cm}^2$.

Worked example 2

Question. $d=120$ m nearest 10 m and $t=9.0$ s nearest 0.1 s. Find upper speed.

Worked reasoning

$d < 125$ and $t \geq 8.95$. The upper bound is $125/8.95 \approx 13.97 \text{ m/s}$.

Self-explanation prompt

Which line earns the main method mark? Which line checks interpretation or accuracy?

Chapter 16 • Apply and reason

Limits of accuracy and bounds

Common misconception

For division, using both upper values does not usually maximise the quotient; the denominator must move in the opposite direction.

Guided problem

5 marks. Round 423.166 to 3 decimal place(s) and 3 significant figures. Estimate $(423.2 \times 25)/5$ using one significant figure values. Write the error interval for your decimal-place rounded value. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Planning scaffold

1. Name the target. 2. Write the governing relationship or exact comparison. 3. Complete the calculation. 4. Check sign, size, unit and requested accuracy.

Reasoning prompt

Write a short argument explaining why your chosen method must work. Then describe one tempting method that would fail and identify its first invalid step.

Higher-tier communication

Use mathematical symbols to carry the argument, but finish with a sentence that answers the question in context. If the data are inconsistent, prove the inconsistency instead of forcing a numerical answer.

Stretch the problem

Change one datum so that a different method or conclusion is needed. Solve your new version and state precisely which part of the original reasoning changed.

Chapter 16 • Independent practice

HN16 • Questions 1-8 • 4-6 marks each

Q1 | 5 MARKS

Round 423.166 to 3 decimal place(s) and 3 significant figures. Estimate $(423.2 \times 25)/5$ using one significant figure values. Write the error interval for your decimal-place rounded value. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q2 | 5 MARKS

Round 445.096 to 3 decimal place(s) and 2 significant figures. Estimate $(445.1 \times 21)/8$ using one significant figure values. Write the error interval for your decimal-place rounded value. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q3 | 4 MARKS

Round 127.6 to the nearest whole number. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q4 | 4 MARKS

Round 133.9 to 1 significant figure and 1 decimal place. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q5 | 4 MARKS

Round 140.2 to 1 significant figure and 1 decimal place. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q6 | 4 MARKS

Round 146.5 to 1 significant figure and 1 decimal place. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q7 | 4 MARKS

Round 152.8 to 1 significant figure and 1 decimal place. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

Q8 | 4 MARKS

Round 247.726 to 3 decimal place(s) and 3 significant figures. Estimate $(247.7 \times 22)/6$ using one significant figure values. Write the error interval for your decimal-place rounded value. Use the appropriate extreme values to justify whether the required result is an upper bound or a lower bound.

For every question worth 5 marks or more, show the setup and a final check or justification.

Worked answers • Chapter 1

HN1 • Ordering real numbers

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

Balance = $-146.80 + 235.50 - 3(41.75) - 9.20 = -146.80 + 235.50 - 125.25 - 9.20 = -£45.75$. The negative sign means the account is £45.75 overdrawn. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 6

Let i incorrect and o omitted. $i+o=8$. Score equation $40+70-4i-o=91$ gives $4i+o=19$. Subtract $i+o=8$ to get $3i=11$, not an integer. Therefore the stated data are impossible; at least one recorded value is wrong. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 5

Numerator = $-64 - (-32) = -32$. Denominator $-3^2 + 5 = -9 + 5 = -4$. Quotient = $(-32)/(-4) = 8$. Here -3^2 means $-(3^2)$; brackets would make the base -3. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

$0.58^2 = 0.3364 > 1/3$, so $\sqrt{1/3} < 0.58$; the lower inequality is false. Also $0.57^2 = 0.3249 < 1/3$. Correct bounds are $0.57 < \sqrt{1/3} < 0.58$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 6

The extremes depend on p. If 2p is maximum and -9 minimum, $2p+9=41$ gives $p=16$, valid. If 14 is maximum and 2p minimum, $14-2p=41$ gives $p=-13.5$, but p itself -13.5 is greater than $2p=-27$, valid. Other extreme assignments either contradict ordering or repeat these. Thus $p=16$ or $p=-13.5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Odd power: $(-1)^{101} = -1$. Even power: $(-1)^{202} = 1$. $(-2)^4 = 16$, while $-2^4 = -16$. Expression = $-1 + 1 - 16 - 16 = -32$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 5

$x^2 < 30$ gives $-\sqrt{30} < x < \sqrt{30}$, so integer x is -5 to 5. $|x-2| > 6$ gives $x < -4$ or $x > 8$. With x negative, intersection is $x = -5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

