

GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 2: ALGEBRA

Mapped to the Eduqas GCSE Mathematics specification • HA1-HA25

25 chapters • concept teaching • worked methods • reasoning • practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

Independent publication notice

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How to learn from this notebook

Each chapter moves from meaning to method, then unfamiliar application and Higher-tier independent practice.

1 • Understand

Name the object, domain and conditions before manipulating symbols.

2 • Build

Predict each next line of the worked example and justify equivalence.

3 • Connect

Link symbolic, graphical, numerical and contextual representations.

4 • Practise

Every assessed task carries 4–6 marks and requires visible reasoning.

5 • Diagnose

Classify errors as interpretation, transformation, arithmetic, graph reading or conclusion.

Calculator and graph discipline

Write the equation or function entered, choose an appropriate window or table interval, and interpret displayed values. Technology supplies evidence; your written argument supplies meaning.

Unit map • Eduqas Higher Algebra

Independent teaching summaries of HA1–HA25.

Ch.	Ref.	Lesson
1	HA1	Algebraic notation
2	HA2	Substitution
3	HA3	Expressions, equations, formulae and identities
4	HA4	Algebraic manipulation
5	HA5	Formulae and changing the subject
6	HA6	Identities, arguments and proof
7	HA7	Functions, inverses and composites
8	HA8	Coordinates
9	HA9	Straight-line graphs
10	HA10	Gradients and intercepts
11	HA11	Quadratic roots and turning points
12	HA12	Recognising function graphs
13	HA13	Graph transformations
14	HA14	Graphs in real contexts
15	HA15	Gradients and areas under graphs
16	HA16	Circle equations and tangents
17	HA17	Linear equations
18	HA18	Quadratic equations
19	HA19	Simultaneous equations
20	HA20	Numerical methods and iteration
21	HA21	Algebraic modelling
22	HA22	Linear and quadratic inequalities
23	HA23	Generating sequences
24	HA24	Recognising important sequences
25	HA25	Linear and quadratic nth terms

Marks are independent teaching allocations, not official Eduqas marks.

Higher Algebra method bank

Domain first

State excluded or restricted values before cancelling, inverting or taking roots.

Preserve equivalence

Apply an operation to complete sides or complete expressions.

Choose the revealing form

Factor form shows roots; completed-square form shows a turning point; $y=mx+c$ shows gradient.

Connect representations

Move deliberately between equation, graph, table, mapping and context.

Exactness

Retain fractions and surds until an approximation is requested.

Multiple solutions

Recover every branch created by a square, quadratic or intersection.

Verification

Substitute into original equations and inspect graphical or contextual plausibility.

Proof

Begin generally, transform validly and state the conclusion in the requested language.

Chapter 1 • Algebraic notation

HA1 • Read coefficients, indices, products, quotients and brackets unambiguously.

input

representation

transformation

check

Understand the idea

Algebra compresses relationships. Position matters: $3x^2$ means three lots of x^2 , while $(3x)^2$ squares both factors.

Why this works

Translate symbols into operations before calculating; use brackets to expose every substituted negative value.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 1 • Build the method

Algebraic notation

Reliable routine

Translate symbols into operations before calculating; use brackets to expose every substituted negative value.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Interpret $4a^2b/3$ when $a=-3$ and $b=2$.

Worked reasoning

$4(-3)^2(2)/3=24$. The square acts on the bracketed value -3.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 1 • Apply and reason

Algebraic notation

Common misconception

Assuming $3x^2$ and $(3x)^2$ are the same expression.

Guided Higher problem

4 marks. For $x=3$, evaluate $3x^2 - x + 10$. Show substitution with brackets and check the sign of the squared term.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 1 • Independent practice

HA1 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $x=3$, evaluate $3x^2 - x + 10$. Show substitution with brackets and check the sign of the squared term.

Q2 | 4 MARKS

For $x=-3$, evaluate $6x^2 - 4x + 5$. Show substitution with brackets and check the sign of the squared term.

Q3 | 4 MARKS

For $x=0$, evaluate $2x^2 - 2x + 8$. Show substitution with brackets and check the sign of the squared term.

Q4 | 4 MARKS

For $x=3$, evaluate $5x^2 - 5x + 3$. Show substitution with brackets and check the sign of the squared term.

Q5 | 4 MARKS

For $x=-3$, evaluate $8x^2 - 3x + 6$. Show substitution with brackets and check the sign of the squared term.

Q6 | 4 MARKS

For $x=0$, evaluate $4x^2 - x + 9$. Show substitution with brackets and check the sign of the squared term.

Q7 | 4 MARKS

For $x=3$, evaluate $7x^2 - 4x + 4$. Show substitution with brackets and check the sign of the squared term.

Q8 | 4 MARKS

For $x=-3$, evaluate $3x^2 - 2x + 7$. Show substitution with brackets and check the sign of the squared term.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 2 • Substitution

HA2 • Substitute into formulae while preserving signs, powers and units.



Understand the idea

Substitution replaces a symbol with a complete value. A negative or fractional value must behave as one object.

Why this works

Write one line with brackets around every replacement, then follow operation priority and check the unit.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 2 • Build the method

Substitution

Reliable routine

Write one line with brackets around every replacement, then follow operation priority and check the unit.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Find $2x^2 - 3xy$ when $x = -2$ and $y = 5$.

Worked reasoning

$$2(-2)^2 - 3(-2)(5) = 8 + 30 = 38.$$

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 2 • Apply and reason

Substitution

Common misconception

Dropping brackets around a negative input before applying a power.

Guided Higher problem

4 marks. For $x=3$, evaluate $3x^2 - x + 10$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 2 • Independent practice

HA2 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $x=3$, evaluate $3x^2 - x + 10$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q2 | 4 MARKS

For $x=-3$, evaluate $6x^2 - 4x + 5$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q3 | 4 MARKS

For $x=0$, evaluate $2x^2 - 2x + 8$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q4 | 4 MARKS

For $x=3$, evaluate $5x^2 - 5x + 3$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q5 | 4 MARKS

For $x=-3$, evaluate $8x^2 - 3x + 6$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q6 | 4 MARKS

For $x=0$, evaluate $4x^2 - x + 9$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q7 | 4 MARKS

For $x=3$, evaluate $7x^2 - 4x + 4$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Q8 | 4 MARKS

For $x=-3$, evaluate $3x^2 - 2x + 7$. Show substitution with brackets and check the sign of the squared term. Include one exact substitution check.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 3 • Expressions, equations, formulae and identities

HA3 • Distinguish mathematical statements by what they assert.



Understand the idea

An expression has no equality sign; an equation is true for selected values; an identity is true throughout its domain; a formula connects quantities.

Why this works

Classify first, then perform only operations that preserve the intended relationship.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 3 • Build the method

Expressions, equations, formulae and identities

Reliable routine

Classify first, then perform only operations that preserve the intended relationship.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Is $3(x+2)=3x+6$ an equation or identity?

Worked reasoning

Expansion gives the right side for every x , so it is an identity.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 3 • Apply and reason

Expressions, equations, formulae and identities

Common misconception

Treating an identity sign as though there were one unknown value to solve.

Guided Higher problem

4 marks. Prove algebraically that the difference between the squares of $9k+1$ and $9k-1$ is divisible by 36 for every integer k . Classify each principal algebraic statement used as an expression, equation, formula or identity.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 3 • Independent practice

HA3 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Prove algebraically that the difference between the squares of $9k+1$ and $9k-1$ is divisible by 36 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q2 | 4 MARKS

Prove algebraically that the difference between the squares of $4k+1$ and $4k-1$ is divisible by 16 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q3 | 4 MARKS

Prove algebraically that the difference between the squares of $7k+1$ and $7k-1$ is divisible by 28 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q4 | 4 MARKS

Prove algebraically that the difference between the squares of $2k+1$ and $2k-1$ is divisible by 8 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q5 | 4 MARKS

Prove algebraically that the difference between the squares of $5k+1$ and $5k-1$ is divisible by 20 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q6 | 4 MARKS

Prove algebraically that the difference between the squares of $8k+1$ and $8k-1$ is divisible by 32 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q7 | 4 MARKS

Prove algebraically that the difference between the squares of $3k+1$ and $3k-1$ is divisible by 12 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Q8 | 4 MARKS

Prove algebraically that the difference between the squares of $6k+1$ and $6k-1$ is divisible by 24 for every integer k .
Classify each principal algebraic statement used as an expression, equation, formula or identity.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 4 • Algebraic manipulation

HA4 • Simplify, expand, factorise and work with algebraic fractions and surds.

input

representation

transformation

check

Understand the idea

Equivalent manipulation changes appearance without changing value on the allowed domain. Factorising reverses expansion.

Why this works

Name the structure, take common factors first, then use identities or denominator restrictions as required.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 4 • Build the method

Algebraic manipulation

Reliable routine

Name the structure, take common factors first, then use identities or denominator restrictions as required.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Factorise $6x^2 - x - 2$.

Worked reasoning

Product -12 and sum -1 give $6x^2 + 3x - 4x - 2 = (3x - 2)(2x + 1)$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 4 • Apply and reason

Algebraic manipulation

Common misconception

Cancelling terms across addition instead of cancelling common factors.

Guided Higher problem

4 marks. Expand $2(x - 2)(x + 4)$, then factorise your result to verify the original structure.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 4 • Independent practice

HA4 • Questions 1–8 • 4–6 marks each

Q1 | 4 MARKS

Expand $2(x - 2)(x + 4)$, then factorise your result to verify the original structure.

Q2 | 4 MARKS

Expand $x - 1(x + 3)$, then factorise your result to verify the original structure.

Q3 | 4 MARKS

Expand $2(x - 2)(x + 2)$, then factorise your result to verify the original structure.

Q4 | 4 MARKS

Expand $x - 1(x + 8)$, then factorise your result to verify the original structure.

Q5 | 4 MARKS

Expand $2(x - 2)(x + 7)$, then factorise your result to verify the original structure.

Q6 | 4 MARKS

Expand $x - 1(x + 6)$, then factorise your result to verify the original structure.

Q7 | 4 MARKS

Expand $2(x - 2)(x + 5)$, then factorise your result to verify the original structure.

Q8 | 4 MARKS

Expand $x - 1(x + 4)$, then factorise your result to verify the original structure.

Show the governing algebra or graph for every 5–6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 5 • Formulae and changing the subject

HA5 • Rearrange multi-stage formulae involving fractions, roots and repeated subjects.



Understand the idea

Changing the subject isolates one symbol using inverse operations applied to the whole equation.

Why this works

Clear fractions if helpful, collect every subject term, factor the subject, then divide by its complete coefficient.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 5 • Build the method

Formulae and changing the subject

Reliable routine

Clear fractions if helpful, collect every subject term, factor the subject, then divide by its complete coefficient.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Make x the subject of $y = (3x-2)/(x+4)$.

