

GCSE MATHEMATICS

HIGHER TIER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 5: PROBABILITY

Mapped to the Eduqas GCSE Mathematics specification • HP1-HP9
18 lessons • representations • reasoning • practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

Independent publication notice

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How to learn from this notebook

Every lesson identifies the sample space and denominator before calculation, then connects representations to Higher-tier reasoning.

1 • Define the event

Translate words into precise events, intersections, unions or complements.

2 • Choose the denominator

Use the full sample space unless a condition restricts it.

3 • Select a representation

Use a list, table, Venn diagram, frequency tree or probability tree.

4 • Combine correctly

Multiply along paths and add mutually exclusive paths.

5 • Verify

Check probabilities lie from 0 to 1 and exhaustive branches sum to 1.

Conditional discipline

The event after “given” defines the reduced sample space. Never reverse a conditional without rebuilding its denominator.

Unit map • Eduqas Higher Probability

Independent teaching summaries of HP1–HP9, expanded into distinct assessment forms.

Ch.	Ref.	Lesson
1	HP1	Frequency trees
2	HP2b	Fairness and randomness
3	HP2c	Expected frequency
4	HP2a	Experimental probability
5	HP4a	Probability scale and complements
6	HP4b	Addition rule and mutually exclusive events
7	HP3	Relative frequency and convergence
8	HP5a	Systematic listing
9	HP6a	Sets and Venn diagrams
10	HP5b	Sample spaces and possibility diagrams
11	HP7a	Independent events and multiplication
12	HP8b	Trees with replacement
13	HP7b	Dependent events and two-stage trees
14	HP7c	Repeated events and combined paths
15	HP8a	Conditional probability from tables
16	HP8b	Conditional probability from Venn diagrams
17	HP8c	Conditional probability from trees
18	HP9d	Reverse and algebraic probability

Marks are independent teaching allocations, not official Eduqas marks.

Higher Probability method bank

Name events

State exactly what A, B and complements represent.

Build the sample space

List or model every elementary outcome once.

Check equal likelihood

Outcome counting is valid only when elementary outcomes are equally likely.

Multiply paths

An AND sequence follows one complete route.

Add routes

An OR event combines disjoint successful routes.

Update without replacement

Change both relevant counts after the first draw.

Restrict when conditioned

The given event supplies the new denominator.

Test validity

Probabilities are between 0 and 1; branch totals and complete spaces sum to 1.

Chapter 1 • Frequency trees

HP1 • Organise changing counts through successive categories.



Understand the idea

A frequency tree tracks numbers, not probabilities. At every split, child frequencies add to the parent frequency, so missing values can be found additively.

Why this works

Label the starting total, complete each split using subtraction, then follow only branches relevant to the question and check every node total.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 1 • Build the method

Frequency trees

Reliable routine

Label the starting total, complete each split using subtraction, then follow only branches relevant to the question and check every node total.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. Of 180 students, 105 study a language. Of these, 63 study French. Of the others, 30 study French. How many do not study French?

Worked reasoning

Language but not French: $105 - 63 = 42$. No language: $180 - 105 = 75$, of whom $75 - 30 = 45$ do not study French. Total not French $= 42 + 45 = 87$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 1 • Apply and reason

Frequency trees

Common misconception

Multiplying frequencies as though every branch label were a probability.

Guided Higher problem

4 marks. A frequency tree classifies 330 people first as A or not A, then as B or not B. There are 85 in A, 40 in both A and B, and 60 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 1 • Independent practice

HP1 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A frequency tree classifies 330 people first as A or not A, then as B or not B. There are 85 in A, 40 in both A and B, and 60 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q2 | 4 MARKS

A frequency tree classifies 360 people first as A or not A, then as B or not B. There are 88 in A, 43 in both A and B, and 66 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q3 | 4 MARKS

A frequency tree classifies 390 people first as A or not A, then as B or not B. There are 91 in A, 26 in both A and B, and 32 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q4 | 4 MARKS

A frequency tree classifies 420 people first as A or not A, then as B or not B. There are 94 in A, 29 in both A and B, and 38 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q5 | 4 MARKS

A frequency tree classifies 450 people first as A or not A, then as B or not B. There are 97 in A, 32 in both A and B, and 44 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q6 | 4 MARKS

A frequency tree classifies 480 people first as A or not A, then as B or not B. There are 100 in A, 35 in both A and B, and 50 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q7 | 4 MARKS

A frequency tree classifies 510 people first as A or not A, then as B or not B. There are 103 in A, 38 in both A and B, and 56 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Q8 | 4 MARKS

A frequency tree classifies 540 people first as A or not A, then as B or not B. There are 106 in A, 41 in both A and B, and 62 in not A and B. Complete every branch frequency and find $P(A \text{ given } B)$.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 2 • Fairness and randomness

HP2b • Evaluate whether evidence supports a fair model.



Understand the idea

A fair device gives equally likely relevant outcomes. Randomness does not require short-run counts to be exactly equal; evidence should be judged using sample size and plausible variation.

Why this works

State the fair-model probability, compare it with observed relative frequency, consider the trial count, and give a cautious conclusion.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 2 • Build the method

Fairness and randomness

Reliable routine

State the fair-model probability, compare it with observed relative frequency, consider the trial count, and give a cautious conclusion.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A coin gives 61 heads in 100 tosses. Does this prove it is biased?

Worked reasoning

No. The result differs from 0.5, but one sample of 100 does not prove bias; more trials or a formal test would be needed.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 2 • Apply and reason

Fairness and randomness

Common misconception

Declaring a device unfair whenever observed counts are unequal.

Guided Higher problem

4 marks. A spinner has 14 equal sectors: 5 red, 5 blue and 4 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 5 red sectors but makes winning and losing equally likely.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 2 • Independent practice

HP2b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A spinner has 14 equal sectors: 5 red, 5 blue and 4 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 5 red sectors but makes winning and losing equally likely.

Q2 | 4 MARKS

A spinner has 9 equal sectors: 2 red, 4 blue and 3 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 2 red sectors but makes winning and losing equally likely.

Q3 | 4 MARKS

A spinner has 10 equal sectors: 5 red, 3 blue and 2 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 5 red sectors but makes winning and losing equally likely.

Q4 | 4 MARKS

A spinner has 12 equal sectors: 2 red, 9 blue and 1 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 2 red sectors but makes winning and losing equally likely.

Q5 | 4 MARKS

A spinner has 17 equal sectors: 5 red, 8 blue and 4 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 5 red sectors but makes winning and losing equally likely.

Q6 | 4 MARKS

A spinner has 12 equal sectors: 2 red, 7 blue and 3 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 2 red sectors but makes winning and losing equally likely.

Q7 | 4 MARKS

A spinner has 13 equal sectors: 5 red, 6 blue and 2 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 5 red sectors but makes winning and losing equally likely.

Q8 | 4 MARKS

A spinner has 8 equal sectors: 2 red, 5 blue and 1 green. A game pays a prize for red. Find the theoretical probability of winning, compare it with $\frac{1}{3}$, and state one precise redesign that keeps all 2 red sectors but makes winning and losing equally likely.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 3 • Expected frequency

HP2c • Convert probabilities into predicted counts and reverse the process.



Understand the idea

Expected frequency is probability multiplied by number of trials. It is a model-based prediction, not a guarantee that the count will occur exactly.

Why this works

Check the probability and trial count refer to the same event, multiply, and interpret non-integer results as expectations.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 3 • Build the method

Expected frequency

Reliable routine

Check the probability and trial count refer to the same event, multiply, and interpret non-integer results as expectations.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A defect probability is 0.018. Find the expected number in 2500 items.

Worked reasoning

$2500 \times 0.018 = 45$ items.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 3 • Apply and reason

Expected frequency

Common misconception

Rounding a probability before multiplying and changing the expectation.

Guided Higher problem

4 marks. A quality model gives probability $\frac{1}{2}$ that an item needs reworking. For 1000 items, calculate the expected number needing rework. The observed number is 498; compare observation with expectation and give a cautious conclusion.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 3 • Independent practice

HP2c • Questions 1–8 • 4–6 marks each

Q1 | 4 MARKS

A quality model gives probability $\frac{1}{2}$ that an item needs reworking. For 1000 items, calculate the expected number needing rework. The observed number is 498; compare observation with expectation and give a cautious conclusion.

Q2 | 4 MARKS

A quality model gives probability $\frac{1}{5}$ that an item needs reworking. For 1150 items, calculate the expected number needing rework. The observed number is 231; compare observation with expectation and give a cautious conclusion.

Q3 | 4 MARKS

A quality model gives probability $\frac{1}{2}$ that an item needs reworking. For 1300 items, calculate the expected number needing rework. The observed number is 647; compare observation with expectation and give a cautious conclusion.

Q4 | 4 MARKS

A quality model gives probability $\frac{1}{5}$ that an item needs reworking. For 1450 items, calculate the expected number needing rework. The observed number is 290; compare observation with expectation and give a cautious conclusion.

Q5 | 4 MARKS

A quality model gives probability $\frac{1}{2}$ that an item needs reworking. For 1600 items, calculate the expected number needing rework. The observed number is 803; compare observation with expectation and give a cautious conclusion.

Q6 | 4 MARKS

A quality model gives probability $\frac{1}{5}$ that an item needs reworking. For 1750 items, calculate the expected number needing rework. The observed number is 349; compare observation with expectation and give a cautious conclusion.