The integers are -8, -7, -6. Signed square sum = $(-8)^2 - (-7)^2 + (-6)^2 = 64 - 49 + 36 = 51$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 2

HN2 • Four operations and place value

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

Common-denominator addition gives $\frac{8}{7}$. Multiplication gives $\frac{16}{49}$. Division means multiply by the reciprocal: $\frac{4}{7} \times \frac{7}{4} = 1$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Common-denominator addition gives 1. Multiplication gives $\frac{3}{16}$. Division means multiply by the reciprocal: $\frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Common-denominator addition gives $\frac{39}{56}$. Multiplication gives $\frac{1}{14}$. Division means multiply by the reciprocal: $\frac{4}{7} \times 8 = \frac{32}{7}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Common-denominator addition gives $\frac{13}{36}$. Multiplication gives $\frac{1}{36}$. Division means multiply by the reciprocal: $\frac{1}{4} \times 9 = \frac{9}{4}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Common-denominator addition gives $\frac{50}{63}$. Multiplication gives $\frac{8}{63}$. Division means multiply by the reciprocal: $\frac{4}{7} \times \frac{9}{2} = \frac{18}{7}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Common-denominator addition gives $\frac{7}{20}$. Multiplication gives $\frac{1}{40}$. Division means multiply by the reciprocal: $\frac{1}{4} \times 10 = \frac{5}{2}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Common-denominator addition gives $\frac{47}{70}$. Multiplication gives $\frac{2}{35}$. Division means multiply by the reciprocal: $\frac{4}{7} \times 10 = \frac{40}{7}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Common-denominator addition gives $\frac{47}{44}$. Multiplication gives $\frac{9}{44}$. Division means multiply by the reciprocal: $\frac{1}{4} \times \frac{11}{9} = \frac{11}{36}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 3

HN3 • Inverse operations and calculation priority

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 5

Add $7/10$: $(3/5)x = 11/6 + 7/10 = 55/30 + 21/30 = 76/30 = 38/15$. Multiply by reciprocal $5/3$: $x = 38/15 \times 5/3 = 38/9$. Substitution gives $38/15 - 7/10 = 76/30 - 21/30 = 55/30 = 11/6$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 5

0.6 recurring $= 2/3$ and $0.25 = 1/4$, so numerator $= 11/12$. Denominator $= 6/5 - 5/8 = 48/40 - 25/40 = 23/40$. Quotient $= 11/12 \times 40/23 = 110/69$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 5

Distances are 72 km and 108 km. Times: $72/48 = 3/2$ h and $108/72 = 3/2$ h. Total $= 3$ hours. Averaging the two speeds would not generally give the correct time. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

First quantity $= 3/7 \times 14/15 = 2/5$. Second $= 5/9 \times 27/28 = 15/28$. Divide: $2/5 \times 28/15 = 56/75$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 5

The numerator is $31/24$ and the divisor is $13/24$, which is non-zero. Multiply by the reciprocal: $31/24 \div 13/24 = 31/13$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 5

The numerator is $59/50$ and the divisor is $31/50$, which is non-zero. Multiply by the reciprocal: $59/50 \div 31/50 = 59/31$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 5

The numerator is $47/80$ and the divisor is $23/80$, which is non-zero. Multiply by the reciprocal: $47/80 \div 23/80 = 47/23$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 5

The denominator requires $x \neq 1$. Cross-multiply: $15 = 7(x-1)$. Hence $x-1 = 15/7$, so $x = 22/7$, which is not excluded. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 4

HN4 • Prime structure, HCF and LCM

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 6

Multiples of 4 that are 2 mod 5 are 12, 32, 52, 72, 92, ... Check modulo 7: 52 leaves remainder 3. Therefore the smallest is 52. A systematic equivalent is $n=20k+12$; solve $20k+12 \equiv 3 \pmod{7}$, giving $6k \equiv 5$, $k \equiv 2$, hence $n=52$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 5

A factor has form $2^a 3^b 5^c$. To be a multiple of $60=2^2 \times 3 \times 5$, require $a=2,3,4$ (3 choices), $b=1,2,3$ (3 choices), $c=1$ (1 choice). Total $3 \times 3 \times 1 = 9$ factors. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 5

For each prime, the LCM uses the larger exponent and the HCF the smaller. For 2, $\max(a,3)=5$ and $\min(a,3)=3$, so $a=5$. For 3, $\max(2,b)=4$ and $\min(2,b)=2$, so $b=4$. Thus $a=5$, $b=4$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 6