Worked reasoning

$y(x+4) = 3x-2$, so $x(y-3) = -2-4y$ and $x = (-2-4y)/(y-3)$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 5 • Apply and reason

Formulae and changing the subject

Common misconception

Moving a term across an equals sign without performing the same operation on both sides.

Guided Higher problem

4 marks. Make x the subject of $y = (3x-1)/(10x+5)$. State the value of y for which your rearranged denominator is zero.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 5 • Independent practice

HA5 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Make x the subject of $y = (3x-1)/(10x+5)$. State the value of y for which your rearranged denominator is zero.

Q2 | 4 MARKS

Make x the subject of $y = (6x-4)/(5x+4)$. State the value of y for which your rearranged denominator is zero.

Q3 | 4 MARKS

Make x the subject of $y = (2x-2)/(8x+3)$. State the value of y for which your rearranged denominator is zero.

Q4 | 4 MARKS

Make x the subject of $y = (5x-5)/(3x+2)$. State the value of y for which your rearranged denominator is zero.

Q5 | 4 MARKS

Make x the subject of $y = (8x-3)/(6x+5)$. State the value of y for which your rearranged denominator is zero.

Q6 | 4 MARKS

Make x the subject of $y = (4x-1)/(9x+4)$. State the value of y for which your rearranged denominator is zero.

Q7 | 4 MARKS

Make x the subject of $y = (7x-4)/(4x+3)$. State the value of y for which your rearranged denominator is zero.

Q8 | 4 MARKS

Make x the subject of $y = (3x-2)/(7x+2)$. State the value of y for which your rearranged denominator is zero.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 6 • Identities, arguments and proof

HA6 • Build general algebraic proofs rather than numerical demonstrations.



Understand the idea

A proof begins with a general representation and uses valid transformations to reach the claimed form. Examples can suggest but cannot establish a universal result.

Why this works

Define an integer n , represent parity or divisibility algebraically, simplify, and finish by naming the required form.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 6 • Build the method

Identities, arguments and proof

Reliable routine

Define an integer n , represent parity or divisibility algebraically, simplify, and finish by naming the required form.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Prove the sum of consecutive odd integers is divisible by 4.

Worked reasoning

Write them $2n+1$ and $2n+3$. Their sum is $4n+4=4(n+1)$, a multiple of 4.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 6 • Apply and reason

Identities, arguments and proof

Common misconception

Using several examples as proof of a statement about every integer.

Guided Higher problem

4 marks. Prove algebraically that the difference between the squares of $9k+1$ and $9k-1$ is divisible by 36 for every integer k .

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 6 • Independent practice

HA6 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Prove algebraically that the difference between the squares of $9k+1$ and $9k-1$ is divisible by 36 for every integer k .

Q2 | 4 MARKS

Prove algebraically that the difference between the squares of $4k+1$ and $4k-1$ is divisible by 16 for every integer k .

Q3 | 4 MARKS

Prove algebraically that the difference between the squares of $7k+1$ and $7k-1$ is divisible by 28 for every integer k .

Q4 | 4 MARKS

Prove algebraically that the difference between the squares of $2k+1$ and $2k-1$ is divisible by 8 for every integer k .

Q5 | 4 MARKS

Prove algebraically that the difference between the squares of $5k+1$ and $5k-1$ is divisible by 20 for every integer k .

Q6 | 4 MARKS

Prove algebraically that the difference between the squares of $8k+1$ and $8k-1$ is divisible by 32 for every integer k .

Q7 | 4 MARKS

Prove algebraically that the difference between the squares of $3k+1$ and $3k-1$ is divisible by 12 for every integer k .

Q8 | 4 MARKS

Prove algebraically that the difference between the squares of $6k+1$ and $6k-1$ is divisible by 24 for every integer k .

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 7 • Functions, inverses and composites

HA7 • Use mappings, domains, composite functions and inverse functions.



Understand the idea

A function assigns exactly one output to each permitted input. Composition feeds one output into another; an inverse reverses the mapping on an appropriate domain.

Why this works

For $fg(x)$, evaluate g first. To invert, write $y=f(x)$, solve for x , then interchange x and y .

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 7 • Build the method

Functions, inverses and composites

Reliable routine

For $fg(x)$, evaluate g first. To invert, write $y=f(x)$, solve for x , then interchange x and y .

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. If $f(x)=3x-5$ and $g(x)=x^2$, find $fg(2)$ and $f^{-1}(x)$.

Worked reasoning

$g(2)=4$, so $f(4)=7$. From $y=3x-5$, $x=(y+5)/3$; $f^{-1}(x)=(x+5)/3$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 7 • Apply and reason

Functions, inverses and composites

Common misconception

Reading $fg(x)$ as multiplication rather than composition.

Guided Higher problem

4 marks. Let $f(x)=3x+1$ and $g(x)=x^2-10$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 7 • Independent practice

HA7 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Let $f(x)=3x+1$ and $g(x)=x^2-10$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q2 | 4 MARKS

Let $f(x)=6x+4$ and $g(x)=x^2-5$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q3 | 4 MARKS

Let $f(x)=2x+2$ and $g(x)=x^2-8$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q4 | 4 MARKS

Let $f(x)=5x+5$ and $g(x)=x^2-3$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q5 | 4 MARKS

Let $f(x)=8x+3$ and $g(x)=x^2-6$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q6 | 4 MARKS

Let $f(x)=4x+1$ and $g(x)=x^2-9$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q7 | 4 MARKS

Let $f(x)=7x+4$ and $g(x)=x^2-4$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Q8 | 4 MARKS

Let $f(x)=3x+2$ and $g(x)=x^2-7$. Find $f(g(x))$, $g(f(x))$ and $f^{-1}(x)$, stating why the two compositions differ.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 8 • Coordinates

HA8 • Use ordered pairs and coordinate geometry in four quadrants.

input

representation

transformation

check

Understand the idea

Coordinates locate a point by horizontal movement then vertical movement. Differences in coordinates encode displacement, midpoint and gradient.

Why this works

Sketch axes, preserve coordinate order, and use differences consistently from the same starting point.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 8 • Build the method

Coordinates

Reliable routine

Sketch axes, preserve coordinate order, and use differences consistently from the same starting point.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Find the midpoint of A(-5,7) and B(3,-1).

Worked reasoning

Average corresponding coordinates: $((-5+3)/2, (7-1)/2) = (-1, 3)$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 8 • Apply and reason

Coordinates

Common misconception

Averaging an x-coordinate with a y-coordinate.

Guided Higher problem

4 marks. A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 8 • Independent practice

HA8 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q2 | 4 MARKS

A line has gradient 4 and passes through (-4,6). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q3 | 4 MARKS

A line has gradient 2 and passes through (-1,2). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q4 | 4 MARKS

A line has gradient 5 and passes through (2,5). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q5 | 4 MARKS

A line has gradient 3 and passes through (-4,8). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q6 | 4 MARKS

A line has gradient 1 and passes through (-1,4). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q7 | 4 MARKS

A line has gradient 4 and passes through (2,7). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Q8 | 4 MARKS

A line has gradient 2 and passes through (-4,3). Find its equation. Then find the equation of the perpendicular line through the same point. State the coordinates used at every decisive stage.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 9 • Straight-line graphs

HA9 • Derive and use equations of parallel and perpendicular lines.

point

gradient

equation

verification

Understand the idea

In $y=mx+c$, m is constant rate of vertical change per horizontal unit and c is the y -intercept. Parallel lines share m ; perpendicular gradients multiply to -1 .

Why this works

Find gradient from two points, substitute one point to find c , then verify with the other point.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 9 • Build the method

Straight-line graphs

Reliable routine

Find gradient from two points, substitute one point to find c , then verify with the other point.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Find the line through $(2,7)$ perpendicular to $y=\frac{1}{2}x+4$.

Worked reasoning

Perpendicular gradient is -2 . Then $7=-4+c$, so $c=11$ and $y=-2x+11$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 9 • Apply and reason

Straight-line graphs

Common misconception

Using a negative gradient rather than the negative reciprocal for a perpendicular line.

Guided Higher problem

4 marks. A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 9 • Independent practice

HA9 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point.

Q2 | 4 MARKS

A line has gradient 4 and passes through (-4,6). Find its equation. Then find the equation of the perpendicular line through the same point.

Q3 | 4 MARKS

A line has gradient 2 and passes through (-1,2). Find its equation. Then find the equation of the perpendicular line through the same point.

Q4 | 4 MARKS

A line has gradient 5 and passes through (2,5). Find its equation. Then find the equation of the perpendicular line through the same point.

Q5 | 4 MARKS

A line has gradient 3 and passes through (-4,8). Find its equation. Then find the equation of the perpendicular line through the same point.

Q6 | 4 MARKS

A line has gradient 1 and passes through (-1,4). Find its equation. Then find the equation of the perpendicular line through the same point.

Q7 | 4 MARKS

A line has gradient 4 and passes through (2,7). Find its equation. Then find the equation of the perpendicular line through the same point.

Q8 | 4 MARKS

A line has gradient 2 and passes through (-4,3). Find its equation. Then find the equation of the perpendicular line through the same point.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 10 • Gradients and intercepts

HA10 • Interpret line parameters graphically and contextually.



Understand the idea

Gradient is change in output divided by change in input; an intercept is the value when the other coordinate is zero. Units reveal contextual meaning.

Why this works

Use two well-separated points, form $\Delta y/\Delta x$ in consistent order, and state the units and interpretation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 10 • Build the method

Gradients and intercepts

Reliable routine

Use two well-separated points, form $\Delta y/\Delta x$ in consistent order, and state the units and interpretation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. A cost rises from £18 at 2 h to £42 at 6 h. Interpret the gradient.

Worked reasoning

$(42-18)/(6-2)=£6$ per hour: each extra hour adds £6.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 10 • Apply and reason

Gradients and intercepts

Common misconception

Calculating change in x divided by change in y .

Guided Higher problem

4 marks. A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 10 • Independent practice

HA10 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A line has gradient 1 and passes through (2,3). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q2 | 4 MARKS

A line has gradient 4 and passes through (-4,6). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q3 | 4 MARKS

A line has gradient 2 and passes through (-1,2). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q4 | 4 MARKS

A line has gradient 5 and passes through (2,5). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q5 | 4 MARKS

A line has gradient 3 and passes through (-4,8). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q6 | 4 MARKS

A line has gradient 1 and passes through (-1,4). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q7 | 4 MARKS

A line has gradient 4 and passes through (2,7). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Q8 | 4 MARKS

A line has gradient 2 and passes through (-4,3). Find its equation. Then find the equation of the perpendicular line through the same point. Interpret the sign and units of the gradient or intercept where a context is present.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 11 • Quadratic roots and turning points

HA11 • Connect factor form, roots, symmetry and completed-square form.



Understand the idea

A quadratic graph is symmetric. Factor form exposes roots; completed-square form exposes the turning point and axis of symmetry.

