

GCSE ONLINE TUTORING × RS REMOTE TUTORING

MATHEMATICS HIGHER

HIGHER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 1: NUMBER

Mapped independently to Qualifications Scotland Higher Mathematics C847 76 • 16 detailed chapters
concept teaching • worked methods • reasoning • guided practice • independent practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

EXAM-BOARD INDEPENDENCE

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COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Reasoning throughout.

1 • UNDERSTAND

Read the definition and big idea. Redraw or annotate the representation, then answer the checkpoint without looking back.

2 • BUILD THE METHOD

Cover each next line of the example. Predict it, reveal it and explain why the transformation or calculation is permitted.

3 • CONNECT AND REASON

Use the misconception analysis and guided task to decide which method fits, not merely how to execute it.

4 • PRACTISE INDEPENDENTLY

Show the decisive setup, accurate progress and a supported conclusion. Do not replace mathematical communication with a calculator display.

5 • DIAGNOSE AND REVISIT

Mark from the worked answer. Record whether an error came from interpretation, structure, arithmetic, accuracy, unit or communication.

COMPREHENSIVE LESSON NOTEBOOK

Number unit map

16 chapters mapped independently to the current course specification.

CH 01	H-N1	Exact forms and surd fluency
CH 02	H-N2	Advanced rationalising
CH 03	H-N3	Indices and exponential structure
CH 04	H-N4	Exponential functions
CH 05	H-N5	Logarithms as inverse functions
CH 06	H-N6	Laws of logarithms
CH 07	H-N7	Exponential equations
CH 08	H-N8	Logarithmic equations
CH 09	H-N9	Finding exponential models
CH 10	H-N10	Power models
CH 11	H-N11	Straight-line transformations
CH 12	H-N12	Exponential growth and decay
CH 13	H-N13	Logarithmic scales and interpretation
CH 14	H-N14	Parameter reconstruction
CH 15	H-N15	Exact numerical proof
CH 16	H-N16	Integrated Higher reasoning

COMPREHENSIVE LESSON NOTEBOOK

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COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Exact forms and surd fluency

Manipulate exact numerical forms confidently before approximation.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N1. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Manipulate exact numerical forms confidently before approximation. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Simplify $2\sqrt{27}-\sqrt{12}$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Build the method

Exact forms and surd fluency

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Simplify $2\sqrt{27}-\sqrt{12}$.

WORKED REASONING

1. $2\sqrt{27}=6\sqrt{3}$.
2. $\sqrt{12}=2\sqrt{3}$.

Conclusion: $4\sqrt{3}$.

WORKED EXAMPLE 2

Simplify $\sqrt{6}\sqrt{15}$.

WORKED REASONING

1. $\sqrt{90}=\sqrt{9\times 10}$.

Conclusion: $3\sqrt{10}$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Connect and reason

Exact forms and surd fluency

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N1, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Show that $(\sqrt{5}+1)(\sqrt{5}-1)$ is rational. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding exact forms and surd fluency.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N1 • Exact forms and surd fluency

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Simplify $2\sqrt{27}-\sqrt{12}$.

[4]

2. Simplify $\sqrt{6}\sqrt{15}$.

[4]

3. Show that $(\sqrt{5}+1)(\sqrt{5}-1)$ is rational.

[4]

4. Simplify $\sqrt{48}/\sqrt{3}$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N1 • Exact forms and surd fluency

1. WORKED ANSWER

Method: $2\sqrt{27}=6\sqrt{3}$. $\sqrt{12}=2\sqrt{3}$. Answer: $4\sqrt{3}$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: $\sqrt{90}=\sqrt{9 \times 10}$. Answer: $3\sqrt{10}$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Use difference of two squares: $5-1$. Answer: 4. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Combine under one root: $\sqrt{16}$. Answer: 4. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Advanced rationalising

Rationalise monomial and binomial surd denominators using conjugates.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N2. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Rationalise monomial and binomial surd denominators using conjugates. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Rationalise $4/(3\sqrt{2})$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Build the method

Advanced rationalising

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Rationalise $4/(3\sqrt{2})$.

WORKED REASONING

1. Multiply by $\sqrt{2}/\sqrt{2}$.

Conclusion: $2\sqrt{2}/3$.

WORKED EXAMPLE 2

Rationalise $5/(\sqrt{3}+1)$.