Q7 | 4 MARKS

A quality model gives probability $\frac{1}{2}$ that an item needs reworking. For 1900 items, calculate the expected number needing rework. The observed number is 952; compare observation with expectation and give a cautious conclusion.

Q8 | 4 MARKS

A quality model gives probability $\frac{1}{5}$ that an item needs reworking. For 2050 items, calculate the expected number needing rework. The observed number is 408; compare observation with expectation and give a cautious conclusion.

Show the probability model and denominator for every 5–6 mark task; finish with a validity check.

Chapter 4 • Experimental probability

HP2a • Estimate probability from observed relative frequency.



Understand the idea

Experimental probability is the long-run proportion observed in trials. It is an estimate and may differ from a theoretical model, especially with a small sample.

Why this works

Identify the relevant successes and total trials, form the fraction, simplify or decimalise, and state that the result is an estimate.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 4 • Build the method

Experimental probability

Reliable routine

Identify the relevant successes and total trials, form the fraction, simplify or decimalise, and state that the result is an estimate.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
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Worked example

Question. A spinner lands on blue 84 times in 240 spins. Estimate $P(\text{blue})$.

Worked reasoning

$P(\text{blue})$ is estimated by $84/240=0.35$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 4 • Apply and reason

Experimental probability

Common misconception

Treating an experimental estimate as an exact theoretical probability.

Guided Higher problem

4 marks. An event occurred 74 times in 230 trials and 327 times in a later set of 920 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 4 • Independent practice

HP2a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

An event occurred 74 times in 230 trials and 327 times in a later set of 920 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q2 | 4 MARKS

An event occurred 91 times in 260 trials and 364 times in a later set of 1040 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q3 | 4 MARKS

An event occurred 96 times in 290 trials and 400 times in a later set of 1160 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q4 | 4 MARKS

An event occurred 115 times in 320 trials and 435 times in a later set of 1280 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q5 | 4 MARKS

An event occurred 119 times in 350 trials and 497 times in a later set of 1400 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q6 | 4 MARKS

An event occurred 122 times in 380 trials and 532 times in a later set of 1520 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q7 | 4 MARKS

An event occurred 144 times in 410 trials and 566 times in a later set of 1640 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Q8 | 4 MARKS

An event occurred 145 times in 440 trials and 598 times in a later set of 1760 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 5 • Probability scale and complements

HP4a • Use values from 0 to 1 and complementary events.



Understand the idea

Probabilities lie from 0 to 1. An event and its complement partition the sample space, so their probabilities sum to 1.

Why this works

Define the event clearly, identify its complement, subtract from 1, and check the two values sum to 1.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 5 • Build the method

Probability scale and complements

Reliable routine

Define the event clearly, identify its complement, subtract from 1, and check the two values sum to 1.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. $P(\text{a bus is late}) = 0.17$. Find $P(\text{it is not late})$.

Worked reasoning

$1 - 0.17 = 0.83$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 5 • Apply and reason

Probability scale and complements

Common misconception

Subtracting from 100 when probabilities are written as decimals.

Guided Higher problem

4 marks. Events A and B are mutually exclusive. $P(A)=1/2$ and $P(B)=1/10$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 5 • Independent practice

HP4a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/2$ and $P(B)=1/10$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q2 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/5$ and $P(B)=1/5$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q3 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/2$ and $P(B)=3/10$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q4 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/5$ and $P(B)=2/5$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q5 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/2$ and $P(B)=1/2$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q6 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/5$ and $P(B)=1/10$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q7 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/2$ and $P(B)=1/5$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Q8 | 4 MARKS

Events A and B are mutually exclusive. $P(A)=1/5$ and $P(B)=3/10$. Find $P(A \text{ or } B)$, $P(\text{neither A nor B})$, and explain why all four values lie on the probability scale.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 6 • Addition rule and mutually exclusive events

HP4b • Find probabilities of unions with and without overlap.



Understand the idea

For any events A and B, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. If events are mutually exclusive, the intersection is zero.

Why this works

Decide whether overlap is possible, write the general rule first, substitute, and check the result is at most 1.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 6 • Build the method

Addition rule and mutually exclusive events

Reliable routine

Decide whether overlap is possible, write the general rule first, substitute, and check the result is at most 1.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. $P(A)=0.62$, $P(B)=0.45$ and $P(A \text{ and } B)=0.20$. Find $P(A \text{ or } B)$.

Worked reasoning

$$0.62+0.45-0.20=0.87.$$

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 6 • Apply and reason

Addition rule and mutually exclusive events

Common misconception

Adding overlapping probabilities without subtracting the intersection.

Guided Higher problem

4 marks. $P(A)=7/10$, $P(B)=1/2$ and $P(A \text{ intersection } B)=1/5$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 6 • Independent practice

HP4b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

$P(A)=7/10$, $P(B)=1/2$ and $P(A \text{ intersection } B)=1/5$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive.

Q2 | 4 MARKS

$P(A)=3/5$, $P(B)=3/10$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive.

Q3 | 4 MARKS

$P(A)=1/2$, $P(B)=1/2$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive.

Q4 | 4 MARKS

$P(A)=2/5$, $P(B)=3/10$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive.

Q5 | 4 MARKS

$P(A)=7/10$, $P(B)=1/2$ and $P(A \text{ intersection } B)=1/5$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive. After completing the calculation, give one independent check that supports your result.

Q6 | 4 MARKS

$P(A)=3/5$, $P(B)=3/10$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive. After completing the calculation, give one independent check that supports your result.

Q7 | 4 MARKS

$P(A)=1/2$, $P(B)=1/2$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive. After completing the calculation, give one independent check that supports your result.

Q8 | 4 MARKS

$P(A)=2/5$, $P(B)=3/10$ and $P(A \text{ intersection } B)=1/10$. Find $P(A \text{ union } B)$, $P(\text{neither})$, and decide with justification whether A and B are mutually exclusive. After completing the calculation, give one independent check that supports your result.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 7 • Relative frequency and convergence

HP3 • Use accumulating evidence to refine probability estimates.



Understand the idea

As independent trials accumulate under stable conditions, relative frequency often settles near the underlying probability. This is convergence, not a promise of monotonic improvement.

Why this works

Calculate cumulative relative frequencies, compare estimates at different sample sizes, and privilege the larger relevant sample while noting assumptions.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 7 • Build the method

Relative frequency and convergence

Reliable routine

Calculate cumulative relative frequencies, compare estimates at different sample sizes, and privilege the larger relevant sample while noting assumptions.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
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Worked example

Question. After 200 trials an event occurred 74 times; after 1000 trials it occurred 362 times. Which estimate is usually more reliable?

Worked reasoning

$74/200=0.37$ and $362/1000=0.362$. The second is usually more reliable because it uses more trials under the same conditions.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 7 • Apply and reason

Relative frequency and convergence

Common misconception

Assuming each new relative frequency must be closer than the previous one.

Guided Higher problem

4 marks. An event occurred 74 times in 230 trials and 327 times in a later set of 920 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 7 • Independent practice

HP3 • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

An event occurred 74 times in 230 trials and 327 times in a later set of 920 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q2 | 4 MARKS

An event occurred 91 times in 260 trials and 364 times in a later set of 1040 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q3 | 4 MARKS

An event occurred 96 times in 290 trials and 400 times in a later set of 1160 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q4 | 4 MARKS

An event occurred 115 times in 320 trials and 435 times in a later set of 1280 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q5 | 4 MARKS

An event occurred 119 times in 350 trials and 497 times in a later set of 1400 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q6 | 4 MARKS

An event occurred 122 times in 380 trials and 532 times in a later set of 1520 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q7 | 4 MARKS

An event occurred 144 times in 410 trials and 566 times in a later set of 1640 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Q8 | 4 MARKS

An event occurred 145 times in 440 trials and 598 times in a later set of 1760 trials. Calculate both relative frequencies, compare them, and decide which is the more reliable estimate of the long-run probability. Compare two sample sizes and explain which estimate should usually be more stable.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 8 • Systematic listing

HP5a • Enumerate outcomes without omissions or repetitions.



Understand the idea

A systematic list fixes one choice while cycling through all choices for the others. The product rule predicts how many outcomes should appear and helps verify completeness.

Why this works

Define the choices, choose an order, list in blocks or a table, and compare the count with the product rule.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 8 • Build the method

Systematic listing

Reliable routine

Define the choices, choose an order, list in blocks or a table, and compare the count with the product rule.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. How many four-digit codes use one letter from A,B,C followed by three digits, each 0–4, with repetition allowed?

Worked reasoning

There are $3 \times 5 \times 5 \times 5 = 375$ codes.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 8 • Apply and reason

Systematic listing

Common misconception

Adding numbers of choices instead of multiplying successive independent choices.

