

GCSE ONLINE TUTORING × RS REMOTE TUTORING

MATHEMATICS HIGHER

HIGHER

COMPREHENSIVE LESSON NOTEBOOK

UNIT 4: CALCULUS

Mapped independently to Qualifications Scotland Higher Mathematics C847 76 • 16 detailed chapters
concept teaching • worked methods • reasoning • guided practice • independent practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

EXAM-BOARD INDEPENDENCE

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COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Reasoning throughout.

1 • UNDERSTAND

Read the definition and big idea. Redraw or annotate the representation, then answer the checkpoint without looking back.

2 • BUILD THE METHOD

Cover each next line of the example. Predict it, reveal it and explain why the transformation or calculation is permitted.

3 • CONNECT AND REASON

Use the misconception analysis and guided task to decide which method fits, not merely how to execute it.

4 • PRACTISE INDEPENDENTLY

Show the decisive setup, accurate progress and a supported conclusion. Do not replace mathematical communication with a calculator display.

5 • DIAGNOSE AND REVISIT

Mark from the worked answer. Record whether an error came from interpretation, structure, arithmetic, accuracy, unit or communication.

COMPREHENSIVE LESSON NOTEBOOK

Calculus unit map

16 chapters mapped independently to the current course specification.

CH 01	H-C1	Differentiating powers
CH 02	H-C2	Differentiating sine and cosine
CH 03	H-C3	Chain rule
CH 04	H-C4	Tangents and normals
CH 05	H-C5	Increasing and decreasing functions
CH 06	H-C6	Stationary points and their nature
CH 07	H-C7	Sketching algebraic functions
CH 08	H-C8	Optimisation
CH 09	H-C9	Rates of change
CH 10	H-C10	Integrating powers
CH 11	H-C11	Integrating linear composites
CH 12	H-C12	Definite integrals
CH 13	H-C13	Areas under and between curves
CH 14	H-C14	Differential equations and initial conditions
CH 15	H-C15	Greatest and least values on closed intervals
CH 16	H-C16	Integrated Higher calculus

COMPREHENSIVE LESSON NOTEBOOK

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COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Differentiating powers

Differentiate algebraic functions expressed as powers of x .

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C1. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Differentiate algebraic functions expressed as powers of x . The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Differentiate $f(x)=5x^4-3x^2+7$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Build the method

Differentiating powers

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Differentiate $f(x)=5x^4-3x^2+7$.

WORKED REASONING

1. Use $d/dx(x^n)=nx^{(n-1)}$; constants differentiate to zero.

Conclusion: $f'(x)=20x^3-6x$.

WORKED EXAMPLE 2

Differentiate $y=4\sqrt{x}+3/x$.

WORKED REASONING

1. Rewrite using powers $x^{(1/2)}$ and x^{-1} .

Conclusion: $dy/dx=2x^{(-1/2)}-3x^{(-2)}$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Connect and reason

Differentiating powers

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C1, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find $f'(2)$ for $f(x)=x^3-4x$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding differentiating powers.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C1 • Differentiating powers

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Differentiate $f(x)=5x^4-3x^2+7$.

[4]

2. Differentiate $y=4\sqrt{x}+3/x$.

[4]

3. Find $f'(2)$ for $f(x)=x^3-4x$.

[4]

4. Find where $y=x^2-6x$ has gradient 4.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C1 • Differentiating powers

1. WORKED ANSWER

Method: Use $\frac{d}{dx}(x^n) = nx^{(n-1)}$; constants differentiate to zero. Answer: $f'(x) = 20x^3 - 6x$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Rewrite using powers $x^{(1/2)}$ and x^{-1} . Answer: $\frac{dy}{dx} = 2x^{(-1/2)} - 3x^{(-2)}$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: $f'(x) = 3x^{2-4}$; substitute $x=2$. Answer: 8. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: $\frac{dy}{dx} = 2x - 6$; set equal to 4. Answer: $x=5$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Differentiating sine and cosine

Differentiate $k \sin x$ and $k \cos x$ in radians.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C2. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Differentiate $k \sin x$ and $k \cos x$ in radians. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Differentiate $y=7\sin x$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Build the method

Differentiating sine and cosine

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Differentiate $y=7\sin x$.