Why this works

Choose the form that displays the requested feature, then connect equivalent forms by expansion or factorisation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 11 • Build the method

Quadratic roots and turning points

Reliable routine

Choose the form that displays the requested feature, then connect equivalent forms by expansion or factorisation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Write x^2-6x+5 in completed-square form.

Worked reasoning

$x^2-6x+5=(x-3)^2-9+5=(x-3)^2-4$; turning point (3,-4).

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 11 • Apply and reason

Quadratic roots and turning points

Common misconception

Reading the turning point as $(-3, -4)$ from $(x-3)^2-4$.

Guided Higher problem

4 marks. For $y=x^2 + 4x - 5$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 11 • Independent practice

HA11 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $y = x^2 + 4x - 5$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q2 | 4 MARKS

For $y = x^2 - 2x - 8$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q3 | 4 MARKS

For $y = x^2 + 3x - 10$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q4 | 5 MARKS

For $y = x^2 - 3x - 10$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Verify the result using an independent inverse operation or a second representation.

Q5 | 4 MARKS

For $y = x^2 + 2x - 15$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q6 | 4 MARKS

For $y = x^2 + x - 2$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q7 | 4 MARKS

For $y = x^2 + x - 20$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Q8 | 4 MARKS

For $y = x^2 - 4$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 12 • Recognising function graphs

HA12 • Identify linear, quadratic, cubic, reciprocal, exponential and trigonometric shapes.

input

representation

transformation

check

Understand the idea

Families of functions have invariant structural features such as symmetry, asymptotes, turning behaviour, periodicity and intercept patterns.

Why this works

Inspect domain, symmetry, end behaviour, intercepts and asymptotes before naming a graph.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 12 • Build the method

Recognising function graphs

Reliable routine

Inspect domain, symmetry, end behaviour, intercepts and asymptotes before naming a graph.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Distinguish $y=x^3$ and $y=1/x$ near $x=0$.

Worked reasoning

$y=x^3$ passes through the origin; $y=1/x$ is undefined there and has the axes as asymptotes.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 12 • Apply and reason

Recognising function graphs

Common misconception

Naming a graph from one point rather than its full behaviour.

Guided Higher problem

4 marks. For $y = x^2 + 4x - 5$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 12 • Independent practice

HA12 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $y = x^2 + 4x - 5$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q2 | 4 MARKS

For $y = x^2 - 2x - 8$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q3 | 4 MARKS

For $y = x^2 + 3x - 10$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q4 | 5 MARKS

For $y = x^2 - 3x - 10$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving. Verify the result using an independent inverse operation or a second representation.

Q5 | 4 MARKS

For $y = x^2 + 2x - 15$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q6 | 4 MARKS

For $y = x^2 + x - 2$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q7 | 4 MARKS

For $y = x^2 + x - 20$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Q8 | 4 MARKS

For $y = x^2 - 4$, find the roots, y-intercept, axis of symmetry and discriminant. Use them to describe the graph. Identify the graph family and justify its characteristic shape before solving.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 13 • Graph transformations

HA13 • Translate and reflect graphs using function notation.



Understand the idea

Changes outside f alter outputs; changes inside f alter inputs and act in the opposite horizontal direction.

Why this works

Identify whether the change is inside or outside the function, map key points, and verify one coordinate.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 13 • Build the method

Graph transformations

Reliable routine

Identify whether the change is inside or outside the function, map key points, and verify one coordinate.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Describe $y=f(x+3)-2$.

Worked reasoning

The graph of $y=f(x)$ moves 3 left and 2 down.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 13 • Apply and reason

Graph transformations

Common misconception

Moving right for $f(x+a)$; horizontal changes have the opposite apparent direction.

Guided Higher problem

4 marks. The graph $y=f(x)$ is transformed to $y=3f(x+2)+1$. Describe the transformations in a valid order and map the point (2,5).

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 13 • Independent practice

HA13 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=3f(x+2)+1$. Describe the transformations in a valid order and map the point (2,5).

Q2 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=6f(x-1)-2$. Describe the transformations in a valid order and map the point (2,5).

Q3 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=2f(x+3)+1$. Describe the transformations in a valid order and map the point (2,5).

Q4 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=5f(x-0)-2$. Describe the transformations in a valid order and map the point (2,5).

Q5 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=8f(x-3)+1$. Describe the transformations in a valid order and map the point (2,5).

Q6 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=4f(x+1)-2$. Describe the transformations in a valid order and map the point (2,5).

Q7 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=7f(x-2)+1$. Describe the transformations in a valid order and map the point (2,5).

Q8 | 4 MARKS

The graph $y=f(x)$ is transformed to $y=3f(x+2)-2$. Describe the transformations in a valid order and map the point (2,5).

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 14 • Graphs in real contexts

HA14 • Interpret non-standard graphs and obtain approximate solutions.

input

representation

transformation

check

Understand the idea

A contextual graph encodes quantities and their relationship; its axes and units control every valid interpretation. Intersections solve equality conditions.

Why this works

Read labels and scale, state what a coordinate means, draw any comparison line, and report sensible accuracy.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 14 • Build the method

Graphs in real contexts

Reliable routine

Read labels and scale, state what a coordinate means, draw any comparison line, and report sensible accuracy.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
-----------------------------	-----------------------------	-------------------------	----------------------------

Worked example

Question. How does an intersection of distance-time graphs describe two travellers?

Worked reasoning

At the intersection they have the same distance from the reference point at the same time, so they meet.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 14 • Apply and reason

Graphs in real contexts

Common misconception

Describing a steep distance-time segment as high acceleration rather than high speed.

Guided Higher problem

4 marks. For $y=3x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 14 • Independent practice

HA14 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $y=3x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q2 | 4 MARKS

For $y=6x^2+4$, calculate the gradient of the chord from $x=3$ to $x=7$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q3 | 4 MARKS

For $y=2x^2+2$, calculate the gradient of the chord from $x=1$ to $x=5$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q4 | 4 MARKS

For $y=5x^2+5$, calculate the gradient of the chord from $x=4$ to $x=8$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q5 | 4 MARKS

For $y=8x^2+3$, calculate the gradient of the chord from $x=2$ to $x=6$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q6 | 4 MARKS

For $y=4x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q7 | 4 MARKS

For $y=7x^2+4$, calculate the gradient of the chord from $x=3$ to $x=7$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Q8 | 4 MARKS

For $y=3x^2+2$, calculate the gradient of the chord from $x=1$ to $x=5$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates. Explain why a graphical approximation is appropriate and state its likely accuracy.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 15 • Gradients and areas under graphs

HA15 • Estimate instantaneous rates and accumulated quantities.



Understand the idea

A tangent approximates local gradient; area under a rate graph accumulates $\text{rate} \times \text{time}$. Units provide a powerful check.

Why this works

Draw a tangent at the stated point, use a large triangle, or divide the region into manageable shapes for area.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 15 • Build the method

Gradients and areas under graphs

Reliable routine

Draw a tangent at the stated point, use a large triangle, or divide the region into manageable shapes for area.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
-----------------------------	-----------------------------	-------------------------	----------------------------

Worked example

Question. A velocity-time graph is triangular, base 12 s and height 8 m/s. Find distance.

Worked reasoning

Area = $\frac{1}{2} \times 12 \times 8 = 48$ m.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 15 • Apply and reason

Gradients and areas under graphs

Common misconception

Confusing the gradient of a velocity-time graph with the distance travelled.

Guided Higher problem

4 marks. For $y=3x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 15 • Independent practice

HA15 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

For $y=3x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q2 | 4 MARKS

For $y=6x^2+4$, calculate the gradient of the chord from $x=3$ to $x=7$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q3 | 4 MARKS

For $y=2x^2+2$, calculate the gradient of the chord from $x=1$ to $x=5$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q4 | 4 MARKS

For $y=5x^2+5$, calculate the gradient of the chord from $x=4$ to $x=8$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q5 | 4 MARKS

For $y=8x^2+3$, calculate the gradient of the chord from $x=2$ to $x=6$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q6 | 4 MARKS

For $y=4x^2+1$, calculate the gradient of the chord from $x=0$ to $x=4$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q7 | 4 MARKS

For $y=7x^2+4$, calculate the gradient of the chord from $x=3$ to $x=7$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Q8 | 4 MARKS

For $y=3x^2+2$, calculate the gradient of the chord from $x=1$ to $x=5$. Then find the trapezium estimate of the area under the curve using only the two endpoint ordinates.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 16 • Circle equations and tangents

HA16 • Use $x^2+y^2=r^2$ and perpendicular radius-tangent gradients.

point

gradient

equation

verification

Understand the idea

A point on a circle centred at the origin is r units from the origin, so Pythagoras gives $x^2+y^2=r^2$. A tangent is perpendicular to the radius at contact.

Why this works

Check the point lies on the circle, find the radius gradient, take its negative reciprocal, then find the tangent equation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 16 • Build the method

Circle equations and tangents

Reliable routine

Check the point lies on the circle, find the radius gradient, take its negative reciprocal, then find the tangent equation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
--------------------------------	--------------------------------	----------------------------	-------------------------------

Worked example

Question. Find the tangent to $x^2 + y^2 = 25$ at (3,4).

Worked reasoning

Radius gradient = $4/3$, tangent gradient = $-3/4$. Using (3,4): $y - 4 = -\frac{3}{4}(x - 3)$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 16 • Apply and reason

Circle equations and tangents

Common misconception

Finding a tangent before confirming the stated point lies on the circle.

Guided Higher problem

4 marks. A circle has equation $(x+2)^2 + (y+2)^2 = 36$. State its centre and radius, verify $(4, -2)$ lies on it, and find the tangent there.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 16 • Independent practice

HA16 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A circle has equation $(x+2)^2+(y+2)^2=36$. State its centre and radius, verify (4,-2) lies on it, and find the tangent there.

Q2 | 4 MARKS

A circle has equation $(x-1)^2+(y-1)^2=9$. State its centre and radius, verify (4,1) lies on it, and find the tangent there.

Q3 | 4 MARKS

A circle has equation $(x+3)^2+(y+1)^2=36$. State its centre and radius, verify (3,-1) lies on it, and find the tangent there.

Q4 | 4 MARKS

A circle has equation $(x-0)^2+(y-2)^2=9$. State its centre and radius, verify (3,2) lies on it, and find the tangent there.

Q5 | 4 MARKS

A circle has equation $(x-3)^2+(y-0)^2=36$. State its centre and radius, verify (9,0) lies on it, and find the tangent there.

Q6 | 4 MARKS

A circle has equation $(x+1)^2+(y+2)^2=9$. State its centre and radius, verify (2,-2) lies on it, and find the tangent there.

Q7 | 4 MARKS

A circle has equation $(x-2)^2+(y-1)^2=36$. State its centre and radius, verify (8,1) lies on it, and find the tangent there.