Guided Higher problem

4 marks. A code uses one of 6 letters followed by one of 4 digits. Exactly 4 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 8 • Independent practice

HP5a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A code uses one of 6 letters followed by one of 4 digits. Exactly 4 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q2 | 4 MARKS

A code uses one of 5 letters followed by one of 5 digits. Exactly 4 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q3 | 4 MARKS

A code uses one of 4 letters followed by one of 6 digits. Exactly 2 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q4 | 4 MARKS

A code uses one of 3 letters followed by one of 7 digits. Exactly 1 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q5 | 4 MARKS

A code uses one of 6 letters followed by one of 8 digits. Exactly 4 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q6 | 4 MARKS

A code uses one of 5 letters followed by one of 4 digits. Exactly 3 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q7 | 4 MARKS

A code uses one of 4 letters followed by one of 5 digits. Exactly 2 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Q8 | 4 MARKS

A code uses one of 3 letters followed by one of 6 digits. Exactly 1 specified letter-digit pairs are forbidden. Use the product rule to find the number of valid codes, then explain how a systematic list would verify that no outcome was counted twice.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 9 • Sets and Venn diagrams

HP6a • Translate set notation and populate disjoint regions.



Understand the idea

A Venn diagram partitions a universal set into non-overlapping regions. Intersections must be entered before only-regions to avoid double counting.

Why this works

Start with the deepest intersection, subtract it from given set totals, fill outside last, and verify the universal total.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 9 • Build the method

Sets and Venn diagrams

Reliable routine

Start with the deepest intersection, subtract it from given set totals, fill outside last, and verify the universal total.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. In a group of 80, 45 take A, 38 take B and 21 take both. How many take neither?

Worked reasoning

A only=24, B only=17, union= $24+21+17=62$, so neither=18.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 9 • Apply and reason

Sets and Venn diagrams

Common misconception

Placing the full set totals directly into the only-regions.

Guided Higher problem

4 marks. Of 195 members, 70 are in set A, 53 are in set B and 33 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 9 • Independent practice

HP6a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Of 195 members, 70 are in set A, 53 are in set B and 33 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q2 | 4 MARKS

Of 210 members, 73 are in set A, 59 are in set B and 18 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q3 | 4 MARKS

Of 225 members, 76 are in set A, 65 are in set B and 21 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q4 | 4 MARKS

Of 240 members, 79 are in set A, 71 are in set B and 24 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q5 | 4 MARKS

Of 255 members, 57 are in set A, 52 are in set B and 27 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q6 | 4 MARKS

Of 270 members, 60 are in set A, 58 are in set B and 30 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q7 | 4 MARKS

Of 285 members, 63 are in set A, 64 are in set B and 33 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Q8 | 4 MARKS

Of 300 members, 66 are in set A, 70 are in set B and 18 are in A intersection B. Fill every Venn region, then calculate $P((A \cup B) \text{ complement})$ and $P(A \text{ intersection } B \text{ complement})$.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 10 • Sample spaces and possibility diagrams

HP5b • Represent all equally or unequally likely outcomes.



Understand the idea

A sample space lists possible elementary outcomes. Counting outcomes gives probabilities only when those elementary outcomes are equally likely.

Why this works

Construct the grid or table, label ordered outcomes, mark the event region, and justify equal likelihood before counting.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 10 • Build the method

Sample spaces and possibility diagrams

Reliable routine

Construct the grid or table, label ordered outcomes, mark the event region, and justify equal likelihood before counting.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. Two fair six-sided dice are rolled. Find $P(\text{total } 9)$.

Worked reasoning

Four of 36 ordered outcomes give 9: (3,6),(4,5),(5,4),(6,3), so $P=1/9$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 10 • Apply and reason

Sample spaces and possibility diagrams

Common misconception

Treating the possible totals 2–12 as equally likely.

Guided Higher problem

4 marks. Two fair 4-sided spinners numbered 1 to 4 are spun. Construct a possibility-space argument to find $P(\text{total at least } 6)$. Then find $P(\text{the two scores are equal or the total is at least } 6)$.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 10 • Independent practice

HP5b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Two fair 4-sided spinners numbered 1 to 4 are spun. Construct a possibility-space argument to find $P(\text{total at least 6})$. Then find $P(\text{the two scores are equal or the total is at least 6})$.

Q2 | 4 MARKS

Two fair 7-sided spinners numbered 1 to 7 are spun. Construct a possibility-space argument to find $P(\text{total at least 10})$. Then find $P(\text{the two scores are equal or the total is at least 10})$.

Q3 | 4 MARKS

Two fair 5-sided spinners numbered 1 to 5 are spun. Construct a possibility-space argument to find $P(\text{total at least 8})$. Then find $P(\text{the two scores are equal or the total is at least 8})$.

Q4 | 4 MARKS

Two fair 8-sided spinners numbered 1 to 8 are spun. Construct a possibility-space argument to find $P(\text{total at least 14})$. Then find $P(\text{the two scores are equal or the total is at least 14})$.

Q5 | 4 MARKS

Two fair 6-sided spinners numbered 1 to 6 are spun. Construct a possibility-space argument to find $P(\text{total at least 10})$. Then find $P(\text{the two scores are equal or the total is at least 10})$.

Q6 | 4 MARKS

Two fair 4-sided spinners numbered 1 to 4 are spun. Construct a possibility-space argument to find $P(\text{total at least 7})$. Then find $P(\text{the two scores are equal or the total is at least 7})$.

Q7 | 4 MARKS

Two fair 7-sided spinners numbered 1 to 7 are spun. Construct a possibility-space argument to find $P(\text{total at least 12})$. Then find $P(\text{the two scores are equal or the total is at least 12})$.

Q8 | 4 MARKS

Two fair 5-sided spinners numbered 1 to 5 are spun. Construct a possibility-space argument to find $P(\text{total at least 5})$. Then find $P(\text{the two scores are equal or the total is at least 5})$.

Show the probability model and denominator for every 5–6 mark task; finish with a validity check.

Chapter 11 • Independent events and multiplication

HP7a • Recognise and calculate independent intersections.



Understand the idea

Events are independent when knowing one occurred does not change the probability of the other. Then $P(A \text{ and } B) = P(A)P(B)$.

Why this works

Justify independence from the model or information, multiply along the joint event, and distinguish it from mutual exclusivity.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 11 • Build the method

Independent events and multiplication

Reliable routine

Justify independence from the model or information, multiply along the joint event, and distinguish it from mutual exclusivity.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A fair coin and fair die are used. Find $P(\text{head and a multiple of 3})$.

Worked reasoning

$$(1/2) \times (2/6) = 1/6.$$

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 11 • Apply and reason

Independent events and multiplication

Common misconception

Assuming mutually exclusive events are independent.

Guided Higher problem

4 marks. Events A and B are independent with $P(A)=1/2$ and $P(B)=3/10$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 11 • Independent practice

HP7a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

Events A and B are independent with $P(A)=1/2$ and $P(B)=3/10$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q2 | 4 MARKS

Events A and B are independent with $P(A)=1/5$ and $P(B)=2/5$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q3 | 4 MARKS

Events A and B are independent with $P(A)=1/2$ and $P(B)=1/2$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q4 | 4 MARKS

Events A and B are independent with $P(A)=1/5$ and $P(B)=3/5$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q5 | 4 MARKS

Events A and B are independent with $P(A)=1/2$ and $P(B)=7/10$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q6 | 4 MARKS

Events A and B are independent with $P(A)=1/5$ and $P(B)=3/10$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Q7 | 4 MARKS

Events A and B are independent with $P(A)=1/2$ and $P(B)=2/5$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

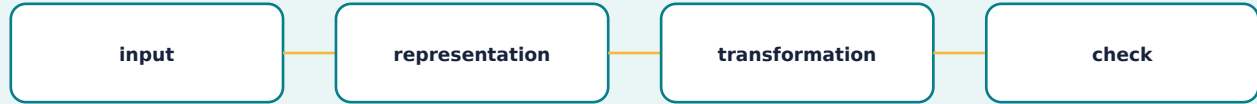
Q8 | 4 MARKS

Events A and B are independent with $P(A)=1/5$ and $P(B)=1/2$. Find $P(A \text{ and } B)$, $P(\text{exactly one of } A \text{ and } B)$, and verify independence using a conditional probability statement.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 12 • Trees with replacement

HP8b • Model repeated draws when the population is restored.



Understand the idea

With replacement, the population returns to its original composition after each draw, so corresponding later branch probabilities remain unchanged.

Why this works

Label the first stage, repeat the same branch probabilities at the next stage, multiply along paths, and add disjoint successful paths.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 12 • Build the method

Trees with replacement

Reliable routine

Label the first stage, repeat the same branch probabilities at the next stage, multiply along paths, and add disjoint successful paths.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A bag has 3 red and 2 blue counters. A counter is replaced after each draw. Find $P(\text{one of each in two draws})$.

Worked reasoning

$$P(\text{RB or BR}) = (3/5)(2/5) + (2/5)(3/5) = 12/25.$$

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 12 • Apply and reason

Trees with replacement

Common misconception

Updating the second-stage probabilities even though the counter is replaced.

Guided Higher problem

4 marks. A biased coin has $P(\text{head})=1/2$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 12 • Independent practice

HP8b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A biased coin has $P(\text{head})=1/2$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged.

Q2 | 4 MARKS

A biased coin has $P(\text{head})=1/5$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged.

Q3 | 4 MARKS

A biased coin has $P(\text{head})=1/2$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. After completing the calculation, give one independent check that supports your result.

Q4 | 4 MARKS

A biased coin has $P(\text{head})=1/5$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. After completing the calculation, give one independent check that supports your result.

Q5 | 4 MARKS

A biased coin has $P(\text{head})=1/2$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. Identify the key condition you used and explain why it applies here.

Q6 | 4 MARKS

A biased coin has $P(\text{head})=1/5$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. Identify the key condition you used and explain why it applies here.