WORKED REASONING

1. Multiply by the conjugate.
2. The denominator is $3-1=2$.

Conclusion: $5(\sqrt{3}-1)/2$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Connect and reason

Advanced rationalising

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N2, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Rationalise $2/(3-\sqrt{5})$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding advanced rationalising.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N2 • Advanced rationalising

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Rationalise $4/(3\sqrt{2})$.

[4]

2. Rationalise $5/(\sqrt{3}+1)$.

[4]

3. Rationalise $2/(3-\sqrt{5})$.

[4]

4. Simplify $(\sqrt{7}+\sqrt{2})/(\sqrt{7}-\sqrt{2})$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N2 • Advanced rationalising

1. WORKED ANSWER

Method: Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$. Answer: $\frac{2\sqrt{2}}{3}$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Multiply by the conjugate. The denominator is $3-1=2$. Answer: $\frac{5(\sqrt{3}-1)}{2}$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Multiply by $3+\sqrt{5}$. The denominator is $9-5=4$, then simplify. Answer: $\frac{(3+\sqrt{5})}{2}$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Multiply by the conjugate of the denominator. Expand and collect exactly. Answer: $\frac{(9+2\sqrt{14})}{5}$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Indices and exponential structure

Use rational indices and exponent laws to transform numerical expressions.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N3. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use rational indices and exponent laws to transform numerical expressions. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Simplify $32^{(3/5)}$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Build the method

Indices and exponential structure

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Simplify $32^{(3/5)}$.

WORKED REASONING

1. The fifth root of 32 is 2; cube it.

Conclusion: 8.

WORKED EXAMPLE 2

Evaluate $81^{(-3/4)}$.

WORKED REASONING

1. The fourth root is 3; cube gives 27; invert.

Conclusion: $1/27$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Connect and reason

Indices and exponential structure

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N3, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Simplify $(x^{(3/2)}x^{(5/2)})/x$, $x > 0$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding indices and exponential structure.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N3 • Indices and exponential structure

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Simplify $32^{(3/5)}$.

[4]

2. Evaluate $81^{(-3/4)}$.

[4]

3. Simplify $(x^{(3/2)}x^{(5/2)})/x$, $x > 0$.

[4]

4. Solve $2^{(x+1)}=32$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N3 • Indices and exponential structure

1. WORKED ANSWER

Method: The fifth root of 32 is 2; cube it. Answer: 8. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: The fourth root is 3; cube gives 27; invert. Answer: $1/27$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Add indices then subtract 1. Answer: x^3 . Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Write 32 as 2^5 . Equate indices. Answer: $x=4$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Exponential functions

Interpret $y=ab^x$, including initial value, multiplier and long-term behaviour.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N4. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Interpret $y=ab^x$, including initial value, multiplier and long-term behaviour. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “For $y=5(1.2)^x$, state y-intercept and growth factor.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Build the method

Exponential functions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

For $y=5(1.2)^x$, state y-intercept and growth factor.

WORKED REASONING

1. Set $x=0$.
2. Read the base multiplier.

Conclusion: Intercept 5; growth factor 1.2.

WORKED EXAMPLE 2

Calculate $y=800(0.76)^5$.

WORKED REASONING

1. Substitute $x=5$ and retain full accuracy.

Conclusion: 202.84 approximately.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Connect and reason

Exponential functions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N4, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Explain why $3(0.4)^x$ represents decay. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding exponential functions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N4 • Exponential functions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. For $y=5(1.2)^x$, state y-intercept and growth factor.

[4]

2. Calculate $y=800(0.76)^5$.

[4]

3. Explain why $3(0.4)^x$ represents decay.

[4]

4. Find x when $7(2^x)=56$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N4 • Exponential functions

1. WORKED ANSWER

Method: Set $x=0$. Read the base multiplier. Answer: Intercept 5; growth factor 1.2. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Substitute $x=5$ and retain full accuracy. Answer: 202.84 approximately. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Each unit increase multiplies y by 0.4. Answer: Its positive multiplier 0.4 lies between 0 and 1. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Divide by 7 to get $2^x=8$. Answer: $x=3$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Logarithms as inverse functions

Translate exactly between exponential and logarithmic notation.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N5. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Translate exactly between exponential and logarithmic notation. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Rewrite $2^5=32$ in logarithmic form.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Build the method

Logarithms as inverse functions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Rewrite $2^5=32$ in logarithmic form.