Q8 | 4 MARKS

A circle has equation $(x+2)^2+(y+1)^2=9$. State its centre and radius, verify (1,-1) lies on it, and find the tangent there.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 17 • Linear equations

HA17 • Solve exact linear and fractional equations with validity checks.



Understand the idea

Equivalent equations have the same solution set. Each operation must apply to both sides, and denominators create excluded values.

Why this works

State restrictions, clear denominators, collect unknown terms, solve, then substitute into the original equation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 17 • Build the method

Linear equations

Reliable routine

State restrictions, clear denominators, collect unknown terms, solve, then substitute into the original equation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Solve $5/[2(x-1)] = 3/8$.

Worked reasoning

$x \neq 1$. Cross-multiply: $40 = 6(x-1)$, so $x = 23/3$, which is allowed.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 17 • Apply and reason

Linear equations

Common misconception
Multiplying only one term by a common denominator.

Guided Higher problem

4 marks. Solve $3/(x-4)+1=5$. State the excluded value and verify the solution in the original equation.

Planning scaffold
1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch
Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 17 • Independent practice

HA17 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Solve $3/(x-4)+1=5$. State the excluded value and verify the solution in the original equation.

Q2 | 4 MARKS

Solve $6/(x-1)+4=11$. State the excluded value and verify the solution in the original equation.

Q3 | 4 MARKS

Solve $2/(x-4)+2=5$. State the excluded value and verify the solution in the original equation.

Q4 | 4 MARKS

Solve $5/(x-1)+5=11$. State the excluded value and verify the solution in the original equation.

Q5 | 4 MARKS

Solve $8/(x-4)+3=12$. State the excluded value and verify the solution in the original equation.

Q6 | 4 MARKS

Solve $4/(x-1)+1=6$. State the excluded value and verify the solution in the original equation.

Q7 | 4 MARKS

Solve $7/(x-4)+4=12$. State the excluded value and verify the solution in the original equation.

Q8 | 4 MARKS

Solve $3/(x-1)+2=6$. State the excluded value and verify the solution in the original equation.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 18 • Quadratic equations

HA18 • Solve by factorising, formula, completing the square or graph.



Understand the idea

Quadratic equations may have two, one or no real roots. The discriminant b^2-4ac predicts which case occurs.

Why this works

Set one side to zero, select a method, preserve $\pm$ when square-rooting, and check roots in the original equation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 18 • Build the method

Quadratic equations

Reliable routine

Set one side to zero, select a method, preserve $\pm$ when square-rooting, and check roots in the original equation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
-----------------------------	-----------------------------	-------------------------	----------------------------

Worked example

Question. Solve $2x^2-3x-4=0$.

Worked reasoning

Formula gives $x=[3\pm\sqrt{(9+32)}]/4=(3\pm\sqrt{41})/4$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 18 • Apply and reason

Quadratic equations

Common misconception

Forgetting the negative b or the $\pm$ in the quadratic formula.

Guided Higher problem

4 marks. Solve $1x^2 + 3x - 5 = 0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 18 • Independent practice

HA18 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Solve $1x^2+3x-5=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q2 | 4 MARKS

Solve $1x^2+6x-2=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q3 | 4 MARKS

Solve $1x^2+2x-5=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q4 | 4 MARKS

Solve $1x^2+5x-2=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q5 | 4 MARKS

Solve $1x^2+8x-5=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q6 | 4 MARKS

Solve $1x^2+4x-2=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q7 | 4 MARKS

Solve $1x^2+7x-5=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Q8 | 4 MARKS

Solve $1x^2+3x-2=0$ using the quadratic formula. Give exact roots and three-decimal approximations.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 19 • Simultaneous equations

HA19 • Solve linear-linear and linear-quadratic systems.



Understand the idea

A simultaneous solution satisfies every equation and represents a common intersection. Substitution can turn a line-curve system into a quadratic.

Why this works

Isolate one variable, substitute, solve all resulting roots, recover the paired values and check both equations.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 19 • Build the method

Simultaneous equations

Reliable routine

Isolate one variable, substitute, solve all resulting roots, recover the paired values and check both equations.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
-----------------------------	-----------------------------	-------------------------	----------------------------

Worked example

Question. Solve $y=x+1$ and $y=x^2-3x+1$.

Worked reasoning

Equate: $x+1=x^2-3x+1$, so $x(x-4)=0$. Solutions are (0,1) and (4,5).

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 19 • Apply and reason

Simultaneous equations

Common misconception

Finding both x-roots but pairing them with only one y-value.

Guided Higher problem

4 marks. Solve simultaneously $y=4x-2$ and $x^2+y^2=5$. Give every intersection and verify both equations.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 19 • Independent practice

HA19 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Solve simultaneously $y=4x-2$ and $x^2+y^2=5$. Give every intersection and verify both equations.

Q2 | 4 MARKS

Solve simultaneously $y=3x+1$ and $x^2+y^2=185$. Give every intersection and verify both equations.

Q3 | 4 MARKS

Solve simultaneously $y=2x-3$ and $x^2+y^2=5$. Give every intersection and verify both equations.

Q4 | 4 MARKS

Solve simultaneously $y=x$ and $x^2+y^2=50$. Give every intersection and verify both equations.

Q5 | 4 MARKS

Solve simultaneously $y=4x+3$ and $x^2+y^2=234$. Give every intersection and verify both equations.

Q6 | 4 MARKS

Solve simultaneously $y=3x-1$ and $x^2+y^2=5$. Give every intersection and verify both equations.

Q7 | 4 MARKS

Solve simultaneously $y=2x+2$ and $x^2+y^2=116$. Give every intersection and verify both equations.

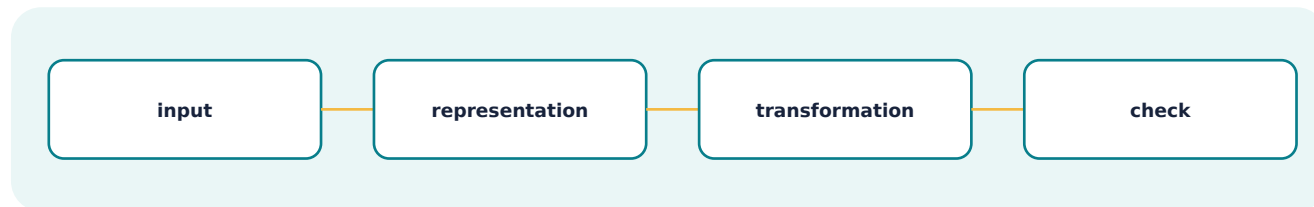
Q8 | 4 MARKS

Solve simultaneously $y=x-2$ and $x^2+y^2=4$. Give every intersection and verify both equations.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 20 • Numerical methods and iteration

HA20 • Approximate roots by search, bisection and iterative rearrangement.



Understand the idea

A sign change brackets a continuous root. Iteration generates a sequence; convergence must be checked rather than assumed.

Why this works

Locate an interval, refine systematically, retain enough digits, and verify the final approximation in the original relation.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 20 • Build the method

Numerical methods and iteration

Reliable routine

Locate an interval, refine systematically, retain enough digits, and verify the final approximation in the original relation.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Given $f(2) < 0$ and $f(3) > 0$, what can be concluded?

Worked reasoning

If f is continuous on $[2, 3]$, at least one root lies between 2 and 3; uniqueness requires further evidence.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 20 • Apply and reason

Numerical methods and iteration

Common misconception

Claiming a sign change proves exactly one root.

Guided Higher problem

4 marks. Use $x_{n+1} = 0.5(x_n + 3/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 20 • Independent practice

HA20 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 3/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q2 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 6/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q3 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 2/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q4 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 5/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q5 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 8/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q6 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 4/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q7 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 7/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies.

Q8 | 4 MARKS

Use $x_{(n+1)} = 0.5(x_n + 3/x_n)$ with $x_1 = 1.0$ to find x_4 . Give six decimal places and explain what equation the fixed point satisfies. After completing the calculation, give one independent check that supports your result.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 21 • Algebraic modelling

HA21 • Translate a context into algebra and evaluate the model.



Understand the idea

A model deliberately simplifies reality. Variables, constraints and units are part of the mathematics, not decoration.

Why this works

Define variables, form relationships from the context, solve, reject impossible values and evaluate assumptions.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 21 • Build the method

Algebraic modelling

Reliable routine

Define variables, form relationships from the context, solve, reject impossible values and evaluate assumptions.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. A rectangle has area 60 and length 7 more than width. Find dimensions.

Worked reasoning

Let width $x > 0$. $x(x+7)=60$ gives $(x+12)(x-5)=0$, so width 5 and length 12.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 21 • Apply and reason

Algebraic modelling

Common misconception

Keeping a negative algebraic root that cannot represent a length.

Guided Higher problem

4 marks. A rectangle has perimeter 36 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 12 cm gives at least 90% of the maximum.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 21 • Independent practice

HA21 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A rectangle has perimeter 36 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 12 cm gives at least 90% of the maximum.

Q2 | 4 MARKS

A rectangle has perimeter 30 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 7 cm gives at least 90% of the maximum.

Q3 | 4 MARKS

A rectangle has perimeter 36 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 10 cm gives at least 90% of the maximum.

Q4 | 4 MARKS

A rectangle has perimeter 30 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 5 cm gives at least 90% of the maximum.

Q5 | 4 MARKS

A rectangle has perimeter 36 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 8 cm gives at least 90% of the maximum.

Q6 | 4 MARKS

A rectangle has perimeter 30 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 11 cm gives at least 90% of the maximum.

Q7 | 4 MARKS

A rectangle has perimeter 36 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 6 cm gives at least 90% of the maximum.

Q8 | 4 MARKS

A rectangle has perimeter 30 cm. Its width is x cm. Form a quadratic for its area, find the maximum area exactly, and decide whether width 9 cm gives at least 90% of the maximum.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 22 • Linear and quadratic inequalities

HA22 • Solve inequalities and represent their solution sets.



Understand the idea

An inequality describes a region of values. Multiplying by a negative reverses order; a quadratic sign chart uses roots to partition the number line.

Why this works

Move to one side, find critical values, test intervals, then use open or closed endpoints correctly.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 22 • Build the method

Linear and quadratic inequalities

Reliable routine

Move to one side, find critical values, test intervals, then use open or closed endpoints correctly.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Solve $x^2 - 5x + 6 < 0$.

Worked reasoning

$(x-2)(x-3) < 0$. The product is negative between the roots, so $2 < x < 3$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 22 • Apply and reason

Linear and quadratic inequalities

Common misconception

Assuming a quadratic inequality is solved by treating it as a linear inequality.