Write the integers as $12a$ and $12b$ with $\text{HCF}(a,b)=1$. Then $a+b=7$. Positive unordered pairs are (1,6), (2,5), (3,4), all coprime. Hence the pairs are (12,72), (24,60), (36,48). Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 6

If n has exactly three factors, they include 1 and n , with one factor d between. Since d divides n , the paired factor n/d must also be a factor; to avoid a fourth factor it must equal d . Thus $n=d^2$. If d were composite it would contribute another factor of n , so d must be prime. Hence $n=p^2$ for prime p . Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 6

Among four consecutive integers there are two even numbers, one of which is divisible by 4; together they supply a factor $2 \times 4 = 8$. Also one of any three consecutive integers is divisible by 3, so the product contains factor 3. Since 8 and 3 are coprime, the product is divisible by 24. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 6

Consider modulo 3. The three numbers $p, p+10, p+14$ have residues $p, p+1, p+2$ modulo 3, so one is divisible by 3. For all three to be prime, that divisible number must equal 3. If $p=3$, the numbers are 3, 13, 17, all prime. If $p+10=3$ or $p+14=3$, p is negative. Thus $p=3$ only. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 6

Write numbers $6a, 6b$ with $\text{HCF}(a,b)=1$. Then $36ab=7560$, so $ab=210=2 \times 3 \times 5 \times 7$. To keep a, b coprime, allocate whole prime powers between them. Factor pairs closest to $\sqrt{210}$ are 14 and 15, and $\text{HCF}(14,15)=1$. Thus numbers are 84 and 90, difference 6. Any closer factor pair would contradict 14×15 being the closest around $\sqrt{210}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 5

HN5 • Systematic listing and product rule

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 6

There are $7!$ total arrangements. For each placement of the other digits, the six relative orders of 1,2,3 are equally represented, and exactly one is 1 before 2 before 3. Count $= 7!/3! = 5040/6 = 840$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 6

Let $x=a-1, y=b-1, z=c-1$, so $x+y+z=9$ with non-negative integers. Place 9 identical units and 2 separators: choose separator positions among 11, giving $11 \times 10 \div 2 = 55$ solutions. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Digit sum must be divisible by 3. Valid ordered pairs are from residue-compatible digits: 12,15,21,24,42,45,51,54. There are 8. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 6

Arrange four towns in $4! = 24$ orders. Each tour and its reverse describe the same undirected cycle from home, so divide by 2: 12 distinct tours. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 6

Let $f(n) = f(n-1) + f(n-2)$, according to whether the last move is 1 or 2. Starting $f(1)=1, f(2)=2$ gives 3,5,8,13,21,34. Therefore $f(8)=34$ sequences. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Unrestricted codes $= 4 \times 5^2 = 100$. Each forbidden digit-pair combines with any of 4 letters, so exclude $4 \times 3 = 12$. Valid codes $= 88$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Unrestricted codes $= 3 \times 8^2 = 192$. Each forbidden digit-pair combines with any of 3 letters, so exclude $3 \times 5 = 15$. Valid codes $= 177$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Unrestricted journeys $= 4 \times 2 \times 6 = 48$. There are 4 unavailable complete combinations, so valid journeys $= 44$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 6

HN6 • Powers, roots and estimation

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 5

$64=2^6$. First inequality gives $x+1 \leq 6$, so $x \leq 5$. Second gives $6 < x+3$, so $x > 3$. Thus $x=4$ or 5 . Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Odd roots preserve sign because $(-4)^3 = -64$, so cube root $(-64) = -4$. Even roots of negative numbers are not real, but odd roots are real. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 5

Power then subtract divisor exponents: $x^{(6 \times 2 - 5)} = x^7$; $y^{(4 \times 2 - 1)} = y^9$. Rewrite negative exponents as reciprocals. The simplified structure is $x^7 \times y^9$, with $x, y \neq 0$ where required. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

Power then subtract divisor exponents: $x^{(7 \times 2 - 6)} = x^8$; $y^{(3 \times 2 - 1)} = y^5$. Rewrite negative exponents as reciprocals. The simplified structure is $x^8 \times y^5$, with $x, y \neq 0$ where required. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 5