WORKED REASONING

1. The logarithm is the exponent.

Conclusion: $\log_2(32)=5$.

WORKED EXAMPLE 2

Evaluate $\log_3(1/27)$.

WORKED REASONING

1. $1/27=3^{-3}$.

Conclusion: -3.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Connect and reason

Logarithms as inverse functions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N5, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $\log_5(x)=3$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding logarithms as inverse functions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N5 • Logarithms as inverse functions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Rewrite $2^5=32$ in logarithmic form.

[4]

2. Evaluate $\log_3(1/27)$.

[4]

3. Solve $\log_5(x)=3$.

[4]

4. State the domain of $y=\log_2(x)$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N5 • Logarithms as inverse functions

1. WORKED ANSWER

Method: The logarithm is the exponent. Answer: $\log_2(32)=5$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: $1/27=3^{-3}$. Answer: -3. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Rewrite as $x=5^3$. Answer: $x=125$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: A real logarithm requires a positive argument. Answer: $x>0$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Laws of logarithms

Combine and expand logarithmic expressions using product, quotient and power laws.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N6. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Combine and expand logarithmic expressions using product, quotient and power laws. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Simplify $\log(a)+\log(b)$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Build the method

Laws of logarithms

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Simplify $\log(a) + \log(b)$.

WORKED REASONING

1. Use the product law.

Conclusion: $\log(ab)$, for $a > 0$ and $b > 0$.

WORKED EXAMPLE 2

Simplify $2\log(x) - \log(y)$.

WORKED REASONING

1. Use the power law then quotient law.

Conclusion: $\log(x^2/y)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Connect and reason

Laws of logarithms

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N6, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Expand $\log((a^3b)/c)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding laws of logarithms.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N6 • Laws of logarithms

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Simplify $\log(a) + \log(b)$.

[4]

2. Simplify $2\log(x) - \log(y)$.

[4]

3. Expand $\log(a^3b/c)$.

[4]

4. Given $\log(2)=p$ and $\log(3)=q$, express $\log(72)$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N6 • Laws of logarithms

1. WORKED ANSWER

Method: Use the product law. Answer: $\log(ab)$, for $a > 0$ and $b > 0$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Use the power law then quotient law. Answer: $\log(x^2/y)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Apply quotient, product and power laws. Answer: $3\log(a) + \log(b) - \log(c)$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: $72 = 2^3 \times 3^2$. Answer: $3p + 2q$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Exponential equations

Solve equations by matching bases or taking logarithms.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N7. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Solve equations by matching bases or taking logarithms. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $3^{(2x-1)}=27$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Build the method

Exponential equations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Solve $3^{(2x-1)}=27$.

WORKED REASONING

1. Write 27 as 3^3 .
2. Equate $2x-1=3$.

Conclusion: $x=2$.

WORKED EXAMPLE 2

Solve $5^x=18$, giving x to 3 significant figures.

WORKED REASONING

1. Take logs: $x=\log(18)/\log(5)$.

Conclusion: $x=1.80$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Connect and reason

Exponential equations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N7, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $2e^{(0.3t)}=11$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding exponential equations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N7 • Exponential equations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $3^{(2x-1)}=27$.

[4]

2. Solve $5^x=18$, giving x to 3 significant figures.

[4]

3. Solve $2e^{(0.3t)}=11$.

[4]

4. Solve $4^x=2^{(x+6)}$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N7 • Exponential equations

1. WORKED ANSWER

Method: Write 27 as 3^3 . Equate $2x-1=3$. Answer: $x=2$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Take logs: $x=\log(18)/\log(5)$. Answer: $x=1.80$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Divide by 2, take natural logs, then divide by 0.3. Answer: $t=\ln(5.5)/0.3$, approximately 5.68. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Write $4^x=2^{(2x)}$. Equate indices. Answer: $x=6$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Logarithmic equations

Use log laws, domain restrictions and inverse operations to solve equations.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N8. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use log laws, domain restrictions and inverse operations to solve equations. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $\log_2(x)=6$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Build the method

Logarithmic equations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Solve $\log_2(x)=6$.

WORKED REASONING

1. Rewrite in exponential form.

Conclusion: $x=64$.

WORKED EXAMPLE 2

Solve $\log(x)+\log(x-9)=1$, base 10.