WORKED REASONING

1. Use radians.

Conclusion: $\frac{dy}{dx}=7\cos x$.

WORKED EXAMPLE 2

Differentiate $y=5\cos x-2\sin x$.

WORKED REASONING

1. Differentiate term by term.

Conclusion: $\frac{dy}{dx}=-5\sin x-2\cos x$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Connect and reason

Differentiating sine and cosine

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C2, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find the gradient of $y=3\sin x$ at $x=\pi/3$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding differentiating sine and cosine.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C2 • Differentiating sine and cosine

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Differentiate $y=7\sin x$.

[4]

2. Differentiate $y=5\cos x-2\sin x$.

[4]

3. Find the gradient of $y=3\sin x$ at $x=\pi/3$.

[4]

4. Solve $dy/dx=0$ for $y=\cos x$ on $0\leq x\leq 2\pi$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C2 • Differentiating sine and cosine

1. WORKED ANSWER

Method: Use radians. Answer: $dy/dx = 7\cos x$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Differentiate term by term. Answer: $dy/dx = -5\sin x - 2\cos x$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: $dy/dx = 3\cos x$; $\cos(\pi/3) = 1/2$. Answer: $3/2$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: $dy/dx = -\sin x$, so solve $\sin x = 0$ on the stated interval. Answer: $x = 0, \pi$ and 2π . Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Chain rule

Differentiate composite algebraic and trigonometric functions.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C3. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Differentiate composite algebraic and trigonometric functions. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Differentiate $(3x-2)^5$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Build the method

Chain rule

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Differentiate $(3x-2)^5$.

WORKED REASONING

1. Differentiate the outer power and multiply by derivative 3.

Conclusion: $15(3x-2)^4$.

WORKED EXAMPLE 2

Differentiate $\sin(4x+1)$.

WORKED REASONING

1. Apply the chain rule.

Conclusion: $4\cos(4x+1)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Connect and reason

Chain rule

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C3, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Differentiate $\cos(5x-2)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding chain rule.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C3 • Chain rule

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Differentiate $(3x-2)^5$.

[4]

2. Differentiate $\sin(4x+1)$.

[4]

3. Differentiate $\cos(5x-2)$.

[4]

4. Differentiate $(x^2+3)^4$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C3 • Chain rule

1. WORKED ANSWER

Method: Differentiate the outer power and multiply by derivative 3. Answer: $15(3x-2)^4$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Apply the chain rule. Answer: $4\cos(4x+1)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Include the inner derivative 5. Answer: $-5\sin(5x-2)$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Outer derivative $4(\dots)^3$ times inner derivative $2x$. Answer: $8x(x^2+3)^3$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Tangents and normals

Determine equations of tangents and normals using differentiation.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C4. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Determine equations of tangents and normals using differentiation. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Find the tangent to $y=x^2+1$ at $x=2$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Build the method

Tangents and normals

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Find the tangent to $y=x^2+1$ at $x=2$.

WORKED REASONING

1. Point is (2,5); gradient is $2x=4$.
2. Use point-gradient form.

Conclusion: $y=4x-3$.

WORKED EXAMPLE 2

Find the normal to $y=x^3$ at $x=1$.

WORKED REASONING

1. Tangent gradient is 3; normal gradient is $-1/3$.

Conclusion: $y-1=-(1/3)(x-1)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Connect and reason

Tangents and normals

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C4, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find points on $y=x^3-3x$ where the tangent is horizontal. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding tangents and normals.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C4 • Tangents and normals

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Find the tangent to $y=x^2+1$ at $x=2$.

[4]

2. Find the normal to $y=x^3$ at $x=1$.

[4]

3. Find points on $y=x^3-3x$ where the tangent is horizontal.

[4]

4. Find the tangent to $y=2\sin x$ at $x=\pi/6$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C4 • Tangents and normals

1. WORKED ANSWER

Method: Point is (2,5); gradient is $2x=4$. Use point-gradient form. Answer: $y=4x-3$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Tangent gradient is 3; normal gradient is $-1/3$. Answer: $y-1=-(1/3)(x-1)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: $dy/dx=3x^2-3=0$. Answer: (1,-2) and (-1,2). Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: The point is $(\pi/6,1)$. The derivative is $2\cos x$, giving gradient $\sqrt{3}$. Answer: $y-1=\sqrt{3}(x-\pi/6)$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Increasing and decreasing functions

Use derivative signs to determine intervals of increase and decrease.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C5. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use derivative signs to determine intervals of increase and decrease. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “For $f(x)=x^2-4x$, find where f is increasing.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Build the method

Increasing and decreasing functions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

For $f(x)=x^2-4x$, find where f is increasing.