Q7 | 4 MARKS

A biased coin has $P(\text{head})=1/2$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. Write one sentence interpreting your answer in the context of the question.

Q8 | 4 MARKS

A biased coin has $P(\text{head})=1/5$ and is tossed three times independently. Use a tree-path argument to find $P(\text{exactly two heads})$ and $P(\text{at least one head})$. Explain why the branch probabilities stay unchanged. Write one sentence interpreting your answer in the context of the question.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 13 • Dependent events and two-stage trees

HP7b • Update probabilities after an outcome changes the available population.



Understand the idea

Without replacement, the first outcome changes both numerator and denominator for the second draw. Tree branches show the correct conditional probabilities.

Why this works

Write first-stage probabilities, update counts on every second-stage branch, multiply along paths, and add disjoint successful paths.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 13 • Build the method

Dependent events and two-stage trees

Reliable routine

Write first-stage probabilities, update counts on every second-stage branch, multiply along paths, and add disjoint successful paths.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A bag has 5 red and 3 blue counters. Two are drawn without replacement. Find $P(\text{two red})$.

Worked reasoning

$$(5/8) \times (4/7) = 5/14.$$

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 13 • Apply and reason

Dependent events and two-stage trees

Common misconception

Reusing the original denominator on the second draw.

Guided Higher problem

4 marks. A bag contains 7 red and 7 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 13 • Independent practice

HP7b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A bag contains 7 red and 7 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q2 | 4 MARKS

A bag contains 4 red and 6 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q3 | 4 MARKS

A bag contains 7 red and 5 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q4 | 4 MARKS

A bag contains 4 red and 11 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q5 | 4 MARKS

A bag contains 7 red and 10 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q6 | 4 MARKS

A bag contains 4 red and 9 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Q7 | 4 MARKS

A bag contains 7 red and 8 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

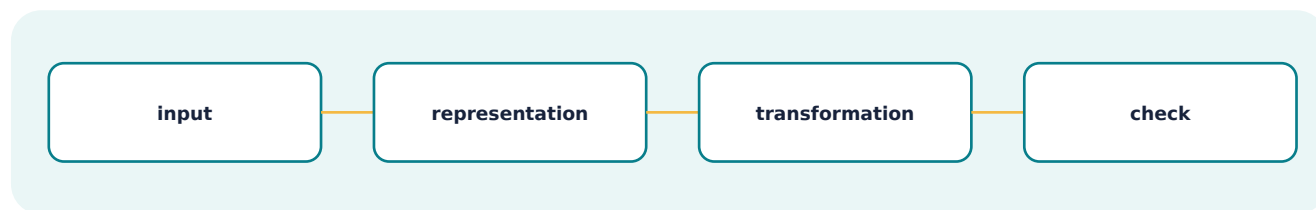
Q8 | 4 MARKS

A bag contains 4 red and 7 blue counters. Two are selected without replacement. Find $P(\text{two red})$ and $P(\text{one of each colour})$. Show the changing denominators and combine all valid orders.

Show the probability model and denominator for every 5–6 mark task; finish with a validity check.

Chapter 14 • Repeated events and combined paths

HP7c • Combine several routes to an event.



Understand the idea

Probabilities multiply along a path and add across mutually exclusive paths. Repeated independent trials can often be organised by a tree or binomial reasoning.

Why this works

Define every successful sequence, calculate each path product, add them, and use a complement when it shortens the work.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 14 • Build the method

Repeated events and combined paths

Reliable routine

Define every successful sequence, calculate each path product, add them, and use a complement when it shortens the work.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A biased coin has $P(H)=0.7$. It is tossed three times. Find $P(\text{exactly two heads})$.

Worked reasoning

There are three sequences, each with probability $0.7^2 \times 0.3$, so $\text{total} = 3 \times 0.147 = 0.441$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 14 • Apply and reason

Repeated events and combined paths

Common misconception

Calculating one ordering but forgetting the other successful sequences.

Guided Higher problem

4 marks. An event has probability $\frac{1}{2}$ on each of 5 independent trials. Find the probability of exactly 3 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 14 • Independent practice

HP7c • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

An event has probability $\frac{1}{2}$ on each of 5 independent trials. Find the probability of exactly 3 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation.

Q2 | 4 MARKS

An event has probability $\frac{1}{5}$ on each of 4 independent trials. Find the probability of exactly 2 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation.

Q3 | 5 MARKS

An event has probability $\frac{1}{2}$ on each of 5 independent trials. Find the probability of exactly 3 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. Verify the result using an independent inverse operation or a second representation.

Q4 | 5 MARKS

An event has probability $\frac{1}{5}$ on each of 4 independent trials. Find the probability of exactly 2 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. Verify the result using an independent inverse operation or a second representation.

Q5 | 5 MARKS

An event has probability $\frac{1}{2}$ on each of 5 independent trials. Find the probability of exactly 3 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. Identify one plausible incorrect approach and explain precisely why it fails.

Q6 | 5 MARKS

An event has probability $\frac{1}{5}$ on each of 4 independent trials. Find the probability of exactly 2 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. Identify one plausible incorrect approach and explain precisely why it fails.

Q7 | 5 MARKS

An event has probability $\frac{1}{2}$ on each of 5 independent trials. Find the probability of exactly 3 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. State the key condition that makes your method valid, then test that condition.

Q8 | 5 MARKS

An event has probability $\frac{1}{5}$ on each of 4 independent trials. Find the probability of exactly 2 occurrences and of at least one occurrence. Explain the combinatorial multiplier in the exact calculation. State the key condition that makes your method valid, then test that condition.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 15 • Conditional probability from tables

HP8a • Restrict the denominator to the stated condition.



Understand the idea

$P(A \text{ given } B)$ asks for the proportion satisfying A within the reduced population B . The condition after “given” determines the denominator.

Why this works

Circle the conditioned row or column total, locate the intersection numerator, divide, and label the result.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 15 • Build the method

Conditional probability from tables

Reliable routine

Circle the conditioned row or column total, locate the intersection numerator, divide, and label the result.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. Of 120 students, 50 travel by bus; 18 of those are late. Find $P(\text{late given bus})$.

Worked reasoning

$18/50=0.36$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 15 • Apply and reason

Conditional probability from tables

Common misconception

Dividing by the grand total rather than the conditioned subgroup.

Guided Higher problem

4 marks. A two-way table represents 310 people. There are 80 in A, 102 in B and 40 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 15 • Independent practice

HP8a • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A two-way table represents 310 people. There are 80 in A, 102 in B and 40 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q2 | 4 MARKS

A two-way table represents 340 people. There are 83 in A, 73 in B and 43 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q3 | 4 MARKS

A two-way table represents 370 people. There are 86 in A, 79 in B and 46 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q4 | 4 MARKS

A two-way table represents 400 people. There are 89 in A, 85 in B and 27 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q5 | 4 MARKS

A two-way table represents 430 people. There are 92 in A, 91 in B and 30 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q6 | 4 MARKS

A two-way table represents 460 people. There are 95 in A, 97 in B and 33 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q7 | 4 MARKS

A two-way table represents 490 people. There are 98 in A, 103 in B and 36 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Q8 | 4 MARKS

A two-way table represents 520 people. There are 66 in A, 74 in B and 39 in both. Calculate $P(A \text{ given } B)$, $P(B \text{ given } A)$, and explain why these conditional probabilities need not be equal.

Show the probability model and denominator for every 5–6 mark task; finish with a validity check.

Chapter 16 • Conditional probability from Venn diagrams

HP8b • Use a set as the restricted sample space.



Understand the idea

Conditioning on B discards everything outside B. Within B, the favourable region for A is A intersection B.

Why this works

Identify the conditioned set, total its regions, identify the favourable intersection, and divide.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 16 • Build the method

Conditional probability from Venn diagrams

Reliable routine

Identify the conditioned set, total its regions, identify the favourable intersection, and divide.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. A Venn diagram has 14 in A only, 9 in both and 21 in B only. Find $P(A \text{ given } B)$.

Worked reasoning

Within B there are $9+21=30$; 9 are also in A, so probability $=9/30=3/10$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 16 • Apply and reason

Conditional probability from Venn diagrams

Common misconception

Using the union total as the denominator.

Guided Higher problem

4 marks. A Venn diagram has universal total 215, $n(A)=75$, $n(B)=60$ and $n(A \text{ intersection } B)=35$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 16 • Independent practice

HP8b • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A Venn diagram has universal total 215, $n(A)=75$, $n(B)=60$ and $n(A \text{ intersection } B)=35$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q2 | 4 MARKS

A Venn diagram has universal total 230, $n(A)=78$, $n(B)=66$ and $n(A \text{ intersection } B)=38$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q3 | 4 MARKS

A Venn diagram has universal total 245, $n(A)=81$, $n(B)=72$ and $n(A \text{ intersection } B)=21$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q4 | 4 MARKS

A Venn diagram has universal total 260, $n(A)=84$, $n(B)=78$ and $n(A \text{ intersection } B)=24$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q5 | 4 MARKS

A Venn diagram has universal total 275, $n(A)=87$, $n(B)=84$ and $n(A \text{ intersection } B)=27$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q6 | 4 MARKS

A Venn diagram has universal total 290, $n(A)=60$, $n(B)=62$ and $n(A \text{ intersection } B)=30$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q7 | 4 MARKS

A Venn diagram has universal total 305, $n(A)=63$, $n(B)=68$ and $n(A \text{ intersection } B)=33$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Q8 | 4 MARKS

A Venn diagram has universal total 320, $n(A)=66$, $n(B)=74$ and $n(A \text{ intersection } B)=36$. Find $P(A \text{ given } B)$, $P(A \text{ complement given } B)$, and $P(B \text{ given } A \text{ union } B)$.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 17 • Conditional probability from trees

HP8c • Read later branch probabilities as conditional values.