WORKED REASONING

1. Combine to $\log(x(x-9))=1$.
2. Solve $x^2-9x-10=0$ and reject $x=-1$.

Conclusion: $x=10$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Connect and reason

Logarithmic equations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N8, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $\ln(3x)=2$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding logarithmic equations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N8 • Logarithmic equations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $\log_2(x)=6$.

[4]

2. Solve $\log(x)+\log(x-9)=1$, base 10.

[4]

3. Solve $\ln(3x)=2$.

[4]

4. Solve $\log_3(x+1)-\log_3(x-1)=1$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N8 • Logarithmic equations

1. WORKED ANSWER

Method: Rewrite in exponential form. Answer: $x=64$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Combine to $\log(x(x-9))=1$. Solve $x^2-9x-10=0$ and reject $x=-1$. Answer: $x=10$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Use the inverse exponential. Answer: $x=e^{2/3}$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Combine as $\log_3((x+1)/(x-1))=1$. Solve $\text{ratio}=3$ and check $x>1$. Answer: $x=2$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Finding exponential models

Determine a and b in $y=ab^x$ from data or points.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N9. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Determine a and b in $y=ab^x$ from data or points. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “An exponential model $y=ab^x$ passes through $(0,6)$ and $(2,24)$. Find positive a and b .”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Build the method

Finding exponential models

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

An exponential model $y=ab^x$ passes through (0,6) and (2,24). Find positive a and b .

WORKED REASONING

1. At $x=0$, $a=6$.
2. Then $24=6b^2$.

Conclusion: $a=6$, $b=2$.

WORKED EXAMPLE 2

Values are $y=80$ at $x=1$ and $y=20$ at $x=3$. Find $y=ab^x$ with $b>0$.

WORKED REASONING

1. Divide equations to get $b^2=1/4$.
2. Use $b=1/2$ and $a=160$.

Conclusion: $y=160(0.5)^x$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Connect and reason

Finding exponential models

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N9, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A population is 1200 and grows 7% per year. Write a model after t years. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding finding exponential models.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N9 • Finding exponential models

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. An exponential model $y=ab^x$ passes through (0,6) and (2,24). Find positive a and b .

[4]

2. Values are $y=80$ at $x=1$ and $y=20$ at $x=3$. Find $y=ab^x$ with $b>0$.

[4]

3. A population is 1200 and grows 7% per year. Write a model after t years.

[4]

4. For $y=450(0.82)^t$, interpret 450 and 0.82.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N9 • Finding exponential models

1. WORKED ANSWER

Method: At $x=0$, $a=6$. Then $24=6b^2$. Answer: $a=6$, $b=2$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Divide equations to get $b^2=1/4$. Use $b=1/2$ and $a=160$. Answer: $y=160(0.5)^x$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Initial value is 1200 and multiplier 1.07. Answer: $P=1200(1.07)^t$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Set $t=0$ and convert retention to decrease. Answer: 450 is the initial value; 0.82 means 18% decay per unit time. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Power models

Use logarithms to identify and fit relationships of the form $y=ax^b$.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N10. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use logarithms to identify and fit relationships of the form $y=ax^b$. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “If $y=ax^b$ and $y=12$ when $x=2$, $y=108$ when $x=6$, find b .”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Build the method

Power models

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

If $y = ax^b$ and $y = 12$ when $x = 2$, $y = 108$ when $x = 6$, find b .

WORKED REASONING

1. Divide: $9 = (6/2)^b = 3^b$.

Conclusion: $b = 2$.

WORKED EXAMPLE 2

Continue the previous model to find a .

WORKED REASONING

1. $12 = a(2^2)$, so $a = 3$.

Conclusion: $a = 3$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Connect and reason

Power models

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N10, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Linearise $y=4x^{1.7}$ using logarithms. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding power models.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N10 • Power models

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. If $y=ax^b$ and $y=12$ when $x=2$, $y=108$ when $x=6$, find b .