Guided Higher problem

4 marks. Solve $(x+4)(x-2) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 22 • Independent practice

HA22 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Solve $(x+4)(x-2) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q2 | 4 MARKS

Solve $(x+1)(x-4) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q3 | 4 MARKS

Solve $(x+5)(x+1) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q4 | 4 MARKS

Solve $(x+2)(x-1) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q5 | 4 MARKS

Solve $(x-1)(x-7) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q6 | 4 MARKS

Solve $(x+3)(x-2) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q7 | 4 MARKS

Solve $(x-0)(x-4) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Q8 | 4 MARKS

Solve $(x+4)(x+1) \leq 0$. Represent the solution on a number line and list all integer solutions between -10 and 10.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 23 • Generating sequences

HA23 • Use term-to-term, recurrence and position-to-term rules.



Understand the idea

A recurrence explains how to generate the next term; an n th-term formula gives direct access to any position. Initial conditions are essential.

Why this works

Identify differences or ratios, state the rule with its start value, and verify it against several terms.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 23 • Build the method

Generating sequences

Reliable routine

Identify differences or ratios, state the rule with its start value, and verify it against several terms.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Find the n th term of 7,11,15,19,...

Worked reasoning

Difference 4 gives $4n+c$. At $n=1$, $4+c=7$, so $c=3$ and term is $4n+3$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 23 • Apply and reason

Generating sequences

Common misconception
Giving “add 4” as an n th-term formula.

Guided Higher problem

4 marks. A quadratic sequence begins [7, 17, 35, 61, 95]. Find its n th term using differences, then determine whether 385 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 23 • Independent practice

HA23 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A quadratic sequence begins [7, 17, 35, 61, 95]. Find its n th term using differences, then determine whether 385 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q2 | 4 MARKS

A quadratic sequence begins [6, 16, 32, 54, 82]. Find its n th term using differences, then determine whether 312 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q3 | 4 MARKS

A quadratic sequence begins [4, 7, 14, 25, 40]. Find its n th term using differences, then determine whether 175 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q4 | 4 MARKS

A quadratic sequence begins [3, 6, 11, 18, 27]. Find its n th term using differences, then determine whether 102 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q5 | 4 MARKS

A quadratic sequence begins [12, 27, 50, 81, 120]. Find its n th term using differences, then determine whether 435 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q6 | 4 MARKS

A quadratic sequence begins [4, 12, 26, 46, 72]. Find its n th term using differences, then determine whether 292 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q7 | 4 MARKS

A quadratic sequence begins [9, 17, 29, 45, 65]. Find its n th term using differences, then determine whether 225 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Q8 | 4 MARKS

A quadratic sequence begins [1, 2, 5, 10, 17]. Find its n th term using differences, then determine whether 82 is the 10th term. Generate the required terms from the stated rule before drawing a conclusion.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 24 • Recognising important sequences

HA24 • Identify arithmetic, geometric, quadratic and special sequences.



Understand the idea

First differences identify arithmetic patterns; constant second differences identify quadratics; constant ratios identify geometric sequences.

Why this works

Compute structured difference or ratio rows before selecting a model.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 24 • Build the method

Recognising important sequences

Reliable routine

Compute structured difference or ratio rows before selecting a model.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Classify 3,8,15,24,35,...

Worked reasoning

First differences 5,7,9,11 have constant second difference 2, so the sequence is quadratic.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 24 • Apply and reason

Recognising important sequences

Common misconception
Calling every increasing sequence arithmetic.

Guided Higher problem

4 marks. A quadratic sequence begins [7, 17, 35, 61, 95]. Find its n th term using differences, then determine whether 385 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 24 • Independent practice

HA24 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A quadratic sequence begins [7, 17, 35, 61, 95]. Find its n th term using differences, then determine whether 385 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q2 | 4 MARKS

A quadratic sequence begins [6, 16, 32, 54, 82]. Find its n th term using differences, then determine whether 312 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q3 | 4 MARKS

A quadratic sequence begins [4, 7, 14, 25, 40]. Find its n th term using differences, then determine whether 175 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q4 | 4 MARKS

A quadratic sequence begins [3, 6, 11, 18, 27]. Find its n th term using differences, then determine whether 102 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q5 | 4 MARKS

A quadratic sequence begins [12, 27, 50, 81, 120]. Find its n th term using differences, then determine whether 435 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q6 | 4 MARKS

A quadratic sequence begins [4, 12, 26, 46, 72]. Find its n th term using differences, then determine whether 292 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q7 | 4 MARKS

A quadratic sequence begins [9, 17, 29, 45, 65]. Find its n th term using differences, then determine whether 225 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Q8 | 4 MARKS

A quadratic sequence begins [1, 2, 5, 10, 17]. Find its n th term using differences, then determine whether 82 is the 10th term. Name the sequence type and cite the relevant difference or ratio structure.

Show the governing algebra or graph for every 5–6 mark task; unsupported answers cannot demonstrate the complete method.

Chapter 25 • Linear and quadratic nth terms

HA25 • Derive and use nth terms, including membership tests.

structure

critical values

solution set

check

Understand the idea

For an^2+bn+c , the constant second difference is $2a$. Remaining coefficients follow by subtraction or simultaneous conditions.

Why this works

Find a from second differences, remove an^2 , fit the remaining linear sequence, then test the formula.

Vocabulary and notation

Define every new symbol in your own words. State its domain, its inverse operation and one representation that makes its meaning visible.

Checkpoint

Construct one valid example and one non-example. Explain exactly which condition the non-example breaks.

Chapter 25 • Build the method

Linear and quadratic nth terms

Reliable routine

Find a from second differences, remove an^2 , fit the remaining linear sequence, then test the formula.

Interpret target and domain	Represent equation or graph	Transform valid algebra	Verify original conditions
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Worked example

Question. Find the n th term of 4,11,22,37,...

Worked reasoning

Second difference is 4, so $a=2$. Subtract $2n^2$: 2,3,4,5,...= $n+1$. Hence $2n^2+n+1$.

Variation

Change one coefficient, condition or representation. Predict the structural effect before recalculating, then verify it.

Self-explanation

Identify the line containing the decisive algebraic transformation and explain why it preserves the solution or value.

Chapter 25 • Apply and reason

Linear and quadratic nth terms

Common misconception

Using the second difference itself as the coefficient of n^2 instead of halving it.

Guided Higher problem

4 marks. A quadratic sequence begins [7, 17, 35, 61, 95]. Find its nth term using differences, then determine whether 385 is the 10th term.

Planning scaffold

1. Define variables and restrictions. 2. Choose a representation. 3. Show connected valid steps. 4. Recover every solution. 5. Check and interpret.

Reasoning prompt

Explain why the chosen form or representation exposes the information needed. Give one alternative method and compare its efficiency.

Stretch

Modify the task so that it has a different number of valid solutions. Solve the modified version and justify the change.

Chapter 25 • Independent practice

HA25 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A quadratic sequence begins [7, 17, 35, 61, 95]. Find its n th term using differences, then determine whether 385 is the 10th term.

Q2 | 4 MARKS

A quadratic sequence begins [6, 16, 32, 54, 82]. Find its n th term using differences, then determine whether 312 is the 10th term.

Q3 | 4 MARKS

A quadratic sequence begins [4, 7, 14, 25, 40]. Find its n th term using differences, then determine whether 175 is the 10th term.

Q4 | 4 MARKS

A quadratic sequence begins [3, 6, 11, 18, 27]. Find its n th term using differences, then determine whether 102 is the 10th term.

Q5 | 4 MARKS

A quadratic sequence begins [12, 27, 50, 81, 120]. Find its n th term using differences, then determine whether 435 is the 10th term.

Q6 | 4 MARKS

A quadratic sequence begins [4, 12, 26, 46, 72]. Find its n th term using differences, then determine whether 292 is the 10th term.

Q7 | 4 MARKS

A quadratic sequence begins [9, 17, 29, 45, 65]. Find its n th term using differences, then determine whether 225 is the 10th term.

Q8 | 4 MARKS

A quadratic sequence begins [1, 2, 5, 10, 17]. Find its n th term using differences, then determine whether 82 is the 10th term.

Show the governing algebra or graph for every 5-6 mark task; unsupported answers cannot demonstrate the complete method.

Worked answers • Chapter 1

HA1 • Algebraic notation

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Substitute $x=3$: $3(3)^2-1(3)+10=34$. Therefore the exact value is 34. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Substitute $x=-3$: $6(-3)^2-4(-3)+5=71$. Therefore the exact value is 71. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Substitute $x=0$: $2(0)^2-2(0)+8=8$. Therefore the exact value is 8. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Substitute $x=3$: $5(3)^2-5(3)+3=33$. Therefore the exact value is 33. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Substitute $x=-3$: $8(-3)^2-3(-3)+6=87$. Therefore the exact value is 87. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Substitute $x=0$: $4(0)^2-1(0)+9=9$. Therefore the exact value is 9. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Substitute $x=3$: $7(3)^2-4(3)+4=55$. Therefore the exact value is 55. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Substitute $x=-3$: $3(-3)^2-2(-3)+7=40$. Therefore the exact value is 40. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 2

HA2 • Substitution

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Substitute $x=3$: $3(3)^2-1(3)+10=34$. Therefore the exact value is 34. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Substitute $x=-3$: $6(-3)^2-4(-3)+5=71$. Therefore the exact value is 71. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Substitute $x=0$: $2(0)^2-2(0)+8=8$. Therefore the exact value is 8. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Substitute $x=3$: $5(3)^2-5(3)+3=33$. Therefore the exact value is 33. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Substitute $x=-3$: $8(-3)^2-3(-3)+6=87$. Therefore the exact value is 87. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Substitute $x=0$: $4(0)^2-1(0)+9=9$. Therefore the exact value is 9. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Substitute $x=3$: $7(3)^2-4(3)+4=55$. Therefore the exact value is 55. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Substitute $x=-3$: $3(-3)^2-2(-3)+7=40$. Therefore the exact value is 40. Confirm the substitution by evaluating in a second valid order. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 3

HA3 • Expressions, equations, formulae and identities

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Use difference of squares: $[(9k+1)^2-(9k-1)^2]=[(2)][2(9)k]=36k$. Since k is an integer, the result is divisible by 36. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Use difference of squares: $[(4k+1)^2-(4k-1)^2]=[(2)][2(4)k]=16k$. Since k is an integer, the result is divisible by 16. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Use difference of squares: $[(7k+1)^2-(7k-1)^2]=[(2)][2(7)k]=28k$. Since k is an integer, the result is divisible by 28. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Use difference of squares: $[(2k+1)^2-(2k-1)^2]=[(2)][2(2)k]=8k$. Since k is an integer, the result is divisible by 8. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Use difference of squares: $[(5k+1)^2-(5k-1)^2]=[(2)][2(5)k]=20k$. Since k is an integer, the result is divisible by 20. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Use difference of squares: $[(8k+1)^2-(8k-1)^2]=[(2)][2(8)k]=32k$. Since k is an integer, the result is divisible by 32. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Use difference of squares: $[(3k+1)^2-(3k-1)^2]=[(2)][2(3)k]=12k$. Since k is an integer, the result is divisible by 12. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Use difference of squares: $[(6k+1)^2-(6k-1)^2]=[(2)][2(6)k]=24k$. Since k is an integer, the result is divisible by 24. The classification must use the defining feature of each algebraic object. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 4