Understand the idea

A second-stage branch is already conditioned on the path used to reach its node. Reverse conditionals require comparing complete path probabilities, not reading a branch backwards.

Why this works

Label the condition, locate or calculate relevant path probabilities, sum any matching paths, then form the restricted ratio.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 17 • Build the method

Conditional probability from trees

Reliable routine

Label the condition, locate or calculate relevant path probabilities, sum any matching paths, then form the restricted ratio.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. $P(A)=0.4$, $P(B \text{ given } A)=0.7$ and $P(B \text{ given not } A)=0.2$. Find $P(A \text{ and } B)$.

Worked reasoning

$P(A \text{ and } B)=0.4 \times 0.7=0.28$.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 17 • Apply and reason

Conditional probability from trees

Common misconception

Reading $P(B \text{ given } A)$ as $P(A \text{ given } B)$.

Guided Higher problem

4 marks. $P(\text{condition}) = 3/5$. A test is positive with probability $7/10$ when the condition is present and $1/5$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 17 • Independent practice

HP8c • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

$P(\text{condition}) = 3/5$. A test is positive with probability $7/10$ when the condition is present and $1/5$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree.

Q2 | 4 MARKS

$P(\text{condition}) = 1/2$. A test is positive with probability $7/10$ when the condition is present and $1/10$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree.

Q3 | 4 MARKS

$P(\text{condition}) = 2/5$. A test is positive with probability $7/10$ when the condition is present and $1/5$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree.

Q4 | 4 MARKS

$P(\text{condition}) = 3/10$. A test is positive with probability $7/10$ when the condition is present and $1/10$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree.

Q5 | 4 MARKS

$P(\text{condition}) = 3/5$. A test is positive with probability $7/10$ when the condition is present and $1/5$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree. After completing the calculation, give one independent check that supports your result.

Q6 | 4 MARKS

$P(\text{condition}) = 1/2$. A test is positive with probability $7/10$ when the condition is present and $1/10$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree. After completing the calculation, give one independent check that supports your result.

Q7 | 4 MARKS

$P(\text{condition}) = 2/5$. A test is positive with probability $7/10$ when the condition is present and $1/5$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree. After completing the calculation, give one independent check that supports your result.

Q8 | 4 MARKS

$P(\text{condition}) = 3/10$. A test is positive with probability $7/10$ when the condition is present and $1/10$ when absent. Find $P(\text{positive})$, then $P(\text{condition given positive})$, using a probability tree. After completing the calculation, give one independent check that supports your result.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Chapter 18 • Reverse and algebraic probability

HP9d • Reverse conditions and solve probability equations.



Understand the idea

A reverse conditional uses joint probability divided by the new conditioned total. When probabilities contain an unknown, branch totals and a stated combined probability create an equation whose solutions must lie from 0 to 1.

Why this works

Calculate complete path probabilities, rebuild the required denominator, form any algebraic constraint, solve, and reject invalid probability roots.

Vocabulary and representation

Define the event, complement, union, intersection or condition. State the sample space and identify whether outcomes are equally likely, independent or dependent.

Checkpoint

Give one model where the rule applies and one superficially similar model where a key assumption fails.

Chapter 18 • Build the method

Reverse and algebraic probability

Reliable routine

Calculate complete path probabilities, rebuild the required denominator, form any algebraic constraint, solve, and reject invalid probability roots.

Define event and space	Represent paths or regions	Calculate with correct rule	Verify range and total
------------------------	----------------------------	-----------------------------	------------------------

Worked example

Question. $P(A)=x$ and $P(B)=x+0.1$ for independent events. If $P(A \text{ and } B)=0.12$, find x .

Worked reasoning

$x(x+0.1)=0.12$, so $(x+0.4)(x-0.3)=0$. Only $x=0.3$ is a valid probability.

Variation

Change one probability, count or dependence assumption. Predict which branches or regions must change before recalculating.

Self-explanation

Identify the decisive denominator and justify every multiplication or addition.

Chapter 18 • Apply and reason

Reverse and algebraic probability

Common misconception

Reversing conditional notation directly or retaining an algebraic root outside 0 to 1.

Guided Higher problem

4 marks. A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=3/5$; if A does not occur, $P(\text{win})=2/5$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Planning scaffold

1. Define events. 2. Build the sample space or conditioned subgroup. 3. Label paths or regions. 4. Calculate without premature rounding. 5. Check totals and interpret.

Reasoning prompt

Explain whether the model assumes equal likelihood, independence or stable experimental conditions, and what would change if that assumption failed.

Stretch

Construct a different representation of the same situation and prove it gives the same probability.

Chapter 18 • Independent practice

HP9d • Questions 1-8 • 4-6 marks each

Q1 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=3/5$; if A does not occur, $P(\text{win})=2/5$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Q2 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=3/4$; if A does not occur, $P(\text{win})=1/4$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Q3 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=2/3$; if A does not occur, $P(\text{win})=1/3$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Q4 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=7/9$; if A does not occur, $P(\text{win})=2/9$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Q5 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=5/7$; if A does not occur, $P(\text{win})=2/7$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability.

Q6 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=3/5$; if A does not occur, $P(\text{win})=2/5$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability. After completing the calculation, give one independent check that supports your result.

Q7 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=3/4$; if A does not occur, $P(\text{win})=1/4$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability. After completing the calculation, give one independent check that supports your result.

Q8 | 4 MARKS

A two-stage game uses $P(A)=p$. If A occurs, $P(\text{win})=2/3$; if A does not occur, $P(\text{win})=1/3$. The overall probability of winning is $1/2$. Form and solve an equation for p , then check that your answer is a valid probability. After completing the calculation, give one independent check that supports your result.

Show the probability model and denominator for every 5-6 mark task; finish with a validity check.

Worked answers • Chapter 1

HP1 • Frequency trees

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

A and not B = $85 - 40 = 45$. Not A = $330 - 85 = 245$; not A and not B = $245 - 60 = 185$. Total B = $40 + 60 = 100$. Therefore $P(A \text{ given } B) = 40/100 = 2/5$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

A and not B = $88 - 43 = 45$. Not A = $360 - 88 = 272$; not A and not B = $272 - 66 = 206$. Total B = $43 + 66 = 109$. Therefore $P(A \text{ given } B) = 43/109 = 43/109$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

A and not B = $91 - 26 = 65$. Not A = $390 - 91 = 299$; not A and not B = $299 - 32 = 267$. Total B = $26 + 32 = 58$. Therefore $P(A \text{ given } B) = 26/58 = 13/29$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

A and not B = $94 - 29 = 65$. Not A = $420 - 94 = 326$; not A and not B = $326 - 38 = 288$. Total B = $29 + 38 = 67$. Therefore $P(A \text{ given } B) = 29/67 = 29/67$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

A and not B = $97 - 32 = 65$. Not A = $450 - 97 = 353$; not A and not B = $353 - 44 = 309$. Total B = $32 + 44 = 76$. Therefore $P(A \text{ given } B) = 32/76 = 8/19$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

A and not B = $100 - 35 = 65$. Not A = $480 - 100 = 380$; not A and not B = $380 - 50 = 330$. Total B = $35 + 50 = 85$. Therefore $P(A \text{ given } B) = 35/85 = 7/17$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

A and not B = $103 - 38 = 65$. Not A = $510 - 103 = 407$; not A and not B = $407 - 56 = 351$. Total B = $38 + 56 = 94$. Therefore $P(A \text{ given } B) = 38/94 = 19/47$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

A and not B = $106 - 41 = 65$. Not A = $540 - 106 = 434$; not A and not B = $434 - 62 = 372$. Total B = $41 + 62 = 103$. Therefore $P(A \text{ given } B) = 41/103 = 41/103$.
Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 2

HP2b • Fairness and randomness

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$P(\text{win}) = 5/14 = 5/14$. It is greater than $1/3$. Keeping 5 winning sectors requires exactly 5 losing sectors, so remove 4 losing sector(s). The redesigned spinner then has 10 equal sectors and $P(\text{win}) = 5/10 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$P(\text{win}) = 2/9 = 2/9$. It is less than $1/3$. Keeping 2 winning sectors requires exactly 2 losing sectors, so remove 5 losing sector(s). The redesigned spinner then has 4 equal sectors and $P(\text{win}) = 2/4 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$P(\text{win}) = 5/10 = 1/2$. It is greater than $1/3$. Keeping 5 winning sectors requires exactly 5 losing sectors, so make no change. The redesigned spinner then has 10 equal sectors and $P(\text{win}) = 5/10 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$P(\text{win}) = 2/12 = 1/6$. It is less than $1/3$. Keeping 2 winning sectors requires exactly 2 losing sectors, so remove 8 losing sector(s). The redesigned spinner then has 4 equal sectors and $P(\text{win}) = 2/4 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$P(\text{win}) = 5/17 = 5/17$. It is less than $1/3$. Keeping 5 winning sectors requires exactly 5 losing sectors, so remove 7 losing sector(s). The redesigned spinner then has 10 equal sectors and $P(\text{win}) = 5/10 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$P(\text{win}) = 2/12 = 1/6$. It is less than $1/3$. Keeping 2 winning sectors requires exactly 2 losing sectors, so remove 8 losing sector(s). The redesigned spinner then has 4 equal sectors and $P(\text{win}) = 2/4 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$P(\text{win}) = 5/13 = 5/13$. It is greater than $1/3$. Keeping 5 winning sectors requires exactly 5 losing sectors, so remove 3 losing sector(s). The redesigned spinner then has 10 equal sectors and $P(\text{win}) = 5/10 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$P(\text{win}) = 2/8 = 1/4$. It is less than $1/3$. Keeping 2 winning sectors requires exactly 2 losing sectors, so remove 4 losing sector(s). The redesigned spinner then has 4 equal sectors and $P(\text{win}) = 2/4 = 1/2$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 3