Power then subtract divisor exponents: $x^{(3 \times 2 - 1)} = x^5$; $y^{(6 \times 2 - 5)} = y^7$. Rewrite negative exponents as reciprocals. The simplified structure is $x^5 \times y^7$, with $x, y \neq 0$ where required. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Equal powers with common base 2 give $3x+5=13$. Hence $x=8/3$. Substitution makes both exponents 13. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Equal powers with common base 4 give $2x+2=12$. Hence $x=5$. Substitution makes both exponents 12. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 5

$13^2=169 < 190 < 196=14^2$. The midpoint is 13.5; its square is 182.25. Since 190 is above this midpoint square, the root is closer to 14. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 7

HN7 • Integer and fractional indices

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

The 2th root of 4 is 2. Raising to power 5 gives $2^5=32$. The negative-index value is its reciprocal, $1/32$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

The 2th root of 25 is 5. Raising to power 3 gives $5^3=125$. The negative-index value is its reciprocal, $1/125$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

The 2th root of 64 is 8. Raising to power 3 gives $8^3=512$. The negative-index value is its reciprocal, $1/512$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

The 3th root of 1331 is 11. Raising to power 2 gives $11^2=121$. The negative-index value is its reciprocal, $1/121$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

The 3th root of 125 is 5. Raising to power 4 gives $5^4=625$. The negative-index value is its reciprocal, $1/625$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 5

Combined exponent= $7/4+(3/4)-1/2=2$. Therefore the expression is z^2 ; division and any negative powers require $z \neq 0$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 5

Combined exponent= $9/5+(-4/5)-2/5=3/5$. Therefore the expression is $z^{3/5}$; division and any negative powers require $z \neq 0$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 5

Combined exponent= $15/10+(-3/10)-1/5=1$. Therefore the expression is z^1 ; division and any negative powers require $z \neq 0$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 8

HN8 • Exact fractions, surds and multiples of pi

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

Both are positive, so squaring preserves order. $(5\sqrt{3})^2 = 75$ and $(3\sqrt{8})^2 = 72$. Hence $5\sqrt{3} > 3\sqrt{8}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Area = $(4 + \sqrt{3})(4 - \sqrt{3}) = 16 - 3 = 13 \text{ cm}^2$. Conjugate factors cancel the surd cross-terms. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$. Therefore the expression is $(4 \times 2 - 2)\sqrt{5} = 6\sqrt{5}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$. Therefore the expression is $(5 \times 3 - 3)\sqrt{7} = 12\sqrt{7}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$. Therefore the expression is $(4 \times 5 - 1)\sqrt{3} = 19\sqrt{3}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 5

Expansion gives $10 - 2\sqrt{3} + 5\sqrt{3} - 3$. Collecting terms gives $7 + 3\sqrt{3}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 5

Expansion gives $20 - 5\sqrt{6} + 4\sqrt{6} - 6$. Collecting terms gives $14 - \sqrt{6}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 5

Expansion gives $35 - 7\sqrt{2} + 5\sqrt{2} - 2$. Collecting terms gives $33 - 2\sqrt{2}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 9

HN9 • Standard form

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

$1.9\text{e}+06=1.9\times10^6$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 3.8×10^7 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$1400=1.4\times10^3$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 7×10^4 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$1.7\text{e}+06=1.7\times10^6$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 5.1×10^7 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$1200=1.2\times10^3$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 7.2×10^4 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$1.5\text{e}+06=1.5\times10^6$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 6×10^7 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$1800=1.8\times10^3$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 3.6×10^4 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$1.3\text{e}+06=1.3\times10^6$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 6.5×10^7 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$1600=1.6\times10^3$. Multiply coefficients and add powers, then move the decimal if the coefficient is at least 10. The normalised result is 4.8×10^4 , with $1\leq\text{coefficient}<10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 10

HN10 • Terminating and recurring decimals

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

$10x=8.444\dots$ and $100x=84.444\dots$. Subtract: $90x=76$, so $x=38/45$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$10x=3.333\dots$ and $100x=33.333\dots$. Subtract: $90x=30$, so $x=1/3$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$10x=6.222\dots$ and $100x=62.222\dots$. Subtract: $90x=56$, so $x=28/45$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$10x=1.888\dots$ and $100x=18.888\dots$. Subtract: $90x=17$, so $x=17/90$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$10x=4.777\dots$ and $100x=47.777\dots$. Subtract: $90x=43$, so $x=43/90$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$10x=7.666\dots$ and $100x=76.666\dots$. Subtract: $90x=69$, so $x=23/30$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$10x=2.555\dots$ and $100x=25.555\dots$. Subtract: $90x=23$, so $x=23/90$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$10x=5.444\dots$ and $100x=54.444\dots$. Subtract: $90x=49$, so $x=49/90$. Division reproduces the stated recurring decimal. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 11