HA4 • Algebraic manipulation

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Expansion gives $2x^2 + 4x - 16$. To reverse, split or identify the middle term and factorise: $2x^2 + 4x - 16 = (2x - 2)(x + 4)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Expansion gives $x^2 + 2x - 3$. To reverse, split or identify the middle term and factorise: $x^2 + 2x - 3 = (x - 1)(x + 3)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Expansion gives $2x^2 - 8$. To reverse, split or identify the middle term and factorise: $2x^2 - 8 = (2x - 2)(x + 2)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Expansion gives $x^2 + 7x - 8$. To reverse, split or identify the middle term and factorise: $x^2 + 7x - 8 = (x - 1)(x + 8)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Expansion gives $2x^2 + 10x - 28$. To reverse, split or identify the middle term and factorise: $2x^2 + 10x - 28 = (2x - 2)(x + 7)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Expansion gives $x^2 + 5x - 6$. To reverse, split or identify the middle term and factorise: $x^2 + 5x - 6 = (x - 1)(x + 6)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Expansion gives $2x^2 + 6x - 20$. To reverse, split or identify the middle term and factorise: $2x^2 + 6x - 20 = (2x - 2)(x + 5)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Expansion gives $x^2 + 3x - 4$. To reverse, split or identify the middle term and factorise: $x^2 + 3x - 4 = (x - 1)(x + 4)$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 5

HA5 • Formulae and changing the subject

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$y(10x+5)=3x-1$; $10xy+5y=3x-1$; $x(10y-3)=-1-5y$. Hence $x=(1+5y)/(3-10y)$, requiring $y \neq 3/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$y(5x+4)=6x-4$; $5xy+4y=6x-4$; $x(5y-6)=-4-4y$. Hence $x=(4+4y)/(6-5y)$, requiring $y \neq 6/5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$y(8x+3)=2x-2$; $8xy+3y=2x-2$; $x(8y-2)=-2-3y$. Hence $x=(2+3y)/(2-8y)$, requiring $y \neq 1/4$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$y(3x+2)=5x-5$; $3xy+2y=5x-5$; $x(3y-5)=-5-2y$. Hence $x=(5+2y)/(5-3y)$, requiring $y \neq 5/3$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$y(6x+5)=8x-3$; $6xy+5y=8x-3$; $x(6y-8)=-3-5y$. Hence $x=(3+5y)/(8-6y)$, requiring $y \neq 4/3$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$y(9x+4)=4x-1$; $9xy+4y=4x-1$; $x(9y-4)=-1-4y$. Hence $x=(1+4y)/(4-9y)$, requiring $y \neq 4/9$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$y(4x+3)=7x-4$; $4xy+3y=7x-4$; $x(4y-7)=-4-3y$. Hence $x=(4+3y)/(7-4y)$, requiring $y \neq 7/4$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$y(7x+2)=3x-2$; $7xy+2y=3x-2$; $x(7y-3)=-2-2y$. Hence $x=(2+2y)/(3-7y)$, requiring $y \neq 3/7$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 6

HA6 • Identities, arguments and proof

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Use difference of squares: $[(9k+1)^2-(9k-1)^2]=[(2)][2(9)k]=36k$. Since k is an integer, the result is divisible by 36. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Use difference of squares: $[(4k+1)^2-(4k-1)^2]=[(2)][2(4)k]=16k$. Since k is an integer, the result is divisible by 16. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Use difference of squares: $[(7k+1)^2-(7k-1)^2]=[(2)][2(7)k]=28k$. Since k is an integer, the result is divisible by 28. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Use difference of squares: $[(2k+1)^2-(2k-1)^2]=[(2)][2(2)k]=8k$. Since k is an integer, the result is divisible by 8. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Use difference of squares: $[(5k+1)^2-(5k-1)^2]=[(2)][2(5)k]=20k$. Since k is an integer, the result is divisible by 20. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Use difference of squares: $[(8k+1)^2-(8k-1)^2]=[(2)][2(8)k]=32k$. Since k is an integer, the result is divisible by 32. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Use difference of squares: $[(3k+1)^2-(3k-1)^2]=[(2)][2(3)k]=12k$. Since k is an integer, the result is divisible by 12. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Use difference of squares: $[(6k+1)^2-(6k-1)^2]=[(2)][2(6)k]=24k$. Since k is an integer, the result is divisible by 24. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 7

HA7 • Functions, inverses and composites

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$f(g(x)) = 3(x^2 - 10) + 1 = 3x^2 - 29$. $g(f(x)) = (3x + 1)^2 - 10 = 9x^2 + 6x - 9$. From $y = 3x + 1$, $f^{-1}(-1)(x) = (x - 1)/3$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$f(g(x)) = 6(x^2 - 5) + 4 = 6x^2 - 26$. $g(f(x)) = (6x + 4)^2 - 5 = 36x^2 + 48x + 11$. From $y = 6x + 4$, $f^{-1}(-1)(x) = (x - 4)/6$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$f(g(x)) = 2(x^2 - 8) + 2 = 2x^2 - 14$. $g(f(x)) = (2x + 2)^2 - 8 = 4x^2 + 8x - 4$. From $y = 2x + 2$, $f^{-1}(-1)(x) = (x - 2)/2$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$f(g(x)) = 5(x^2 - 3) + 5 = 5x^2 - 10$. $g(f(x)) = (5x + 5)^2 - 3 = 25x^2 + 50x + 22$. From $y = 5x + 5$, $f^{-1}(-1)(x) = (x - 5)/5$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$f(g(x)) = 8(x^2 - 6) + 3 = 8x^2 - 45$. $g(f(x)) = (8x + 3)^2 - 6 = 64x^2 + 48x + 3$. From $y = 8x + 3$, $f^{-1}(-1)(x) = (x - 3)/8$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$f(g(x)) = 4(x^2 - 9) + 1 = 4x^2 - 35$. $g(f(x)) = (4x + 1)^2 - 9 = 16x^2 + 8x - 8$. From $y = 4x + 1$, $f^{-1}(-1)(x) = (x - 1)/4$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$f(g(x)) = 7(x^2 - 4) + 4 = 7x^2 - 24$. $g(f(x)) = (7x + 4)^2 - 4 = 49x^2 + 56x + 12$. From $y = 7x + 4$, $f^{-1}(-1)(x) = (x - 4)/7$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$f(g(x)) = 3(x^2 - 7) + 2 = 3x^2 - 19$. $g(f(x)) = (3x + 2)^2 - 7 = 9x^2 + 12x - 3$. From $y = 3x + 2$, $f^{-1}(-1)(x) = (x - 2)/3$. Composition order changes the operations. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 8

HA8 • Coordinates

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = x + 1$. A perpendicular gradient is $-1/1$; its intercept is 5, so $y = -x + 5$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 4x + 22$. A perpendicular gradient is $-1/4$; its intercept is 5, so $y = -1/4x + 5$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 2x + 4$. A perpendicular gradient is $-1/2$; its intercept is $3/2$, so $y = -1/2x + 3/2$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 5x - 5$. A perpendicular gradient is $-1/5$; its intercept is $27/5$, so $y = -1/5x + 27/5$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 3x + 20$. A perpendicular gradient is $-1/3$; its intercept is $20/3$, so $y = -1/3x + 20/3$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = x + 5$. A perpendicular gradient is $-1/1$; its intercept is 3, so $y = -x + 3$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 4x - 1$. A perpendicular gradient is $-1/4$; its intercept is $15/2$, so $y = -1/4x + 15/2$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Using $y - y_1 = m(x - x_1)$, the first line is $y = 2x + 11$. A perpendicular gradient is $-1/2$; its intercept is 1, so $y = -1/2x + 1$. Coordinate evidence must be visible in the reasoning. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 9

HA9 • Straight-line graphs

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=x+1$. A perpendicular gradient is $-1/1$; its intercept is 5, so $y=-x+5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=4x+22$. A perpendicular gradient is $-1/4$; its intercept is 5, so $y=-1/4x+5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=2x+4$. A perpendicular gradient is $-1/2$; its intercept is $3/2$, so $y=-1/2x+3/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=5x-5$. A perpendicular gradient is $-1/5$; its intercept is $27/5$, so $y=-1/5x+27/5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=3x+20$. A perpendicular gradient is $-1/3$; its intercept is $20/3$, so $y=-1/3x+20/3$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=x+5$. A perpendicular gradient is $-1/1$; its intercept is 3, so $y=-x+3$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=4x-1$. A perpendicular gradient is $-1/4$; its intercept is $15/2$, so $y=-1/4x+15/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=2x+11$. A perpendicular gradient is $-1/2$; its intercept is 1, so $y=-1/2x+1$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 10

HA10 • Gradients and intercepts

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=x+1$. A perpendicular gradient is $-1/1$; its intercept is 5, so $y=-x+5$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=4x+22$. A perpendicular gradient is $-1/4$; its intercept is 5, so $y=-1/4x+5$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=2x+4$. A perpendicular gradient is $-1/2$; its intercept is $3/2$, so $y=-1/2x+3/2$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=5x-5$. A perpendicular gradient is $-1/5$; its intercept is $27/5$, so $y=-1/5x+27/5$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=3x+20$. A perpendicular gradient is $-1/3$; its intercept is $20/3$, so $y=-1/3x+20/3$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=x+5$. A perpendicular gradient is $-1/1$; its intercept is 3, so $y=-x+3$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=4x-1$. A perpendicular gradient is $-1/4$; its intercept is $15/2$, so $y=-1/4x+15/2$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Using $y-y_1=m(x-x_1)$, the first line is $y=2x+11$. A perpendicular gradient is $-1/2$; its intercept is 1, so $y=-1/2x+1$. The interpretation should link the algebraic parameter to the graph or context. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 11

HA11 • Quadratic roots and turning points

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Factor form gives roots $x=1$, $x=-5$. The y-intercept is -5. The axis is the midpoint, $x=-2$. The discriminant is $36>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Factor form gives roots $x=4$, $x=-2$. The y-intercept is -8. The axis is the midpoint, $x=1$. The discriminant is $36>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Factor form gives roots $x=2$, $x=-5$. The y-intercept is -10. The axis is the midpoint, $x=-3/2$. The discriminant is $49>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