HP2c • Expected frequency

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Expected frequency = $1000 \times \frac{1}{2} = 500$. The difference is $498 - 500 = -2$. This sample is below the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Expected frequency = $1150 \times \frac{1}{5} = 230$. The difference is $231 - 230 = 1$. This sample is above the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Expected frequency = $1300 \times \frac{1}{2} = 650$. The difference is $647 - 650 = -3$. This sample is below the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Expected frequency = $1450 \times \frac{1}{5} = 290$. The difference is $290 - 290 = 0$. This sample is equal to the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Expected frequency = $1600 \times \frac{1}{2} = 800$. The difference is $803 - 800 = 3$. This sample is above the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Expected frequency = $1750 \times \frac{1}{5} = 350$. The difference is $349 - 350 = -1$. This sample is below the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Expected frequency = $1900 \times \frac{1}{2} = 950$. The difference is $952 - 950 = 2$. This sample is above the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Expected frequency = $2050 \times \frac{1}{5} = 410$. The difference is $408 - 410 = -2$. This sample is below the model expectation, but ordinary random variation means one result alone is insufficient to reject the model. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 4

HP2a • Experimental probability

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

First relative frequency = $74/230 = 0.3217$. Second = $327/920 = 0.3554$. The difference is 0.0337. The second estimate is normally more reliable because it is based on 920 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

First relative frequency = $91/260 = 0.3500$. Second = $364/1040 = 0.3500$. The difference is 0.0000. The second estimate is normally more reliable because it is based on 1040 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

First relative frequency = $96/290 = 0.3310$. Second = $400/1160 = 0.3448$. The difference is 0.0138. The second estimate is normally more reliable because it is based on 1160 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

First relative frequency = $115/320 = 0.3594$. Second = $435/1280 = 0.3398$. The difference is 0.0195. The second estimate is normally more reliable because it is based on 1280 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

First relative frequency = $119/350 = 0.3400$. Second = $497/1400 = 0.3550$. The difference is 0.0150. The second estimate is normally more reliable because it is based on 1400 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

First relative frequency = $122/380 = 0.3211$. Second = $532/1520 = 0.3500$. The difference is 0.0289. The second estimate is normally more reliable because it is based on 1520 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

First relative frequency = $144/410 = 0.3512$. Second = $566/1640 = 0.3451$. The difference is 0.0061. The second estimate is normally more reliable because it is based on 1640 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

First relative frequency = $145/440 = 0.3295$. Second = $598/1760 = 0.3398$. The difference is 0.0102. The second estimate is normally more reliable because it is based on 1760 trials, although neither guarantees the true probability. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 5

HP4a • Probability scale and complements

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/2 + 1/10 = 3/5$. The complement is $1 - 3/5 = 2/5$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/5 + 1/5 = 2/5$. The complement is $1 - 2/5 = 3/5$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/2 + 3/10 = 4/5$. The complement is $1 - 4/5 = 1/5$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/5 + 2/5 = 3/5$. The complement is $1 - 3/5 = 2/5$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/2 + 1/2 = 1$. The complement is $1 - 1 = 0$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/5 + 1/10 = 3/10$. The complement is $1 - 3/10 = 7/10$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/2 + 1/5 = 7/10$. The complement is $1 - 7/10 = 3/10$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

For mutually exclusive events, $P(A \text{ or } B) = 1/5 + 3/10 = 1/2$. The complement is $1 - 1/2 = 1/2$. Each probability is between 0 and 1, and the event plus its complement totals 1. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 6

HP4b • Addition rule and mutually exclusive events

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 7/10 + 1/2 - 1/5 = 1$. $P(\text{neither}) = 1 - 1 = 0$. They are not mutually exclusive because $P(A \cap B) = 1/5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/5 + 3/10 - 1/10 = 4/5$. $P(\text{neither}) = 1 - 4/5 = 1/5$. They are not mutually exclusive because $P(A \cap B) = 1/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/2 + 1/2 - 1/10 = 9/10$. $P(\text{neither}) = 1 - 9/10 = 1/10$. They are not mutually exclusive because $P(A \cap B) = 1/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 2/5 + 3/10 - 1/10 = 3/5$. $P(\text{neither}) = 1 - 3/5 = 2/5$. They are not mutually exclusive because $P(A \cap B) = 1/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 7/10 + 1/2 - 1/5 = 1$. $P(\text{neither}) = 1 - 1 = 0$. They are not mutually exclusive because $P(A \cap B) = 1/5$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/5 + 3/10 - 1/10 = 4/5$. $P(\text{neither}) = 1 - 4/5 = 1/5$. They are not mutually exclusive because $P(A \cap B) = 1/10$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/2 + 1/2 - 1/10 = 9/10$. $P(\text{neither}) = 1 - 9/10 = 1/10$. They are not mutually exclusive because $P(A \cap B) = 1/10$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 2/5 + 3/10 - 1/10 = 3/5$. $P(\text{neither}) = 1 - 3/5 = 2/5$. They are not mutually exclusive because $P(A \cap B) = 1/10$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 7

HP3 • Relative frequency and convergence

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

First relative frequency = $74/230 = 0.3217$. Second = $327/920 = 0.3554$. The difference is 0.0337. The second estimate is normally more reliable because it is based on 920 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

First relative frequency = $91/260 = 0.3500$. Second = $364/1040 = 0.3500$. The difference is 0.0000. The second estimate is normally more reliable because it is based on 1040 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

First relative frequency = $96/290 = 0.3310$. Second = $400/1160 = 0.3448$. The difference is 0.0138. The second estimate is normally more reliable because it is based on 1160 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

First relative frequency = $115/320 = 0.3594$. Second = $435/1280 = 0.3398$. The difference is 0.0195. The second estimate is normally more reliable because it is based on 1280 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

First relative frequency = $119/350 = 0.3400$. Second = $497/1400 = 0.3550$. The difference is 0.0150. The second estimate is normally more reliable because it is based on 1400 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

First relative frequency = $122/380 = 0.3211$. Second = $532/1520 = 0.3500$. The difference is 0.0289. The second estimate is normally more reliable because it is based on 1520 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

First relative frequency = $144/410 = 0.3512$. Second = $566/1640 = 0.3451$. The difference is 0.0061. The second estimate is normally more reliable because it is based on 1640 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

First relative frequency = $145/440 = 0.3295$. Second = $598/1760 = 0.3398$. The difference is 0.0102. The second estimate is normally more reliable because it is based on 1760 trials, although neither guarantees the true probability. The conclusion must refer to reduced relative variation rather than claiming certainty. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 8

HP5a • Systematic listing

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Before restrictions there are $6 \times 4 = 24$ ordered codes. Removing 4 distinct forbidden pairs leaves 20. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Before restrictions there are $5 \times 5 = 25$ ordered codes. Removing 4 distinct forbidden pairs leaves 21. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Before restrictions there are $4 \times 6 = 24$ ordered codes. Removing 2 distinct forbidden pairs leaves 22. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Before restrictions there are $3 \times 7 = 21$ ordered codes. Removing 1 distinct forbidden pairs leaves 20. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Before restrictions there are $6 \times 8 = 48$ ordered codes. Removing 4 distinct forbidden pairs leaves 44. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Before restrictions there are $5 \times 4 = 20$ ordered codes. Removing 3 distinct forbidden pairs leaves 17. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Before restrictions there are $4 \times 5 = 20$ ordered codes. Removing 2 distinct forbidden pairs leaves 18. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Before restrictions there are $3 \times 6 = 18$ ordered codes. Removing 1 distinct forbidden pairs leaves 17. A verification list groups codes by first letter and lists every allowed digit once within each group, preventing omission or duplication. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 9

HP6a • Sets and Venn diagrams

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

A-only = $70 - 33 = 37$; B-only = $53 - 33 = 20$; neither = $195 - (70 + 53 - 33) = 105$. Thus $P((A \cup B) \text{ complement}) = 7/13$ and $P(A \cap B \text{ complement}) = 37/195$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

A-only = $73 - 18 = 55$; B-only = $59 - 18 = 41$; neither = $210 - (73 + 59 - 18) = 96$. Thus $P((A \cup B) \text{ complement}) = 16/35$ and $P(A \cap B \text{ complement}) = 11/42$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

A-only = $76 - 21 = 55$; B-only = $65 - 21 = 44$; neither = $225 - (76 + 65 - 21) = 105$. Thus $P((A \cup B) \text{ complement}) = 7/15$ and $P(A \cap B \text{ complement}) = 11/45$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