HN11 • Fractions in ratio problems

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

Reserved amount= $4/6 \times 480 = \text{GBP } 320$. Remainder= $\text{GBP } 160$. One ratio part= $160/(8) = \text{GBP } 20$. Shares are $\text{GBP } 40$ and $\text{GBP } 120$; the second is $1/4$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Reserved amount= $5/9 \times 720 = \text{GBP } 400$. Remainder= $\text{GBP } 320$. One ratio part= $320/(8) = \text{GBP } 40$. Shares are $\text{GBP } 200$ and $\text{GBP } 120$; the second is $1/6$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Reserved amount= $1/5 \times 450 = \text{GBP } 90$. Remainder= $\text{GBP } 360$. One ratio part= $360/(9) = \text{GBP } 40$. Shares are $\text{GBP } 120$ and $\text{GBP } 240$; the second is $8/15$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Reserved amount= $1/8 \times 720 = \text{GBP } 90$. Remainder= $\text{GBP } 630$. One ratio part= $630/(9) = \text{GBP } 70$. Shares are $\text{GBP } 420$ and $\text{GBP } 210$; the second is $7/24$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Reserved amount= $1/11 \times 1100 = \text{GBP } 100$. Remainder= $\text{GBP } 1000$. One ratio part= $1000/(10) = \text{GBP } 100$. Shares are $\text{GBP } 400$ and $\text{GBP } 600$; the second is $6/11$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Reserved amount= $1/7 \times 350 = \text{GBP } 50$. Remainder= $\text{GBP } 300$. One ratio part= $300/(5) = \text{GBP } 60$. Shares are $\text{GBP } 120$ and $\text{GBP } 180$; the second is $18/35$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Reserved amount= $2/10 \times 1100 = \text{GBP } 220$. Remainder= $\text{GBP } 880$. One ratio part= $880/(11) = \text{GBP } 80$. Shares are $\text{GBP } 400$ and $\text{GBP } 480$; the second is $24/55$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Reserved amount= $1/6 \times 360 = \text{GBP } 60$. Remainder= $\text{GBP } 300$. One ratio part= $300/(6) = \text{GBP } 50$. Shares are $\text{GBP } 150$ and $\text{GBP } 150$; the second is $5/12$ of the whole. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 12

HN12 • Fractions and percentages as operators

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

40% of GBP 540=GBP 216.00. Increased amount=GBP 756.00. Reverse by dividing by multiplier 1.40: $\text{GBP } 756.00 / 1.40 = \text{GBP } 540.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

15% of GBP 600=GBP 90.00. Increased amount=GBP 690.00. Reverse by dividing by multiplier 1.15: $\text{GBP } 690.00 / 1.15 = \text{GBP } 600.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

30% of GBP 660=GBP 198.00. Increased amount=GBP 858.00. Reverse by dividing by multiplier 1.30: $\text{GBP } 858.00 / 1.30 = \text{GBP } 660.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

5% of GBP 720=GBP 36.00. Increased amount=GBP 756.00. Reverse by dividing by multiplier 1.05: $\text{GBP } 756.00 / 1.05 = \text{GBP } 720.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

20% of GBP 780=GBP 156.00. Increased amount=GBP 936.00. Reverse by dividing by multiplier 1.20: $\text{GBP } 936.00 / 1.20 = \text{GBP } 780.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

35% of GBP 840=GBP 294.00. Increased amount=GBP 1134.00. Reverse by dividing by multiplier 1.35: $\text{GBP } 1134.00 / 1.35 = \text{GBP } 840.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

10% of GBP 900=GBP 90.00. Increased amount=GBP 990.00. Reverse by dividing by multiplier 1.10: $\text{GBP } 990.00 / 1.10 = \text{GBP } 900.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

25% of GBP 960=GBP 240.00. Increased amount=GBP 1200.00. Reverse by dividing by multiplier 1.25: $\text{GBP } 1200.00 / 1.25 = \text{GBP } 960.00$. The decrease would use a different base amount. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 13