Factor form gives roots $x=5$, $x=-2$. The y-intercept is -10. The axis is the midpoint, $x=3/2$. The discriminant is $49>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Accept a correct independent verification using an inverse operation, substitution, alternative representation or equivalent check. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Factor form gives roots $x=3$, $x=-5$. The y-intercept is -15. The axis is the midpoint, $x=-1$. The discriminant is $64>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Factor form gives roots $x=1$, $x=-2$. The y-intercept is -2. The axis is the midpoint, $x=-1/2$. The discriminant is $9>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Factor form gives roots $x=4$, $x=-5$. The y-intercept is -20. The axis is the midpoint, $x=-1/2$. The discriminant is $81>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Factor form gives roots $x=2$, $x=-2$. The y-intercept is -4. The axis is the midpoint, $x=0$. The discriminant is $16>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 12

HA12 • Recognising function graphs

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Factor form gives roots $x=1$, $x=-5$. The y-intercept is -5. The axis is the midpoint, $x=-2$. The discriminant is $36>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Factor form gives roots $x=4$, $x=-2$. The y-intercept is -8. The axis is the midpoint, $x=1$. The discriminant is $36>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Factor form gives roots $x=2$, $x=-5$. The y-intercept is -10. The axis is the midpoint, $x=-3/2$. The discriminant is $49>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

Factor form gives roots $x=5$, $x=-2$. The y-intercept is -10. The axis is the midpoint, $x=3/2$. The discriminant is $49>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Accept a correct independent verification using an inverse operation, substitution, alternative representation or equivalent check. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Factor form gives roots $x=3$, $x=-5$. The y-intercept is -15. The axis is the midpoint, $x=-1$. The discriminant is $64>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Factor form gives roots $x=1$, $x=-2$. The y-intercept is -2. The axis is the midpoint, $x=-1/2$. The discriminant is $9>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Factor form gives roots $x=4$, $x=-5$. The y-intercept is -20. The axis is the midpoint, $x=-1/2$. The discriminant is $81>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Factor form gives roots $x=2$, $x=-2$. The y-intercept is -4. The axis is the midpoint, $x=0$. The discriminant is $16>0$, confirming two distinct real roots; the positive x^2 coefficient means the graph opens upward. Credit a correct family identification supported by its intercept, asymptote, symmetry or periodicity. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 13

HA13 • Graph transformations

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$f(x+2)$ translates the graph 2 units left; multiplying by 3 gives a vertical stretch factor 3; the final term translates it 1 units up. Thus (2,5) maps to (0,16). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$f(x-1)$ translates the graph 1 units right; multiplying by 6 gives a vertical stretch factor 6; the final term translates it 2 units down. Thus (2,5) maps to (3,28). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$f(x+3)$ translates the graph 3 units left; multiplying by 2 gives a vertical stretch factor 2; the final term translates it 1 units up. Thus (2,5) maps to (-1,11). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$f(x-0)$ translates the graph 0 units horizontally; multiplying by 5 gives a vertical stretch factor 5; the final term translates it 2 units down. Thus (2,5) maps to (2,23). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$f(x-3)$ translates the graph 3 units right; multiplying by 8 gives a vertical stretch factor 8; the final term translates it 1 units up. Thus (2,5) maps to (5,41). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$f(x+1)$ translates the graph 1 units left; multiplying by 4 gives a vertical stretch factor 4; the final term translates it 2 units down. Thus (2,5) maps to (1,18). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$f(x-2)$ translates the graph 2 units right; multiplying by 7 gives a vertical stretch factor 7; the final term translates it 1 units up. Thus (2,5) maps to (4,36). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$f(x+2)$ translates the graph 2 units left; multiplying by 3 gives a vertical stretch factor 3; the final term translates it 2 units down. Thus (2,5) maps to (0,13). Horizontal movement changes x-coordinates; vertical operations change y-coordinates. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 14

HA14 • Graphs in real contexts

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Endpoint ordinates are 1 and 49. Chord gradient $= (49-1)/(4-0) = 12$. One trapezium gives area $= 1/2(1+49)(4) = 100$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Endpoint ordinates are 58 and 298. Chord gradient $= (298-58)/(7-3) = 60$. One trapezium gives area $= 1/2(58+298)(4) = 712$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Endpoint ordinates are 4 and 52. Chord gradient $= (52-4)/(5-1) = 12$. One trapezium gives area $= 1/2(4+52)(4) = 112$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Endpoint ordinates are 85 and 325. Chord gradient $= (325-85)/(8-4) = 60$. One trapezium gives area $= 1/2(85+325)(4) = 820$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Endpoint ordinates are 35 and 291. Chord gradient $= (291-35)/(6-2) = 64$. One trapezium gives area $= 1/2(35+291)(4) = 652$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Endpoint ordinates are 1 and 65. Chord gradient $= (65-1)/(4-0) = 16$. One trapezium gives area $= 1/2(1+65)(4) = 132$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Endpoint ordinates are 67 and 347. Chord gradient $= (347-67)/(7-3) = 70$. One trapezium gives area $= 1/2(67+347)(4) = 828$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Endpoint ordinates are 5 and 77. Chord gradient $= (77-5)/(5-1) = 18$. One trapezium gives area $= 1/2(5+77)(4) = 164$ square units. The accuracy statement must be consistent with the graph scale or numerical information. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 15

HA15 • Gradients and areas under graphs

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Endpoint ordinates are 1 and 49. Chord gradient= $(49-1)/(4-0)=12$. One trapezium gives area= $1/2(1+49)(4)=100$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Endpoint ordinates are 58 and 298. Chord gradient= $(298-58)/(7-3)=60$. One trapezium gives area= $1/2(58+298)(4)=712$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Endpoint ordinates are 4 and 52. Chord gradient= $(52-4)/(5-1)=12$. One trapezium gives area= $1/2(4+52)(4)=112$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Endpoint ordinates are 85 and 325. Chord gradient= $(325-85)/(8-4)=60$. One trapezium gives area= $1/2(85+325)(4)=820$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Endpoint ordinates are 35 and 291. Chord gradient= $(291-35)/(6-2)=64$. One trapezium gives area= $1/2(35+291)(4)=652$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Endpoint ordinates are 1 and 65. Chord gradient= $(65-1)/(4-0)=16$. One trapezium gives area= $1/2(1+65)(4)=132$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Endpoint ordinates are 67 and 347. Chord gradient= $(347-67)/(7-3)=70$. One trapezium gives area= $1/2(67+347)(4)=828$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Endpoint ordinates are 5 and 77. Chord gradient= $(77-5)/(5-1)=18$. One trapezium gives area= $1/2(5+77)(4)=164$ square units. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 16

HA16 • Circle equations and tangents

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

The centre is $(-2, -2)$ and radius 6. Substitution gives $6^2 + 0^2 = 36$. The radius to $(4, -2)$ is horizontal, so the tangent is vertical: $x = 4$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

The centre is $(1, 1)$ and radius 3. Substitution gives $3^2 + 0^2 = 9$. The radius to $(4, 1)$ is horizontal, so the tangent is vertical: $x = 4$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

The centre is $(-3, -1)$ and radius 6. Substitution gives $6^2 + 0^2 = 36$. The radius to $(3, -1)$ is horizontal, so the tangent is vertical: $x = 3$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

The centre is $(0, 2)$ and radius 3. Substitution gives $3^2 + 0^2 = 9$. The radius to $(3, 2)$ is horizontal, so the tangent is vertical: $x = 3$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

The centre is $(3, 0)$ and radius 6. Substitution gives $6^2 + 0^2 = 36$. The radius to $(9, 0)$ is horizontal, so the tangent is vertical: $x = 9$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

The centre is $(-1, -2)$ and radius 3. Substitution gives $3^2 + 0^2 = 9$. The radius to $(2, -2)$ is horizontal, so the tangent is vertical: $x = 2$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

The centre is $(2, 1)$ and radius 6. Substitution gives $6^2 + 0^2 = 36$. The radius to $(8, 1)$ is horizontal, so the tangent is vertical: $x = 8$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

The centre is $(-2, -1)$ and radius 3. Substitution gives $3^2 + 0^2 = 9$. The radius to $(1, -1)$ is horizontal, so the tangent is vertical: $x = 1$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 17

HA17 • Linear equations

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Restriction: $x \neq 4$. Then $3/(x-4)=4$, so $x-4=3/4$ and $x=19/4$. Substitution returns 5; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Restriction: $x \neq 1$. Then $6/(x-1)=7$, so $x-1=6/7$ and $x=13/7$. Substitution returns 11; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Restriction: $x \neq 4$. Then $2/(x-4)=3$, so $x-4=2/3$ and $x=14/3$. Substitution returns 5; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Restriction: $x \neq 1$. Then $5/(x-1)=6$, so $x-1=5/6$ and $x=11/6$. Substitution returns 11; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Restriction: $x \neq 4$. Then $8/(x-4)=9$, so $x-4=8/9$ and $x=44/9$. Substitution returns 12; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Restriction: $x \neq 1$. Then $4/(x-1)=5$, so $x-1=4/5$ and $x=9/5$. Substitution returns 6; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Restriction: $x \neq 4$. Then $7/(x-4)=8$, so $x-4=7/8$ and $x=39/8$. Substitution returns 12; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Restriction: $x \neq 1$. Then $3/(x-1)=4$, so $x-1=3/4$ and $x=7/4$. Substitution returns 6; the solution is not excluded. Item-specific marking check
Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 18

HA18 • Quadratic equations

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

With $a=1$, $b=3$, $c=-5$, discriminant $=3^2-4(1)(-5)=29$. Thus $x=(-3 \text{ plus or minus } \sqrt{29})/2$. Exact roots are $-3/2 + \sqrt{29}/2$ and $-\sqrt{29}/2 - 3/2$; approximately 1.193 and -4.193. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

With $a=1$, $b=6$, $c=-2$, discriminant $=6^2-4(1)(-2)=44$. Thus $x=(-6 \text{ plus or minus } \sqrt{44})/2$. Exact roots are $-3 + \sqrt{11}$ and $-\sqrt{11} - 3$; approximately 0.317 and -6.317. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

With $a=1$, $b=2$, $c=-5$, discriminant $=2^2-4(1)(-5)=24$. Thus $x=(-2 \text{ plus or minus } \sqrt{24})/2$. Exact roots are $-1 + \sqrt{6}$ and $-\sqrt{6} - 1$; approximately 1.449 and -3.449. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

With $a=1$, $b=5$, $c=-2$, discriminant $=5^2-4(1)(-2)=33$. Thus $x=(-5 \text{ plus or minus } \sqrt{33})/2$. Exact roots are $-5/2 + \sqrt{33}/2$ and $-\sqrt{33}/2 - 5/2$; approximately 0.372 and -5.372. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

With $a=1$, $b=8$, $c=-5$, discriminant $=8^2-4(1)(-5)=84$. Thus $x=(-8 \text{ plus or minus } \sqrt{84})/2$. Exact roots are $-4 + \sqrt{21}$ and $-\sqrt{21} - 4$; approximately 0.583 and -8.583. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

