

GCSE ONLINE TUTORING × RS REMOTE TUTORING

MATHEMATICS

NATIONAL 5

NATIONAL 5

COMPREHENSIVE LESSON NOTEBOOK

UNIT 2: ALGEBRA

Mapped independently to Qualifications Scotland National 5 Mathematics C847 75 • 15 detailed chapters
concept teaching • worked methods • reasoning • guided practice • independent practice • worked answers

Student: _____

Class: _____ Target grade: _____ Start date: _____

EXAM-BOARD INDEPENDENCE

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COMPREHENSIVE LESSON NOTEBOOK

How to learn from this notebook

Meaning first. Method second. Reasoning throughout.

1 • UNDERSTAND

Read the definition and big idea. Redraw or annotate the representation, then answer the checkpoint without looking back.

2 • BUILD THE METHOD

Cover each next line of the example. Predict it, reveal it and explain why the transformation or calculation is permitted.

3 • CONNECT AND REASON

Use the misconception analysis and guided task to decide which method fits, not merely how to execute it.

4 • PRACTISE INDEPENDENTLY

Show the decisive setup, accurate progress and a supported conclusion. Do not replace mathematical communication with a calculator display.

5 • DIAGNOSE AND REVISIT

Mark from the worked answer. Record whether an error came from interpretation, structure, arithmetic, accuracy, unit or communication.

COMPREHENSIVE LESSON NOTEBOOK

Algebra unit map

15 chapters mapped independently to the current course specification.

CH 01	N5-A1	Expanding brackets
CH 02	N5-A2	Common factors and difference of squares
CH 03	N5-A3	Factorising unitary trinomials
CH 04	N5-A4	Factorising non-unitary trinomials
CH 05	N5-A5	Completing the square
CH 06	N5-A6	Simplifying algebraic fractions
CH 07	N5-A7	Operations with algebraic fractions
CH 08	N5-A8	Straight-line equations
CH 09	N5-A9	Linear equations and inequations
CH 10	N5-A10	Simultaneous equations
CH 11	N5-A11	Changing the subject
CH 12	N5-A12	Quadratic graphs and forms
CH 13	N5-A13	Solving quadratics by factorising
CH 14	N5-A14	Quadratic formula
CH 15	N5-A15	Discriminant and integrated algebra

COMPREHENSIVE LESSON NOTEBOOK

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COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Expanding brackets

Expand and simplify products involving one or two brackets.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A1. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Expand and simplify products involving one or two brackets. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Expand and simplify $4(3x-5)-2(x+7)$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Build the method

Expanding brackets

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Expand and simplify $4(3x-5)-2(x+7)$.

WORKED REASONING

1. Expand each bracket: $12x-20-2x-14$.
2. Collect like terms.

Conclusion: $10x-34$.

WORKED EXAMPLE 2

Expand $(x+4)(x-7)$.

WORKED REASONING

1. Multiply every term in the first bracket by every term in the second.
2. Collect the two middle terms.

Conclusion: $x^2-3x-28$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 1: Connect and reason

Expanding brackets

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A1, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Expand $(2x-3)(x+5)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding expanding brackets.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A1 • Expanding brackets

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Expand and simplify $4(3x-5)-2(x+7)$.

[4]

2. Expand $(x+4)(x-7)$.

[4]

3. Expand $(2x-3)(x+5)$.

[4]

4. Show that $(x+3)(x+2)-x(x+5)$ is constant.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A1 • Expanding brackets

1. WORKED ANSWER

Method: Expand each bracket: $12x-20-2x-14$. Collect like terms. Answer: $10x-34$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Multiply every term in the first bracket by every term in the second. Collect the two middle terms. Answer: $x^2-3x-28$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Products are $2x^2$, $10x$, $-3x$ and -15 . Answer: $2x^2+7x-15$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Expand both products. The x^2 and $5x$ terms cancel. Answer: It simplifies to 6. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Common factors and difference of squares

Factorise by removing a common factor or recognising a difference of two squares.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A2. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Factorise by removing a common factor or recognising a difference of two squares. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Factorise $18x^2y - 12xy^2$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Build the method

Common factors and difference of squares

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Factorise $18x^2y - 12xy^2$.

WORKED REASONING

1. Find the greatest common numerical and algebraic factor.
2. Check by re-expanding.

Conclusion: $6xy(3x - 2y)$.

WORKED EXAMPLE 2

Factorise $x^2 - 49$.

WORKED REASONING

1. Use $a^2 - b^2 = (a - b)(a + b)$.

Conclusion: $(x - 7)(x + 7)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 2: Connect and reason

Common factors and difference of squares

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A2, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Factorise $25p^2 - 4q^2$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding common factors and difference of squares.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A2 • Common factors and difference of squares

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Factorise $18x^2y - 12xy^2$.