HN13 • Measures and compound measures

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

Density= $2700/75=36$ g/cm³. Area= $40/10000=0.004$ m². Pressure= $600/0.004=150000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Density= $3000/50=60$ g/cm³. Area= $25/10000=0.0025$ m². Pressure= $660/0.0025=264000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Density= $3300/65=660/13$ g/cm³. Area= $40/10000=0.004$ m². Pressure= $720/0.004=180000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Density= $3600/40=90$ g/cm³. Area= $25/10000=0.0025$ m². Pressure= $780/0.0025=312000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Density= $3900/55=780/11$ g/cm³. Area= $40/10000=0.004$ m². Pressure= $840/0.004=210000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Density= $4200/70=60$ g/cm³. Area= $25/10000=0.0025$ m². Pressure= $900/0.0025=360000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Density= $4500/45=100$ g/cm³. Area= $40/10000=0.004$ m². Pressure= $960/0.004=240000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Density= $4800/60=80$ g/cm³. Area= $25/10000=0.0025$ m². Pressure= $1020/0.0025=408000$ Pa. Compound units require each base-unit conversion before division. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 14

HN14 • Estimation and checking

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

To 1 d.p.: 233.1. To 3 s.f.: 233. A one-significant-figure estimate is approximately 1000. Error interval: $233.05 \leq x < 233.15$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

To 1 d.p.: 255.0. To 2 s.f.: 260. A one-significant-figure estimate is approximately 857.143. Error interval: $254.95 \leq x < 255.05$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

To 1 d.p.: 277.0. To 3 s.f.: 277. A one-significant-figure estimate is approximately 1200. Error interval: $276.95 \leq x < 277.05$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

To 1 d.p.: 298.9. To 2 s.f.: 300. A one-significant-figure estimate is approximately 750. Error interval: $298.85 \leq x < 298.95$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

To 1 d.p.: 320.8. To 3 s.f.: 321. A one-significant-figure estimate is approximately 1000. Error interval: $320.75 \leq x < 320.85$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

To 1 d.p.: 342.8. To 2 s.f.: 340. A one-significant-figure estimate is approximately 1500. Error interval: $342.75 \leq x < 342.85$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

To 1 d.p.: 364.7. To 3 s.f.: 365. A one-significant-figure estimate is approximately 1142.86. Error interval: $364.65 \leq x < 364.75$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

To 1 d.p.: 386.6. To 2 s.f.: 390. A one-significant-figure estimate is approximately 1600. Error interval: $386.55 \leq x < 386.65$; the upper bound is excluded. Award the checking credit only when a stated estimate is compared explicitly with the final value. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 15

HN15 • Rounding and error intervals

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 4

To 2 d.p.: 328.14. To 2 s.f.: 330. A one-significant-figure estimate is approximately 857.143. Error interval: $328.135 \leq x < 328.145$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

To 2 d.p.: 350.07. To 3 s.f.: 350. A one-significant-figure estimate is approximately 1600. Error interval: $350.065 \leq x < 350.075$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

To 2 d.p.: 372.0. To 2 s.f.: 370. A one-significant-figure estimate is approximately 1000. Error interval: $371.995 \leq x < 372.005$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

To 2 d.p.: 393.93. To 3 s.f.: 394. A one-significant-figure estimate is approximately 1333.33. Error interval: $393.925 \leq x < 393.935$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 5

To 2 d.p.: 415.86. To 2 s.f.: 420. A one-significant-figure estimate is approximately 2000. Error interval: $415.855 \leq x < 415.865$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 5

To 2 d.p.: 437.79. To 3 s.f.: 438. A one-significant-figure estimate is approximately 1142.86. Error interval: $437.785 \leq x < 437.795$; the upper bound is excluded. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

126. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

132. The interval must include the correct inclusive lower endpoint and exclusive upper endpoint. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 16

HN16 • Limits of accuracy and bounds

These are independent teaching solutions, not an official mark scheme. Equivalent correct reasoning should receive credit.

Q1 • 5

To 3 d.p.: 423.166. To 3 s.f.: 423. A one-significant-figure estimate is approximately 1600. Error interval: $423.1655 \leq x < 423.1665$; the upper bound is excluded. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 5

To 3 d.p.: 445.096. To 2 s.f.: 450. A one-significant-figure estimate is approximately 1000. Error interval: $445.0955 \leq x < 445.0965$; the upper bound is excluded. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

128. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

100 and 133.9. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

100 and 140.2. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

100 and 146.5. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

200 and 152.8. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

To 3 d.p.: 247.726. To 3 s.f.: 248. A one-significant-figure estimate is approximately 666.667. Error interval: $247.7255 \leq x < 247.7265$; the upper bound is excluded. The justification must explain the choice of every extreme value used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

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Unit 1 • Higher Number • HN1-HN16

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