With $a=1$, $b=4$, $c=-2$, discriminant $=4^2-4(1)(-2)=24$. Thus $x=(-4 \text{ plus or minus } \sqrt{24})/2$. Exact roots are $-2 + \sqrt{6}$ and $-\sqrt{6} - 2$; approximately 0.449 and -4.449. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

With $a=1$, $b=7$, $c=-5$, discriminant $=7^2-4(1)(-5)=69$. Thus $x=(-7 \text{ plus or minus } \sqrt{69})/2$. Exact roots are $-7/2 + \sqrt{69}/2$ and $-\sqrt{69}/2 - 7/2$; approximately 0.653 and -7.653. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

With $a=1$, $b=3$, $c=-2$, discriminant $=3^2-4(1)(-2)=17$. Thus $x=(-3 \text{ plus or minus } \sqrt{17})/2$. Exact roots are $-3/2 + \sqrt{17}/2$ and $-\sqrt{17}/2 - 3/2$; approximately 0.562 and -3.562. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 19

HA19 • Simultaneous equations

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Substitution gives $17x^2 - 16x - 1 = 0$, with x-values $-1/17, 1$. Hence intersections are $[(-1/17, -38/17), (1, 2)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Substitution gives $10x^2 + 6x - 184 = 0$, with x-values $-23/5, 4$. Hence intersections are $[(-23/5, -64/5), (4, 13)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 185$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Substitution gives $5x^2 - 12x + 4 = 0$, with x-values $2/5, 2$. Hence intersections are $[(2/5, -11/5), (2, 1)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Substitution gives $2x^2 - 50 = 0$, with x-values $-5, 5$. Hence intersections are $[(-5, -5), (5, 5)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 50$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Substitution gives $17x^2 + 24x - 225 = 0$, with x-values $-75/17, 3$. Hence intersections are $[(-75/17, -249/17), (3, 15)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 234$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Substitution gives $10x^2 - 6x - 4 = 0$, with x-values $-2/5, 1$. Hence intersections are $[(-2/5, -11/5), (1, 2)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Substitution gives $5x^2 + 8x - 112 = 0$, with x-values $-28/5, 4$. Hence intersections are $[(-28/5, -46/5), (4, 10)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 116$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Substitution gives $2x^2 - 4x = 0$, with x-values $0, 2$. Hence intersections are $[(0, -2), (2, 0)]$. Substituting each point satisfies the line and gives $x^2 + y^2 = 4$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 20

HA20 • Numerical methods and iteration

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Iterations: $x_2=2.000000$, $x_3=1.750000$, $x_4=1.732143$. At a fixed point $x=0.5(x+3/x)$, so $x^2=3$; the positive iteration approaches $\sqrt{3}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Iterations: $x_2=3.500000$, $x_3=2.607143$, $x_4=2.454256$. At a fixed point $x=0.5(x+6/x)$, so $x^2=6$; the positive iteration approaches $\sqrt{6}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Iterations: $x_2=1.500000$, $x_3=1.416667$, $x_4=1.414216$. At a fixed point $x=0.5(x+2/x)$, so $x^2=2$; the positive iteration approaches $\sqrt{2}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Iterations: $x_2=3.000000$, $x_3=2.333333$, $x_4=2.238095$. At a fixed point $x=0.5(x+5/x)$, so $x^2=5$; the positive iteration approaches $\sqrt{5}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Iterations: $x_2=4.500000$, $x_3=3.138889$, $x_4=2.843781$. At a fixed point $x=0.5(x+8/x)$, so $x^2=8$; the positive iteration approaches $\sqrt{8}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Iterations: $x_2=2.500000$, $x_3=2.050000$, $x_4=2.000610$. At a fixed point $x=0.5(x+4/x)$, so $x^2=4$; the positive iteration approaches $\sqrt{4}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Iterations: $x_2=4.000000$, $x_3=2.875000$, $x_4=2.654891$. At a fixed point $x=0.5(x+7/x)$, so $x^2=7$; the positive iteration approaches $\sqrt{7}$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Iterations: $x_2=2.000000$, $x_3=1.750000$, $x_4=1.732143$. At a fixed point $x=0.5(x+3/x)$, so $x^2=3$; the positive iteration approaches $\sqrt{3}$.
A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 21

HA21 • Algebraic modelling

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Length=18-x, so $A=x(18-x)=-x^2+18x$. Completing the square or symmetry gives maximum at $x=9$, with $A=81 \text{ cm}^2$. For $x=12$, $A=72$, which is 88.9% of the maximum; therefore it does not reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Length=15-x, so $A=x(15-x)=-x^2+15x$. Completing the square or symmetry gives maximum at $x=15/2$, with $A=225/4 \text{ cm}^2$. For $x=7$, $A=56$, which is 99.6% of the maximum; therefore it does reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Length=18-x, so $A=x(18-x)=-x^2+18x$. Completing the square or symmetry gives maximum at $x=9$, with $A=81 \text{ cm}^2$. For $x=10$, $A=80$, which is 98.8% of the maximum; therefore it does reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Length=15-x, so $A=x(15-x)=-x^2+15x$. Completing the square or symmetry gives maximum at $x=15/2$, with $A=225/4 \text{ cm}^2$. For $x=5$, $A=50$, which is 88.9% of the maximum; therefore it does not reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Length=18-x, so $A=x(18-x)=-x^2+18x$. Completing the square or symmetry gives maximum at $x=9$, with $A=81 \text{ cm}^2$. For $x=8$, $A=80$, which is 98.8% of the maximum; therefore it does reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Length=15-x, so $A=x(15-x)=-x^2+15x$. Completing the square or symmetry gives maximum at $x=15/2$, with $A=225/4 \text{ cm}^2$. For $x=11$, $A=44$, which is 78.2% of the maximum; therefore it does not reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Length=18-x, so $A=x(18-x)=-x^2+18x$. Completing the square or symmetry gives maximum at $x=9$, with $A=81 \text{ cm}^2$. For $x=6$, $A=72$, which is 88.9% of the maximum; therefore it does not reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Length=15-x, so $A=x(15-x)=-x^2+15x$. Completing the square or symmetry gives maximum at $x=15/2$, with $A=225/4 \text{ cm}^2$. For $x=9$, $A=54$, which is 96.0% of the maximum; therefore it does reach 90%. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 22

HA22 • Linear and quadratic inequalities

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Critical values are -4 and 2. The product is non-positive between the roots, including endpoints, so $-4 \leq x \leq 2$. Required integers: [-4, -3, -2, -1, 0, 1, 2]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Critical values are -1 and 4. The product is non-positive between the roots, including endpoints, so $-1 \leq x \leq 4$. Required integers: [-1, 0, 1, 2, 3, 4]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Critical values are -5 and -1. The product is non-positive between the roots, including endpoints, so $-5 \leq x \leq -1$. Required integers: [-5, -4, -3, -2, -1]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Critical values are -2 and 1. The product is non-positive between the roots, including endpoints, so $-2 \leq x \leq 1$. Required integers: [-2, -1, 0, 1]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Critical values are 1 and 7. The product is non-positive between the roots, including endpoints, so $1 \leq x \leq 7$. Required integers: [1, 2, 3, 4, 5, 6, 7]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Critical values are -3 and 2. The product is non-positive between the roots, including endpoints, so $-3 \leq x \leq 2$. Required integers: [-3, -2, -1, 0, 1, 2]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Critical values are 0 and 4. The product is non-positive between the roots, including endpoints, so $0 \leq x \leq 4$. Required integers: [0, 1, 2, 3, 4]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Critical values are -4 and -1. The product is non-positive between the roots, including endpoints, so $-4 \leq x \leq -1$. Required integers: [-4, -3, -2, -1]. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 23

HA23 • Generating sequences

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

First differences are [10, 18, 26, 34]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 - 2n + 5$. At $n=10$ this equals 385, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

First differences are [10, 16, 22, 28]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 + 1n + 2$. At $n=10$ this equals 312, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

First differences are [3, 7, 11, 15]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 - 3n + 5$. At $n=10$ this equals 175, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

First differences are [3, 5, 7, 9]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 + 0n + 2$. At $n=10$ this equals 102, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

First differences are [15, 23, 31, 39]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 + 3n + 5$. At $n=10$ this equals 435, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

First differences are [8, 14, 20, 26]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 - 1n + 2$. At $n=10$ this equals 292, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

First differences are [8, 12, 16, 20]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 + 2n + 5$. At $n=10$ this equals 225, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

First differences are [1, 3, 5, 7]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 - 2n + 2$. At $n=10$ this equals 82, so the stated value is the 10th term. The solution must show the generated terms or recurrence steps. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 24

HA24 • Recognising important sequences

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

First differences are [10, 18, 26, 34]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 - 2n + 5$. At $n=10$ this equals 385, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

First differences are [10, 16, 22, 28]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 + n + 2$. At $n=10$ this equals 312, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

First differences are [3, 7, 11, 15]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 - 3n + 5$. At $n=10$ this equals 175, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

First differences are [3, 5, 7, 9]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 + 0n + 2$. At $n=10$ this equals 102, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

First differences are [15, 23, 31, 39]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 + 3n + 5$. At $n=10$ this equals 435, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

First differences are [8, 14, 20, 26]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 - n + 2$. At $n=10$ this equals 292, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

First differences are [8, 12, 16, 20]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 + 2n + 5$. At $n=10$ this equals 225, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

First differences are [1, 3, 5, 7]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 - 2n + 2$. At $n=10$ this equals 82, so the stated value is the 10th term. The classification must be supported by first differences, second differences or common ratios. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 25

HA25 • Linear and quadratic nth terms

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

First differences are [10, 18, 26, 34]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 - 2n + 5$. At $n=10$ this equals 385, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

First differences are [10, 16, 22, 28]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 + 1n + 2$. At $n=10$ this equals 312, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

First differences are [3, 7, 11, 15]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 - 3n + 5$. At $n=10$ this equals 175, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

First differences are [3, 5, 7, 9]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 + 0n + 2$. At $n=10$ this equals 102, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

First differences are [15, 23, 31, 39]; second difference is 8, so the n^2 coefficient is 4. Matching the remaining linear and constant parts gives $4n^2 + 3n + 5$. At $n=10$ this equals 435, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

First differences are [8, 14, 20, 26]; second difference is 6, so the n^2 coefficient is 3. Matching the remaining linear and constant parts gives $3n^2 - 1n + 2$. At $n=10$ this equals 292, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

First differences are [8, 12, 16, 20]; second difference is 4, so the n^2 coefficient is 2. Matching the remaining linear and constant parts gives $2n^2 + 2n + 5$. At $n=10$ this equals 225, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

First differences are [1, 3, 5, 7]; second difference is 2, so the n^2 coefficient is 1. Matching the remaining linear and constant parts gives $1n^2 - 2n + 2$. At $n=10$ this equals 82, so the stated value is the 10th term. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Publication and completion record

Unit 2 • Higher Algebra • HA1-HA25

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