A-only = $79 - 24 = 55$; B-only = $71 - 24 = 47$; neither = $240 - (79 + 71 - 24) = 114$. Thus $P((A \cup B) \text{ complement}) = 19/40$ and $P(A \cap B \text{ complement}) = 11/48$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

A-only = $57 - 27 = 30$; B-only = $52 - 27 = 25$; neither = $255 - (57 + 52 - 27) = 173$. Thus $P((A \cup B) \text{ complement}) = 173/255$ and $P(A \cap B \text{ complement}) = 2/17$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

A-only = $60 - 30 = 30$; B-only = $58 - 30 = 28$; neither = $270 - (60 + 58 - 30) = 182$. Thus $P((A \cup B) \text{ complement}) = 91/135$ and $P(A \cap B \text{ complement}) = 1/9$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

A-only = $63 - 33 = 30$; B-only = $64 - 33 = 31$; neither = $285 - (63 + 64 - 33) = 191$. Thus $P((A \cup B) \text{ complement}) = 191/285$ and $P(A \cap B \text{ complement}) = 2/19$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

A-only = $66 - 18 = 48$; B-only = $70 - 18 = 52$; neither = $300 - (66 + 70 - 18) = 182$. Thus $P((A \cup B) \text{ complement}) = 91/150$ and $P(A \cap B \text{ complement}) = 4/25$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 10

HP5b • Sample spaces and possibility diagrams

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

There are 16 equally likely ordered outcomes. The region with total at least 6 contains 6, so probability = $\frac{3}{8}$. Equality contains 4 outcomes, with 2 also in the high-total region. Union count = $6 + 4 - 2 = 8$, giving $\frac{1}{2}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

There are 49 equally likely ordered outcomes. The region with total at least 10 contains 15, so probability = $\frac{15}{49}$. Equality contains 7 outcomes, with 3 also in the high-total region. Union count = $15 + 7 - 3 = 19$, giving $\frac{19}{49}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

There are 25 equally likely ordered outcomes. The region with total at least 8 contains 6, so probability = $\frac{6}{25}$. Equality contains 5 outcomes, with 2 also in the high-total region. Union count = $6 + 5 - 2 = 9$, giving $\frac{9}{25}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

There are 64 equally likely ordered outcomes. The region with total at least 14 contains 6, so probability = $\frac{3}{32}$. Equality contains 8 outcomes, with 2 also in the high-total region. Union count = $6 + 8 - 2 = 12$, giving $\frac{3}{16}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

There are 36 equally likely ordered outcomes. The region with total at least 10 contains 6, so probability = $\frac{1}{6}$. Equality contains 6 outcomes, with 2 also in the high-total region. Union count = $6 + 6 - 2 = 10$, giving $\frac{5}{18}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

There are 16 equally likely ordered outcomes. The region with total at least 7 contains 3, so probability = $\frac{3}{16}$. Equality contains 4 outcomes, with 1 also in the high-total region. Union count = $3 + 4 - 1 = 6$, giving $\frac{3}{8}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

There are 49 equally likely ordered outcomes. The region with total at least 12 contains 6, so probability = $\frac{6}{49}$. Equality contains 7 outcomes, with 2 also in the high-total region. Union count = $6 + 7 - 2 = 11$, giving $\frac{11}{49}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

There are 25 equally likely ordered outcomes. The region with total at least 5 contains 19, so probability = $\frac{19}{25}$. Equality contains 5 outcomes, with 3 also in the high-total region. Union count = $19 + 5 - 3 = 21$, giving $\frac{21}{25}$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 11

HP7a • Independent events and multiplication

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Independence gives $P(A \text{ and } B) = 1/2 \times 3/10 = 3/20$. Exactly one has two disjoint routes: $1/2 \times 7/10 + 1/2 \times 3/10 = 1/2$. Also $P(A \text{ given } B) = P(A) = 1/2$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Independence gives $P(A \text{ and } B) = 1/5 \times 2/5 = 2/25$. Exactly one has two disjoint routes: $1/5 \times 3/5 + 4/5 \times 2/5 = 11/25$. Also $P(A \text{ given } B) = P(A) = 1/5$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Independence gives $P(A \text{ and } B) = 1/2 \times 1/2 = 1/4$. Exactly one has two disjoint routes: $1/2 \times 1/2 + 1/2 \times 1/2 = 1/2$. Also $P(A \text{ given } B) = P(A) = 1/2$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Independence gives $P(A \text{ and } B) = 1/5 \times 3/5 = 3/25$. Exactly one has two disjoint routes: $1/5 \times 2/5 + 4/5 \times 3/5 = 14/25$. Also $P(A \text{ given } B) = P(A) = 1/5$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Independence gives $P(A \text{ and } B) = 1/2 \times 7/10 = 7/20$. Exactly one has two disjoint routes: $1/2 \times 3/10 + 1/2 \times 7/10 = 1/2$. Also $P(A \text{ given } B) = P(A) = 1/2$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Independence gives $P(A \text{ and } B) = 1/5 \times 3/10 = 3/50$. Exactly one has two disjoint routes: $1/5 \times 7/10 + 4/5 \times 3/10 = 19/50$. Also $P(A \text{ given } B) = P(A) = 1/5$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Independence gives $P(A \text{ and } B) = 1/2 \times 2/5 = 1/5$. Exactly one has two disjoint routes: $1/2 \times 3/5 + 1/2 \times 2/5 = 1/2$. Also $P(A \text{ given } B) = P(A) = 1/2$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Independence gives $P(A \text{ and } B) = 1/5 \times 1/2 = 1/10$. Exactly one has two disjoint routes: $1/5 \times 1/2 + 4/5 \times 1/2 = 1/2$. Also $P(A \text{ given } B) = P(A) = 1/5$, which verifies the independence condition. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 12

HP8b • Trees with replacement

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/2)^2(1/2)$. Hence $P(\text{exactly two})=3/8$. $P(\text{at least one})=1-P(\text{no heads})=1-(1/2)^3=7/8$. Replacement/independent trials keep each branch probability constant. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/5)^2(4/5)$. Hence $P(\text{exactly two})=12/125$. $P(\text{at least one})=1-P(\text{no heads})=1-(4/5)^3=61/125$. Replacement/independent trials keep each branch probability constant. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/2)^2(1/2)$. Hence $P(\text{exactly two})=3/8$. $P(\text{at least one})=1-P(\text{no heads})=1-(1/2)^3=7/8$. Replacement/independent trials keep each branch probability constant. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/5)^2(4/5)$. Hence $P(\text{exactly two})=12/125$. $P(\text{at least one})=1-P(\text{no heads})=1-(4/5)^3=61/125$. Replacement/independent trials keep each branch probability constant. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/2)^2(1/2)$. Hence $P(\text{exactly two})=3/8$. $P(\text{at least one})=1-P(\text{no heads})=1-(1/2)^3=7/8$. Replacement/independent trials keep each branch probability constant. The explanation must name the relevant condition and link it explicitly to the method used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/5)^2(4/5)$. Hence $P(\text{exactly two})=12/125$. $P(\text{at least one})=1-P(\text{no heads})=1-(4/5)^3=61/125$. Replacement/independent trials keep each branch probability constant. The explanation must name the relevant condition and link it explicitly to the method used. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/2)^2(1/2)$. Hence $P(\text{exactly two})=3/8$. $P(\text{at least one})=1-P(\text{no heads})=1-(1/2)^3=7/8$. Replacement/independent trials keep each branch probability constant. The interpretation should include the correct units or geometric/statistical meaning where applicable. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

There are $C(3,2)=3$ paths with two heads, each probability $(1/5)^2(4/5)$. Hence $P(\text{exactly two})=12/125$. $P(\text{at least one})=1-P(\text{no heads})=1-(4/5)^3=61/125$. Replacement/independent trials keep each branch probability constant. The interpretation should include the correct units or geometric/statistical meaning where applicable. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 13

HP7b • Dependent events and two-stage trees

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

$P(RR)=7/14 \times 6/13=3/13$. One of each can be RB or BR: $7/14 \times 7/13 + 7/14 \times 7/13 = 7/13$. The second denominator is 13 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

$P(RR)=4/10 \times 3/9=2/15$. One of each can be RB or BR: $4/10 \times 6/9 + 6/10 \times 4/9 = 8/15$. The second denominator is 9 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

$P(RR)=7/12 \times 6/11=7/22$. One of each can be RB or BR: $7/12 \times 5/11 + 5/12 \times 7/11 = 35/66$. The second denominator is 11 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

$P(RR)=4/15 \times 3/14=2/35$. One of each can be RB or BR: $4/15 \times 11/14 + 11/15 \times 4/14 = 44/105$. The second denominator is 14 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

$P(RR)=7/17 \times 6/16=21/136$. One of each can be RB or BR: $7/17 \times 10/16 + 10/17 \times 7/16 = 35/68$. The second denominator is 16 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

$P(RR)=4/13 \times 3/12=1/13$. One of each can be RB or BR: $4/13 \times 9/12 + 9/13 \times 4/12 = 6/13$. The second denominator is 12 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

$P(RR)=7/15 \times 6/14=1/5$. One of each can be RB or BR: $7/15 \times 8/14 + 8/15 \times 7/14 = 8/15$. The second denominator is 14 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