WORKED REASONING

1. $f'(x)=2x-4>0$.

Conclusion: $x>2$.

WORKED EXAMPLE 2

For $f(x)=x^3-3x$, find increasing intervals.

WORKED REASONING

1. $f'(x)=3(x-1)(x+1)>0$ outside the roots.

Conclusion: $x<-1$ or $x>1$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Connect and reason

Increasing and decreasing functions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C5, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find where $f(x) = -x^2 + 6x$ is decreasing. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding increasing and decreasing functions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C5 • Increasing and decreasing functions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. For $f(x)=x^2-4x$, find where f is increasing.

[4]

2. For $f(x)=x^3-3x$, find increasing intervals.

[4]

3. Find where $f(x)=-x^2+6x$ is decreasing.

[4]

4. Explain what $f'(x)<0$ means geometrically.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C5 • Increasing and decreasing functions

1. WORKED ANSWER

Method: $f'(x) = 2x - 4 > 0$. Answer: $x > 2$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: $f'(x) = 3(x-1)(x+1) > 0$ outside the roots. Answer: $x < -1$ or $x > 1$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: $f'(x) = -2x + 6 < 0$. Answer: $x > 3$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: The function is strictly decreasing on that interval. Answer: The graph slopes downward as x increases. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Stationary points and their nature

Find and classify maxima, minima and stationary points of inflection.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C6. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Find and classify maxima, minima and stationary points of inflection. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Find and classify stationary point of $y=x^2-6x+5$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Build the method

Stationary points and their nature

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Find and classify stationary point of $y=x^2-6x+5$.

WORKED REASONING

1. $dy/dx=2x-6=0$; the gradient changes from negative to positive.

Conclusion: Minimum at (3,-4).

WORKED EXAMPLE 2

Find stationary points of $y=x^3-3x$.

WORKED REASONING

1. Solve $3x^2-3=0$ and test derivative signs.

Conclusion: Maximum at (-1,2); minimum at (1,-2).

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Connect and reason

Stationary points and their nature

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C6, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Classify the stationary point of $y=x^3$ at $x=0$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding stationary points and their nature.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C6 • Stationary points and their nature

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Find and classify stationary point of $y=x^2-6x+5$.

[4]

2. Find stationary points of $y=x^3-3x$.

[4]

3. Classify the stationary point of $y=x^3$ at $x=0$.

[4]

4. Find k if $y=x^2+kx$ has a stationary point at $x=4$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C6 • Stationary points and their nature

1. WORKED ANSWER

Method: $\frac{dy}{dx} = 2x - 6 = 0$; the gradient changes from negative to positive. Answer: Minimum at (3,-4). Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Solve $3x^2 - 3 = 0$ and test derivative signs. Answer: Maximum at (-1,2); minimum at (1,-2). Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: The derivative is non-negative on both sides, so there is no maximum or minimum. Answer: Stationary point of inflection. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: $\frac{dy}{dx} = 2x + k$; substitute $x = 4$ and set to zero. Answer: $k = -8$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Sketching algebraic functions

Combine intercepts, stationary points and end behaviour to sketch curves.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C7. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Combine intercepts, stationary points and end behaviour to sketch curves. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “For $y=x^3-3x$, state all intercepts.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Build the method

Sketching algebraic functions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

For $y = x^3 - 3x$, state all intercepts.

WORKED REASONING

1. Factor $x(x^2 - 3)$.

Conclusion: x-intercepts $-\sqrt{3}, 0, \sqrt{3}$; y-intercept 0.

WORKED EXAMPLE 2

Describe end behaviour of $y = -2x^3 + x$.