[4]

2. Continue the previous model to find a .

[4]

3. Linearise $y=4x^{1.7}$ using logarithms.

[4]

4. A log-log plot has gradient -2 and intercept $\log 5$. State the model.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N10 • Power models

1. WORKED ANSWER

Method: Divide: $9 = (6/2)^b = 3^b$. Answer: $b=2$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: $12 = a(2^2)$, so $a=3$. Answer: $a=3$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Take logs and use the power law. Answer: $\log y = \log 4 + 1.7 \log x$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Gradient gives the power and intercept gives $\log a$. Answer: $y = 5x^{-2}$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Straight-line transformations

Transform exponential and power relationships into linear form and interpret gradient and intercept.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N11. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Transform exponential and power relationships into linear form and interpret gradient and intercept. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “For $y=ab^x$, state a linear graph using common logs.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Build the method

Straight-line transformations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

For $y=ab^x$, state a linear graph using common logs.

WORKED REASONING

1. Take logs of both sides.

Conclusion: Plot $\log y$ against x : $\log y = \log a + x \log b$.

WORKED EXAMPLE 2

A plot of $\log y$ against x has intercept 0.6 and gradient 0.2. Express a and b .

WORKED REASONING

1. Compare with $\log y = \log a + x \log b$.

Conclusion: $a=10^{0.6}$, $b=10^{0.2}$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Connect and reason

Straight-line transformations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N11, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

For $y=ax^b$, which quantities should be plotted for a straight line? Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding straight-line transformations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N11 • Straight-line transformations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. For $y=ab^x$, state a linear graph using common logs. [4]

2. A plot of $\log y$ against x has intercept 0.6 and gradient 0.2. Express a and b . [4]

3. For $y=ax^b$, which quantities should be plotted for a straight line? [4]

4. A log-log line has equation $Y=1.3+2.4X$. State y in terms of x . [4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N11 • Straight-line transformations

1. WORKED ANSWER

Method: Take logs of both sides. Answer: Plot $\log y$ against x : $\log y = \log a + x \log b$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Compare with $\log y = \log a + x \log b$. Answer: $a = 10^{0.6}$, $b = 10^{0.2}$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Then $\log y = \log a + b \log x$. Answer: $\log y$ against $\log x$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Let $Y = \log y$ and $X = \log x$, then invert the logs. Answer: $y = 10^{1.3} x^{2.4}$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Exponential growth and decay

Solve contextual growth and decay problems and interpret parameter values.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N12. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Solve contextual growth and decay problems and interpret parameter values. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “A culture follows $N=900e^{(0.18t)}$. Find N at $t=6$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Build the method

Exponential growth and decay

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

A culture follows $N=900e^{(0.18t)}$. Find N at $t=6$.

WORKED REASONING

1. Substitute $t=6$ and evaluate $900e^{1.08}$.

Conclusion: Approximately 2650.

WORKED EXAMPLE 2

For $N=900e^{(0.18t)}$, find when N first reaches 5000.

WORKED REASONING

1. Divide, take natural logs, then solve.

Conclusion: $t=\ln(5000/900)/0.18$, approximately 9.53.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Connect and reason

Exponential growth and decay

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N12, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A medicine amount follows $A = 75e^{(-0.12t)}$. State the continuous decay rate parameter. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding exponential growth and decay.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N12 • Exponential growth and decay

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. A culture follows $N=900e^{(0.18t)}$. Find N at $t=6$.

[4]

2. For $N=900e^{(0.18t)}$, find when N first reaches 5000.

[4]

3. A medicine amount follows $A=75e^{(-0.12t)}$. State the continuous decay rate parameter.

[4]

4. Find the half-life in $A=A_0e^{(-0.12t)}$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N12 • Exponential growth and decay

1. WORKED ANSWER

Method: Substitute $t=6$ and evaluate $900e^{1.08}$. Answer: Approximately 2650. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Divide, take natural logs, then solve. Answer: $t=\ln(5000/900)/0.18$, approximately 9.53. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Read the coefficient of t in the exponent. Answer: -0.12 per hour. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Set $A/A_0=1/2$ and solve. Answer: $\ln 2/0.12$, approximately 5.78 hours. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Logarithmic scales and interpretation

Use logarithmic relationships to compare multiplicative changes.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N13. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use logarithmic relationships to compare multiplicative changes. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “If $L=10\log_{10}(I/I_0)$, how much does L change when I is multiplied by 100?”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Build the method

Logarithmic scales and interpretation

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

If $L = 10 \log_{10}(I/I_0)$, how much does L change when I is multiplied by 100?

WORKED REASONING

1. $\log(100I/I_0) = 2 + \log(I/I_0)$.
2. Multiply the added 2 by 10.