$P(RR)=4/11 \times 3/10=6/55$. One of each can be RB or BR: $4/11 \times 7/10 + 7/11 \times 4/10 = 28/55$. The second denominator is 10 because the first counter is not replaced. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 14

HP7c • Repeated events and combined paths

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Exactly 3: choose the 3 successful positions in $C(5,3)=10$ ways. Probability= $10 \times (1/2)^3 \times (1/2)^2 = 5/16$. At least one= $1-P(\text{none})=1-(1/2)^5=31/32$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Exactly 2: choose the 2 successful positions in $C(4,2)=6$ ways. Probability= $6 \times (1/5)^2 \times (4/5)^2 = 96/625$. At least one= $1-P(\text{none})=1-(4/5)^4=369/625$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 5

Exactly 3: choose the 3 successful positions in $C(5,3)=10$ ways. Probability= $10 \times (1/2)^3 \times (1/2)^2 = 5/16$. At least one= $1-P(\text{none})=1-(1/2)^5=31/32$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Accept a correct independent verification using an inverse operation, substitution, alternative representation or equivalent check. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 5

Exactly 2: choose the 2 successful positions in $C(4,2)=6$ ways. Probability= $6 \times (1/5)^2 \times (4/5)^2 = 96/625$. At least one= $1-P(\text{none})=1-(4/5)^4=369/625$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Accept a correct independent verification using an inverse operation, substitution, alternative representation or equivalent check. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 5

Exactly 3: choose the 3 successful positions in $C(5,3)=10$ ways. Probability= $10 \times (1/2)^3 \times (1/2)^2 = 5/16$. At least one= $1-P(\text{none})=1-(1/2)^5=31/32$. The explanation must name the relevant condition and link it explicitly to the method used. Accept any mathematically specific incorrect approach with a correct explanation of the failed assumption or operation. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 5

Exactly 2: choose the 2 successful positions in $C(4,2)=6$ ways. Probability= $6 \times (1/5)^2 \times (4/5)^2 = 96/625$. At least one= $1-P(\text{none})=1-(4/5)^4=369/625$. The explanation must name the relevant condition and link it explicitly to the method used. Accept any mathematically specific incorrect approach with a correct explanation of the failed assumption or operation. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 5

Exactly 3: choose the 3 successful positions in $C(5,3)=10$ ways. Probability= $10 \times (1/2)^3 \times (1/2)^2 = 5/16$. At least one= $1-P(\text{none})=1-(1/2)^5=31/32$. The interpretation should include the correct units or geometric/statistical meaning where applicable. Credit a correct validity condition and an explicit check that it holds in this case. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 5

Exactly 2: choose the 2 successful positions in $C(4,2)=6$ ways. Probability= $6 \times (1/5)^2 \times (4/5)^2 = 96/625$. At least one= $1-P(\text{none})=1-(4/5)^4=369/625$. The interpretation should include the correct units or geometric/statistical meaning where applicable. Credit a correct validity condition and an explicit check that it holds in this case. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 15

HP8a • Conditional probability from tables

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Given B restricts the denominator to 102: $P(A \text{ given } B) = 40/102 = 20/51$. Given A restricts it to 80: $P(B \text{ given } A) = 40/80 = 1/2$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Given B restricts the denominator to 73: $P(A \text{ given } B) = 43/73 = 43/73$. Given A restricts it to 83: $P(B \text{ given } A) = 43/83 = 43/83$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Given B restricts the denominator to 79: $P(A \text{ given } B) = 46/79 = 46/79$. Given A restricts it to 86: $P(B \text{ given } A) = 46/86 = 23/43$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Given B restricts the denominator to 85: $P(A \text{ given } B) = 27/85 = 27/85$. Given A restricts it to 89: $P(B \text{ given } A) = 27/89 = 27/89$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Given B restricts the denominator to 91: $P(A \text{ given } B) = 30/91 = 30/91$. Given A restricts it to 92: $P(B \text{ given } A) = 30/92 = 15/46$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Given B restricts the denominator to 97: $P(A \text{ given } B) = 33/97 = 33/97$. Given A restricts it to 95: $P(B \text{ given } A) = 33/95 = 33/95$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Given B restricts the denominator to 103: $P(A \text{ given } B) = 36/103 = 36/103$. Given A restricts it to 98: $P(B \text{ given } A) = 36/98 = 18/49$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Given B restricts the denominator to 74: $P(A \text{ given } B) = 39/74 = 39/74$. Given A restricts it to 66: $P(B \text{ given } A) = 39/66 = 13/22$. The intersection numerator is shared, but the restricted group totals differ. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 16

HP8b • Conditional probability from Venn diagrams

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Within B, 35 are also in A, so $P(A \text{ given } B) = 7/12$. The remaining B-only count is 25, giving $P(A \text{ complement given } B) = 5/12$. Within A union B (100 members), 60 are in B, so $P(B \text{ given } A \text{ union } B) = 3/5$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Within B, 38 are also in A, so $P(A \text{ given } B) = 19/33$. The remaining B-only count is 28, giving $P(A \text{ complement given } B) = 14/33$. Within A union B (106 members), 66 are in B, so $P(B \text{ given } A \text{ union } B) = 33/53$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Within B, 21 are also in A, so $P(A \text{ given } B) = 7/24$. The remaining B-only count is 51, giving $P(A \text{ complement given } B) = 17/24$. Within A union B (132 members), 72 are in B, so $P(B \text{ given } A \text{ union } B) = 6/11$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Within B, 24 are also in A, so $P(A \text{ given } B) = 4/13$. The remaining B-only count is 54, giving $P(A \text{ complement given } B) = 9/13$. Within A union B (138 members), 78 are in B, so $P(B \text{ given } A \text{ union } B) = 13/23$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Within B, 27 are also in A, so $P(A \text{ given } B) = 9/28$. The remaining B-only count is 57, giving $P(A \text{ complement given } B) = 19/28$. Within A union B (144 members), 84 are in B, so $P(B \text{ given } A \text{ union } B) = 7/12$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Within B, 30 are also in A, so $P(A \text{ given } B) = 15/31$. The remaining B-only count is 32, giving $P(A \text{ complement given } B) = 16/31$. Within A union B (92 members), 62 are in B, so $P(B \text{ given } A \text{ union } B) = 31/46$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Within B, 33 are also in A, so $P(A \text{ given } B) = 33/68$. The remaining B-only count is 35, giving $P(A \text{ complement given } B) = 35/68$. Within A union B (98 members), 68 are in B, so $P(B \text{ given } A \text{ union } B) = 34/49$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Within B, 36 are also in A, so $P(A \text{ given } B) = 18/37$. The remaining B-only count is 38, giving $P(A \text{ complement given } B) = 19/37$. Within A union B (104 members), 74 are in B, so $P(B \text{ given } A \text{ union } B) = 37/52$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 17

HP8c • Conditional probability from trees

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $3/5 \times 7/10 + 2/5 \times 1/5 = 1/2$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (3/5 \times 7/10) / 1/2 = 21/25$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $1/2 \times 7/10 + 1/2 \times 1/10 = 2/5$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (1/2 \times 7/10) / 2/5 = 7/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $2/5 \times 7/10 + 3/5 \times 1/5 = 2/5$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (2/5 \times 7/10) / 2/5 = 7/10$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $3/10 \times 7/10 + 7/10 \times 1/10 = 7/25$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (3/10 \times 7/10) / 7/25 = 3/4$. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $3/5 \times 7/10 + 2/5 \times 1/5 = 1/2$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (3/5 \times 7/10) / 1/2 = 21/25$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $1/2 \times 7/10 + 1/2 \times 1/10 = 2/5$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (1/2 \times 7/10) / 2/5 = 7/10$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $2/5 \times 7/10 + 3/5 \times 1/5 = 2/5$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (2/5 \times 7/10) / 2/5 = 7/10$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Positive paths are condition-and-positive plus no-condition-and-positive: $3/10 \times 7/10 + 7/10 \times 1/10 = 7/25$. Restricting to positive outcomes gives $P(\text{condition given positive}) = (3/10 \times 7/10) / 7/25 = 3/4$. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

Worked answers • Chapter 18

HP9d • Reverse and algebraic probability

Independent teaching solutions; equivalent valid methods should receive credit.

Q1 • 4

Total probability gives $p(3/5) + (1-p)(2/5) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q01.

Q2 • 4

Total probability gives $p(3/4) + (1-p)(1/4) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q02.

Q3 • 4

Total probability gives $p(2/3) + (1-p)(1/3) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q03.

Q4 • 4

Total probability gives $p(7/9) + (1-p)(2/9) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q04.

Q5 • 4

Total probability gives $p(5/7) + (1-p)(2/7) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q05.

Q6 • 4

Total probability gives $p(3/5) + (1-p)(2/5) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q06.

Q7 • 4

Total probability gives $p(3/4) + (1-p)(1/4) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q07.

Q8 • 4

Total probability gives $p(2/3) + (1-p)(1/3) = 1/2$. Expanding and solving gives $p = 1/2$. Since $0 \leq 1/2 \leq 1$ and substitution recreates $1/2$, it is valid. A valid independent check uses substitution, an inverse operation, estimation, or the defining property as appropriate. Item-specific marking check Credit the principal method and accurate progress. Reserve any reasoning or checking mark for a statement that answers the explicit demand of Q08.

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