WORKED REASONING

1. The leading term controls end behaviour.

Conclusion: As x tends to infinity, y tends to negative infinity; as x tends to negative infinity, y tends to infinity.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Connect and reason

Sketching algebraic functions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C7, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Sketch information for $y=x^4-4x^2$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding sketching algebraic functions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C7 • Sketching algebraic functions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. For $y=x^3-3x$, state all intercepts.

[4]

2. Describe end behaviour of $y=-2x^3+x$.

[4]

3. Sketch information for $y=x^4-4x^2$.

[4]

4. Why are intercepts alone insufficient for a reliable cubic sketch?

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C7 • Sketching algebraic functions

1. WORKED ANSWER

Method: Factor $x(x^2-3)$. Answer: x-intercepts $-\sqrt{3}, 0, \sqrt{3}$; y-intercept 0. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: The leading term controls end behaviour. Answer: As x tends to infinity, y tends to negative infinity; as x tends to negative infinity, y tends to infinity. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Use symmetry and differentiation. Answer: Even graph; roots $-2, 0, 2$ with 0 repeated; stationary points at $x=0$ and $x=\pm\sqrt{2}$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Use the derivative and leading term as well. Answer: They do not determine turning behaviour or end behaviour. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Optimisation

Build and solve maximum or minimum problems using differentiation.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C8. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Build and solve maximum or minimum problems using differentiation. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “A rectangle has perimeter 40. Find dimensions of maximum area.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Build the method

Optimisation

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

A rectangle has perimeter 40. Find dimensions of maximum area.

WORKED REASONING

1. $A = x(20 - x)$; $A' = 20 - 2x = 0$.

Conclusion: 10 by 10.

WORKED EXAMPLE 2

Find the minimum of $f(x) = x^2 - 8x + 20$.

WORKED REASONING

1. $f' = 2x - 8 = 0$ and the derivative changes from negative to positive.

Conclusion: 4 at $x = 4$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Connect and reason

Optimisation

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C8, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

An open box has square base x and height h with volume 108. Express surface area in x . Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding optimisation.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C8 • Optimisation

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. A rectangle has perimeter 40. Find dimensions of maximum area.

[4]

2. Find the minimum of $f(x)=x^2-8x+20$.

[4]

3. An open box has square base x and height h with volume 108. Express surface area in x .

[4]

4. For $S=x^2+432/x$, find the positive stationary x .

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C8 • Optimisation

1. WORKED ANSWER

Method: $A = x(20 - x)$; $A' = 20 - 2x = 0$. Answer: 10 by 10. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: $f' = 2x - 8 = 0$ and the derivative changes from negative to positive. Answer: 4 at $x = 4$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: $h = 108/x^2$; open surface area is $x^2 + 4xh$. Answer: $S = x^2 + 432/x$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: $S' = 2x - 432/x^2 = 0$, so $2x^3 = 432$. Answer: $x = 6$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Rates of change

Interpret derivatives as instantaneous rates and solve contextual problems.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C9. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Interpret derivatives as instantaneous rates and solve contextual problems. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “ $s=t^3-6t^2+9t$. Find velocity at $t=2$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Build the method

Rates of change

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

$s = t^3 - 6t^2 + 9t$. Find velocity at $t = 2$.

WORKED REASONING

1. $v = ds/dt = 3t^2 - 12t + 9$.

Conclusion: -3.

WORKED EXAMPLE 2

For the same motion find when the particle is at rest.

WORKED REASONING

1. Solve $3(t-1)(t-3) = 0$.

Conclusion: $t = 1$ and $t = 3$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Connect and reason

Rates of change

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C9, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A radius grows at 0.2 cm/s. Express dA/dt for $A = \pi r^2$ in terms of r . Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding rates of change.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C9 • Rates of change

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. $s = t^3 - 6t^2 + 9t$. Find velocity at $t = 2$.

[4]

2. For the same motion find when the particle is at rest.

[4]

3. A radius grows at 0.2 cm/s. Express dA/dt for $A = \pi r^2$ in terms of r .

[4]

4. Explain the units of a derivative dy/dx .

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C9 • Rates of change

1. WORKED ANSWER

Method: $v = ds/dt = 3t^2 - 12t + 9$. Answer: -3. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Solve $3(t-1)(t-3) = 0$. Answer: $t=1$ and $t=3$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Use chain rule: $dA/dt = (dA/dr)(dr/dt)$. Answer: $dA/dt = 0.4\pi r$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: A derivative is a rate of change. Answer: Units of y divided by units of x . Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Integrating powers

Find indefinite integrals of algebraic powers.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C10. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Find indefinite integrals of algebraic powers. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Integrate $6x^2 - 4x + 3$ with respect to x .”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Build the method

Integrating powers

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Integrate $6x^2 - 4x + 3$ with respect to x .