[4]

2. Factorise $x^2 - 49$.

[4]

3. Factorise $25p^2 - 4q^2$.

[4]

4. Factorise $3x^3 - 75x$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A2 • Common factors and difference of squares

1. WORKED ANSWER

Method: Find the greatest common numerical and algebraic factor. Check by re-expanding. Answer: $6xy(3x-2y)$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Use $a^2 - b^2 = (a-b)(a+b)$. Answer: $(x-7)(x+7)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Recognise both terms as squares. Answer: $(5p-2q)(5p+2q)$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: First remove $3x$. Then factorise $x^2 - 25$. Answer: $3x(x-5)(x+5)$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Factorising unitary trinomials

Factorise x^2+bx+c by finding a factor pair.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A3. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Factorise x^2+bx+c by finding a factor pair. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Factorise $x^2+9x+20$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Build the method

Factorising unitary trinomials

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Factorise $x^2 + 9x + 20$.

WORKED REASONING

1. Find two numbers with product 20 and sum 9.

Conclusion: $(x+4)(x+5)$.

WORKED EXAMPLE 2

Factorise $x^2 - 2x - 24$.

WORKED REASONING

1. Find two numbers with product -24 and sum -2.

Conclusion: $(x-6)(x+4)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 3: Connect and reason

Factorising unitary trinomials

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A3, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Factorise $x^2 - 11x + 28$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding factorising unitary trinomials.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A3 • Factorising unitary trinomials

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Factorise $x^2+9x+20$.

[4]

2. Factorise $x^2-2x-24$.

[4]

3. Factorise $x^2-11x+28$.

[4]

4. Find k if $x^2+kx+18=(x+3)(x+6)$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A3 • Factorising unitary trinomials

1. WORKED ANSWER

Method: Find two numbers with product 20 and sum 9. Answer: $(x+4)(x+5)$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Find two numbers with product -24 and sum -2. Answer: $(x-6)(x+4)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Both signs are negative because the product is positive and the sum is negative. Answer: $(x-7)(x-4)$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Expand the right side and compare the coefficient of x . Answer: $k=9$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Factorising non-unitary trinomials

Factorise ax^2+bx+c where the coefficient of x^2 is not 1.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A4. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Factorise ax^2+bx+c where the coefficient of x^2 is not 1. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Factorise $2x^2+7x+3$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Build the method

Factorising non-unitary trinomials

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Factorise $2x^2 + 7x + 3$.

WORKED REASONING

1. Split $7x$ as $6x + x$ or test compatible factor pairs.

Conclusion: $(2x+1)(x+3)$.

WORKED EXAMPLE 2

Factorise $6x^2 - x - 2$.

WORKED REASONING

1. The outer and inner terms combine to $-x$.

Conclusion: $(3x-2)(2x+1)$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 4: Connect and reason

Factorising non-unitary trinomials

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A4, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Factorise $3x^2 - 14x - 5$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding factorising non-unitary trinomials.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A4 • Factorising non-unitary trinomials

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Factorise $2x^2+7x+3$.

[4]

2. Factorise $6x^2-x-2$.

[4]

3. Factorise $3x^2-14x-5$.

[4]

4. Factorise $8x^2+2x-3$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A4 • Factorising non-unitary trinomials

1. WORKED ANSWER

Method: Split 7x as $6x+x$ or test compatible factor pairs. Answer: $(2x+1)(x+3)$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: The outer and inner terms combine to $-x$. Answer: $(3x-2)(2x+1)$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: The middle terms $-15x+x$ give $-14x$. Answer: $(3x+1)(x-5)$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Check the cross terms: $-4x+6x=2x$. Answer: $(4x+3)(2x-1)$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Completing the square

Write a unitary quadratic in completed-square form and identify its minimum or maximum.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A5. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Write a unitary quadratic in completed-square form and identify its minimum or maximum. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Write x^2+8x+3 as $(x+p)^2+q$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Build the method

Completing the square

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Write x^2+8x+3 as $(x+p)^2+q$.

WORKED REASONING

1. Half the coefficient of x .
2. Add and subtract 16.

Conclusion: $(x+4)^2-13$.

WORKED EXAMPLE 2

Write $x^2-6x+11$ in completed-square form.

WORKED REASONING

1. Half -6 is -3; compensate for 9.

Conclusion: $(x-3)^2+2$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 5: Connect and reason

Completing the square

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A5, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

State the minimum value of $x^2 + 10x + 31$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding completing the square.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A5 • Completing the square

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Write x^2+8x+3 as $(x+p)^2+q$.

[4]

2. Write $x^2-6x+11$ in completed-square form.

[4]

3. State the minimum value of $x^2+10x+31$.

[4]

4. Solve $x^2+4x-5=0$ by completing the square.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A5 • Completing the square

1. WORKED ANSWER

Method: Half the coefficient of x . Add and subtract 16. Answer: $(x+4)^2-13$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Half -6 is -3; compensate for 9. Answer: $(x-3)^2+2$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Complete the square: $(x+5)^2+6$. Answer: 6, attained at $x=-5$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Write $(x+2)^2=9$. Take both square roots. Answer: $x=1$ or $x=-5$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Simplifying algebraic fractions

Factorise before cancelling and state excluded values.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A6. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Factorise before cancelling and state excluded values. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Simplify $(6x^2 - 18x)/(3x)$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Build the method

Simplifying algebraic fractions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Simplify $(6x^2 - 18x)/(3x)$.