Conclusion: It increases by 20.

WORKED EXAMPLE 2

Solve $\text{pH} = -\log_{10}(H)$ when $\text{pH} = 3.4$.

WORKED REASONING

1. Undo the negative logarithm.

Conclusion: $H = 10^{-3.4}$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Connect and reason

Logarithmic scales and interpretation

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N13, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Two values differ by 0.7 on a base-10 logarithmic scale. Find their ratio. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding logarithmic scales and interpretation.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N13 • Logarithmic scales and interpretation

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. If $L = 10 \log_{10}(I/I_0)$, how much does L change when I is multiplied by 100?

[4]

2. Solve $pH = -\log_{10}(H)$ when $pH = 3.4$.

[4]

3. Two values differ by 0.7 on a base-10 logarithmic scale. Find their ratio.

[4]

4. Explain why equal steps on a log scale represent equal ratios.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N13 • Logarithmic scales and interpretation

1. WORKED ANSWER

Method: $\log(100/I_0) = 2 + \log(I/I_0)$. Multiply the added 2 by 10. Answer: It increases by 20. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Undo the negative logarithm. Answer: $H = 10^{-3.4}$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: A difference of logs is the log of a ratio. Answer: $10^{0.7}$, approximately 5.01. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Use $\log(kx) = \log x + \log k$. Answer: Adding a constant to a logarithm multiplies the original value by a fixed factor. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Parameter reconstruction

Recover unknown starting values, rates or times from an exponential model.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N14. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Recover unknown starting values, rates or times from an exponential model. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “A value becomes 1458 after 4 years of 5% growth. Find the initial value.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Build the method

Parameter reconstruction

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

A value becomes 1458 after 4 years of 5% growth. Find the initial value.

WORKED REASONING

1. Reverse repeated growth by dividing by the multiplier power.

Conclusion: $1458/(1.05)^4$, approximately 1199.50.

WORKED EXAMPLE 2

A population doubles in 9 years under $P=P_0e^{(kt)}$. Find k .

WORKED REASONING

1. Set $2=e^{(9k)}$ and take logs.

Conclusion: $k=\ln 2/9$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Connect and reason

Parameter reconstruction

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N14, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A machine retains 68% of its value after 5 years. Find the annual retention factor. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding parameter reconstruction.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N14 • Parameter reconstruction

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. A value becomes 1458 after 4 years of 5% growth. Find the initial value.

[4]

2. A population doubles in 9 years under $P = P_0 e^{(kt)}$. Find k .

[4]

3. A machine retains 68% of its value after 5 years. Find the annual retention factor.

[4]

4. A quantity follows $y = 240(1.08)^t$. Find t when $y = 500$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N14 • Parameter reconstruction

1. WORKED ANSWER

Method: Reverse repeated growth by dividing by the multiplier power. Answer: $1458/(1.05)^4$, approximately 1199.50. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Set $2=e^{(9k)}$ and take logs. Answer: $k=\ln 2/9$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: If the annual factor is b , then $b^5=0.68$. Answer: $0.68^{(1/5)}$, approximately 0.9258. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Isolate the exponential then take logs. Answer: $t=\log(500/240)/\log(1.08)$, approximately 9.54. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Exact numerical proof

Construct concise proofs and counterexamples involving powers, surds and logarithms.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N15. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Construct concise proofs and counterexamples involving powers, surds and logarithms. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Prove that $\sqrt{12} + \sqrt{27}$ is irrational.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Build the method

Exact numerical proof

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Prove that $\sqrt{12} + \sqrt{27}$ is irrational.

WORKED REASONING

1. Simplify each surd.
2. A non-zero rational multiple of irrational $\sqrt{3}$ is irrational.

Conclusion: It simplifies to $5\sqrt{3}$, which is irrational.

WORKED EXAMPLE 2

Show that $\log_a(x^3) - \log_a(x) = 2\log_a(x)$, for $a > 0$, a not equal to 1, $x > 0$.

WORKED REASONING

1. State the domain conditions.

Conclusion: Use the quotient law: $\log_a(x^2) = 2\log_a(x)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Connect and reason

Exact numerical proof

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N15, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Disprove: $a^m + a^n = a^{(m+n)}$ for all positive a . Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding exact numerical proof.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N15 • Exact numerical proof

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Prove that $\sqrt{12} + \sqrt{27}$ is irrational.