WORKED REASONING

1. Increase each power by 1 and divide by the new power.

Conclusion: $2x^3 - 2x^2 + 3x + C$.

WORKED EXAMPLE 2

Integrate $5x^{(1/2)}$.

WORKED REASONING

1. Add 1 to the power then divide.

Conclusion: $(10/3)x^{(3/2)} + C$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Connect and reason

Integrating powers

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C10, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Integrate $4/x^2$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding integrating powers.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C10 • Integrating powers

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Integrate $6x^2 - 4x + 3$ with respect to x .

[4]

2. Integrate $5x^{(1/2)}$.

[4]

3. Integrate $4/x^2$.

[4]

4. Why is the constant C required?

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C10 • Integrating powers

1. WORKED ANSWER

Method: Increase each power by 1 and divide by the new power. Answer: $2x^3 - 2x^2 + 3x + C$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Add 1 to the power then divide. Answer: $(10/3)x^{(3/2)} + C$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Rewrite as $4x^{-2}$; the new power is -1. Answer: $-4/x + C$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: C represents that family. Answer: Differentiation removes constants, so infinitely many antiderivatives differ by a constant. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Integrating linear composites

Integrate powers of $px+q$ and stated trigonometric forms.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C11. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Integrate powers of $px+q$ and stated trigonometric forms. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Integrate $(2x+3)^4$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Build the method

Integrating linear composites

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Integrate $(2x+3)^4$.

WORKED REASONING

1. Reverse the chain rule; divide by inner derivative 2 and new power 5.

Conclusion: $(2x+3)^5/10+C$.

WORKED EXAMPLE 2

Integrate $6(3x-1)^2$.

WORKED REASONING

1. Integral of $(3x-1)^2$ is $(3x-1)^3/3$; multiply by 6.

Conclusion: $(2/3)(3x-1)^3+C$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Connect and reason

Integrating linear composites

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C11, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Integrate $4\cos(2x+1)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding integrating linear composites.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C11 • Integrating linear composites

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Integrate $(2x+3)^4$.

[4]

2. Integrate $6(3x-1)^2$.

[4]

3. Integrate $4\cos(2x+1)$.

[4]

4. Integrate $3\sin(5x)$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C11 • Integrating linear composites

1. WORKED ANSWER

Method: Reverse the chain rule; divide by inner derivative 2 and new power 5. Answer: $(2x+3)^5/10+C$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Integral of $(3x-1)^2$ is $(3x-1)^3/9$; multiply by 6. Answer: $(2/3)(3x-1)^3+C$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Divide by the inner derivative 2. Answer: $2\sin(2x+1)+C$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Differentiate the result to check. Answer: $-3/5 \cos(5x)+C$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Definite integrals

Evaluate definite integrals with integer, fractional, surd or radian limits.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C12. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Evaluate definite integrals with integer, fractional, surd or radian limits. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Evaluate integral from 0 to 2 of $3x^2 dx$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Build the method

Definite integrals

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Evaluate integral from 0 to 2 of $3x^2$ dx.

WORKED REASONING

1. Antiderivative x^3 ; evaluate $8-0$.

Conclusion: 8.

WORKED EXAMPLE 2

Evaluate integral from 1 to 4 of $x^{-1/2}$ dx.

WORKED REASONING

1. Antiderivative $2\sqrt{x}$; calculate $4-2$.

Conclusion: 2.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Connect and reason

Definite integrals

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C12, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Evaluate integral from 0 to $\pi/2$ of $\cos x \, dx$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding definite integrals.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C12 • Definite integrals

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Evaluate integral from 0 to 2 of $3x^2$ dx.

[4]

2. Evaluate integral from 1 to 4 of $x^{(-1/2)}$ dx.

[4]

3. Evaluate integral from 0 to $\pi/2$ of $\cos x$ dx.

[4]

4. Explain why a definite integral can be negative.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C12 • Definite integrals

1. WORKED ANSWER

Method: Antiderivative x^3 ; evaluate 8-0. Answer: 8. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Antiderivative $2\sqrt{x}$; calculate 4-2. Answer: 2. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Antiderivative $\sin x$. Answer: 1. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Geometric area requires absolute or split treatment. Answer: It gives signed area; regions below the x-axis contribute negatively. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Areas under and between curves

Use integration to calculate geometric areas.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C13. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use integration to calculate geometric areas. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Find area under $y=x^2$ from $x=0$ to $x=3$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Build the method

Areas under and between curves

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Find area under $y=x^2$ from $x=0$ to $x=3$.

WORKED REASONING

1. Integral $x^3/3$ from 0 to 3.

Conclusion: 9.

WORKED EXAMPLE 2

Find area between $y=4$ and $y=x^2$ from $x=-2$ to $x=2$.

WORKED REASONING

1. Integrate $4-x^2$ over the interval.

Conclusion: 32/3.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Connect and reason

Areas under and between curves

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C13, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find area between $y=x$ and $y=x^2$ from $x=0$ to $x=1$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding areas under and between curves.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C13 • Areas under and between curves

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Find area under $y=x^2$ from $x=0$ to $x=3$.

[4]

2. Find area between $y=4$ and $y=x^2$ from $x=-2$ to $x=2$.

[4]

3. Find area between $y=x$ and $y=x^2$ from $x=0$ to $x=1$.

[4]

4. Why must intersections be found before an area-between-curves calculation?

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C13 • Areas under and between curves

1. WORKED ANSWER

Method: Integral $x^{3/3}$ from 0 to 3. Answer: 9. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Integrate $4-x^2$ over the interval. Answer: $32/3$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Integrate $x-x^2$. Answer: $1/6$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: The integrand must represent upper minus lower. Answer: They determine the limits and which graph is above. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Differential equations and initial conditions

Recover a function from its derivative and use a given point.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C14. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Recover a function from its derivative and use a given point. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Given $\frac{dy}{dx}=6x-4$ and $y=3$ when $x=1$, find y .”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Build the method

Differential equations and initial conditions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Given $\frac{dy}{dx}=6x-4$ and $y=3$ when $x=1$, find y .

WORKED REASONING

1. Integrate to $3x^2-4x+C$.
2. Use the point to find $C=4$.

Conclusion: $y=3x^2-4x+4$.

WORKED EXAMPLE 2

Given $f'(x)=3x^2$ and $f(2)=10$, find $f(x)$.

WORKED REASONING

1. Integrate and use $10=8+C$.

Conclusion: $f(x)=x^3+2$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Connect and reason

Differential equations and initial conditions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C14, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

A velocity is $v=4t-3$ and $s=5$ at $t=0$. Find displacement s . Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding differential equations and initial conditions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C14 • Differential equations and initial conditions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Given $\frac{dy}{dx}=6x-4$ and $y=3$ when $x=1$, find y .

[4]

2. Given $f'(x)=3x^2$ and $f(2)=10$, find $f(x)$.

[4]

3. A velocity is $v=4t-3$ and $s=5$ at $t=0$. Find displacement s .

[4]

4. Explain the role of an initial condition.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C14 • Differential equations and initial conditions

1. WORKED ANSWER

Method: Integrate to $3x^2 - 4x + C$. Use the point to find $C = 4$. Answer: $y = 3x^2 - 4x + 4$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Integrate and use $10 = 8 + C$. Answer: $f(x) = x^3 + 2$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Integrate velocity and apply the initial condition. Answer: $s = 2t^2 - 3t + 5$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Without it the function is not unique. Answer: It selects one member of the antiderivative family by fixing C . Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Greatest and least values on closed intervals

Compare stationary and endpoint values systematically.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C15. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Compare stationary and endpoint values systematically. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Find greatest and least values of $f=x^2-4x+5$ on $[0,5]$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Build the method

Greatest and least values on closed intervals

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Find greatest and least values of $f=x^2-4x+5$ on $[0,5]$.