[4]

2. Show that $\log_a(x^3) - \log_a(x) = 2\log_a(x)$, for $a > 0$, a not equal to 1, $x > 0$.

[4]

3. Disprove: $a^m + a^n = a^{(m+n)}$ for all positive a .

[4]

4. Show that $\frac{1}{\sqrt{5}-2} = \sqrt{5}+2$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N15 • Exact numerical proof

1. WORKED ANSWER

Method: Simplify each surd. A non-zero rational multiple of irrational $\sqrt{3}$ is irrational. Answer: It simplifies to $5\sqrt{3}$, which is irrational. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: State the domain conditions. Answer: Use the quotient law: $\log_a(x^2) = 2\log_a(x)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: One counterexample disproves a universal claim. Answer: For example, $2^1 + 2^1 = 4$ but $2^{(1+1)} = 4$ happens to match; use $2^2 + 2^3 = 12$ while $2^5 = 32$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: The denominator becomes $5 - 4 = 1$. Answer: Multiply by the conjugate. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Integrated Higher reasoning

Connect exact form, modelling and logarithms in multi-stage unfamiliar problems.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-N16. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Connect exact form, modelling and logarithms in multi-stage unfamiliar problems. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher course progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

exact value; approximation; representation; condition; inverse operation; reasonableness; mathematical communication

Three controlling ideas

- Name the mathematical structure and all domain or accuracy conditions.
- Transform one justified step at a time; keep exact values until approximation is required.
- Verify by inverse operation, estimation, substitution, graph behaviour or units.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “A quantity obeys $y=ab^t$. It is 120 at $t=2$ and 405 at $t=5$. Find a and b .”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Build the method

Integrated Higher reasoning

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

A quantity obeys $y=ab^t$. It is 120 at $t=2$ and 405 at $t=5$. Find a and b .

WORKED REASONING

1. Divide to get $b^3=405/120=27/8$.
2. Then $a=120/1.5^2=160/3$.

Conclusion: $b=1.5$ and $a=160/3$.

WORKED EXAMPLE 2

Solve $3(2^x)=17$, giving x to 3 significant figures.

WORKED REASONING

1. Divide by 3, take logarithms and use the power law.
2. Round only after evaluating the full expression.

Conclusion: $x=\log(17/3)/\log(2)$, approximately 2.50.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Connect and reason

Integrated Higher reasoning

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-N16, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A model $y=800e^{(-0.15t)}$ is measured when $y=250$. Find t and give a reasoned 3 s.f. answer. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding integrated higher reasoning.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-N16 • Integrated Higher reasoning

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. A quantity obeys $y=ab^t$. It is 120 at $t=2$ and 405 at $t=5$. Find a and b .

[4]

2. Solve $3(2^x)=17$, giving x to 3 significant figures.

[4]

3. A model $y=800e^{(-0.15t)}$ is measured when $y=250$. Find t and give a reasoned 3 s.f. answer.

[4]

4. Simplify exactly $\log_2(48)-\log_2(3)+\log_2(2^{-2})$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-N16 • Integrated Higher reasoning

1. WORKED ANSWER

Method: Divide to get $b^3 = 405/120 = 27/8$. Then $a = 120/1.5^2 = 160/3$. Answer: $b = 1.5$ and $a = 160/3$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Divide by 3, take logarithms and use the power law. Round only after evaluating the full expression. Answer: $x = \log(17/3)/\log(2)$, approximately 2.50. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Isolate the exponential and use logs. Round only at the end. Answer: $t = \ln(800/250)/0.15$, approximately 7.75. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Combine to $\log_2((48/3) \times 1/4) = \log_2(4)$. Answer: 2. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Number mastery review

Explain, demonstrate, connect and revisit.

I can...

- H-N1 Exact forms and surd fluency: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N2 Advanced rationalising: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N3 Indices and exponential structure: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N4 Exponential functions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N5 Logarithms as inverse functions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N6 Laws of logarithms: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N7 Exponential equations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N8 Logarithmic equations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N9 Finding exponential models: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N10 Power models: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N11 Straight-line transformations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N12 Exponential growth and decay: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N13 Logarithmic scales and interpretation: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N14 Parameter reconstruction: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N15 Exact numerical proof: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-N16 Integrated Higher reasoning: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.

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