WORKED REASONING

1. Check stationary point and both endpoints.

Conclusion: Least 1 at $x=2$; greatest 10 at $x=5$.

WORKED EXAMPLE 2

Find maximum of $f=x^3-3x$ on $[-2,2]$.

WORKED REASONING

1. Check $x=-1,1$ and endpoints.

Conclusion: 2, attained at $x=-1$ and $x=2$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Connect and reason

Greatest and least values on closed intervals

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C15, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Why must endpoints be tested? Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding greatest and least values on closed intervals.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C15 • Greatest and least values on closed intervals

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Find greatest and least values of $f=x^2-4x+5$ on $[0,5]$.

[4]

2. Find maximum of $f=x^3-3x$ on $[-2,2]$.

[4]

3. Why must endpoints be tested?

[4]

4. Find minimum of $f=-x^2+4x$ on $[0,5]$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C15 • Greatest and least values on closed intervals

1. WORKED ANSWER

Method: Check stationary point and both endpoints. Answer: Least 1 at $x=2$; greatest 10 at $x=5$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Check $x=-1, 1$ and endpoints. Answer: 2, attained at $x=-1$ and $x=2$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Stationary points alone are insufficient. Answer: An absolute extremum on a closed interval may occur at an endpoint. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: The stationary point $x=2$ is a maximum; compare endpoints. Answer: -5 at $x=5$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Integrated Higher calculus

Connect differentiation, integration, modelling and verification.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland Higher Mathematics: H-C16. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Connect differentiation, integration, modelling and verification. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this higher calculus progression lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

derivative; gradient; stationary point; integral; rate; area; model; verification

Three controlling ideas

- Identify the quantity, representation, conditions and units before calculating.
- Show a transparent chain from definition or model to the required result.
- Check interpretation, scale, sign, accuracy and contextual meaning independently.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “A curve has derivative $3x^2 - 12x + 9$ and passes through $(0, 2)$. Find its equation.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Build the method

Integrated Higher calculus

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

A curve has derivative $3x^2 - 12x + 9$ and passes through $(0, 2)$. Find its equation.

WORKED REASONING

1. Integrate and use $C=2$.

Conclusion: $y = x^3 - 6x^2 + 9x + 2$.

WORKED EXAMPLE 2

For that curve, find stationary x -values and classify them.

WORKED REASONING

1. Factor derivative $3(x-1)(x-3)$ and test signs.

Conclusion: $x=1$ maximum; $x=3$ minimum.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 16: Connect and reason

Integrated Higher calculus

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For H-C16, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Find the signed area under $y=3x^2-6x$ from $x=0$ to $x=3$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding integrated higher calculus.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

H-C16 • Integrated Higher calculus

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. A curve has derivative $3x^2 - 12x + 9$ and passes through $(0, 2)$. Find its equation.

[4]

2. For that curve, find stationary x -values and classify them.

[4]

3. Find the signed area under $y = 3x^2 - 6x$ from $x = 0$ to $x = 3$.

[4]

4. A rectangle under $y = 12 - x^2$ in the first quadrant has upper corner $(x, 12 - x^2)$. Find x for maximum area.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

H-C16 • Integrated Higher calculus

1. WORKED ANSWER

Method: Integrate and use $C=2$. Answer: $y=x^3-6x^2+9x+2$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Factor derivative $3(x-1)(x-3)$ and test signs. Answer: $x=1$ maximum; $x=3$ minimum. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Antiderivative x^3-3x^2 gives 27-27. Positive and negative regions cancel. Answer: 0. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Area $A=12x-x^3$; $A'=12-3x^2=0$. Use $x>0$ and verify a maximum. Answer: $x=2$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Calculus mastery review

Explain, demonstrate, connect and revisit.

I can...

- H-C1 Differentiating powers: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C2 Differentiating sine and cosine: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C3 Chain rule: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C4 Tangents and normals: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C5 Increasing and decreasing functions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C6 Stationary points and their nature: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C7 Sketching algebraic functions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C8 Optimisation: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C9 Rates of change: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C10 Integrating powers: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C11 Integrating linear composites: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C12 Definite integrals: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C13 Areas under and between curves: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C14 Differential equations and initial conditions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C15 Greatest and least values on closed intervals: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- H-C16 Integrated Higher calculus: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.

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