WORKED REASONING

1. Factorise the numerator as $6x(x-3)$.
2. Cancel the common factor $3x$.

Conclusion: $2(x-3)$, x not equal to 0.

WORKED EXAMPLE 2

Simplify $(x^2 - 25)/(x+5)$.

WORKED REASONING

1. Factorise the numerator.
2. Cancel $x+5$ but retain the original restriction.

Conclusion: $x-5$, x not equal to -5 .

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 6: Connect and reason

Simplifying algebraic fractions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A6, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Simplify $(2x^2 + 7x + 3)/(2x + 1)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding simplifying algebraic fractions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A6 • Simplifying algebraic fractions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Simplify $(6x^2 - 18x)/(3x)$.

[4]

2. Simplify $(x^2 - 25)/(x + 5)$.

[4]

3. Simplify $(2x^2 + 7x + 3)/(2x + 1)$.

[4]

4. Explain why $(x+2)/x$ cannot be simplified to 2.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A6 • Simplifying algebraic fractions

1. WORKED ANSWER

Method: Factorise the numerator as $6x(x-3)$. Cancel the common factor $3x$. Answer: $2(x-3)$, x not equal to 0. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Factorise the numerator. Cancel $x+5$ but retain the original restriction. Answer: $x-5$, x not equal to -5 . Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Factorise the numerator as $(2x+1)(x+3)$. Answer: $x+3$, x not equal to $-1/2$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: For example $x=2$ gives 2, whereas the claimed constant 2 fails at $x=1$, where the value is 3. Answer: Cancellation applies to common factors, not terms joined by addition. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Operations with algebraic fractions

Add, subtract, multiply and divide simple algebraic fractions.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A7. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Add, subtract, multiply and divide simple algebraic fractions. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Simplify $3/x + 2/(x+1)$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Build the method

Operations with algebraic fractions

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Simplify $3/x + 2/(x+1)$.

WORKED REASONING

1. Use common denominator $x(x+1)$.
2. Combine $3(x+1) + 2x$.

Conclusion: $(5x+3)/(x(x+1))$, x not 0 or -1.

WORKED EXAMPLE 2

Simplify $5/(x-2) - 1/x$.

WORKED REASONING

1. Use common denominator $x(x-2)$.

Conclusion: $(4x+2)/(x(x-2))$, x not 0 or 2.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 7: Connect and reason

Operations with algebraic fractions

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A7, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Simplify $(3x/4)(8/x^2)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding operations with algebraic fractions.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A7 • Operations with algebraic fractions

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Simplify $3/x + 2/(x+1)$.

[4]

2. Simplify $5/(x-2) - 1/x$.

[4]

3. Simplify $(3x/4)(8/x^2)$.

[4]

4. Simplify $(x/5)$ divided by $(2x^2/15)$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A7 • Operations with algebraic fractions

1. WORKED ANSWER

Method: Use common denominator $x(x+1)$. Combine $3(x+1)+2x$. Answer: $(5x+3)/(x(x+1))$, x not 0 or -1. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Use common denominator $x(x-2)$. Answer: $(4x+2)/(x(x-2))$, x not 0 or 2. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Multiply then cancel common factors. Answer: $6/x$, x not equal to 0. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Multiply by the reciprocal. Cancel 5 and x . Answer: $3/(2x)$, x not equal to 0. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Straight-line equations

Determine a line equation from points or a point and gradient; use functional notation.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A8. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Determine a line equation from points or a point and gradient; use functional notation. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Find the equation of the line through (2,5) with gradient 3.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Build the method

Straight-line equations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Find the equation of the line through (2,5) with gradient 3.

WORKED REASONING

1. Use $y-5=3(x-2)$.
2. Rearrange.

Conclusion: $y=3x-1$.

WORKED EXAMPLE 2

Find the equation through (-1,4) and (3,12).

WORKED REASONING

1. Gradient= $(12-4)/(3-(-1))=2$.
2. Substitute either point to find the intercept.

Conclusion: $y=2x+6$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 8: Connect and reason

Straight-line equations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A8, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

For $f(x)=5-2x$, find $f(-3)$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding straight-line equations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A8 • Straight-line equations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Find the equation of the line through (2,5) with gradient 3.

[4]

2. Find the equation through (-1,4) and (3,12).

[4]

3. For $f(x)=5-2x$, find $f(-3)$.

[4]

4. A line has equation $4x+2y=10$. State its gradient and y-intercept.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A8 • Straight-line equations

1. WORKED ANSWER

Method: Use $y-5=3(x-2)$. Rearrange. Answer: $y=3x-1$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Gradient $= (12-4)/(3-(-1)) = 2$. Substitute either point to find the intercept. Answer: $y=2x+6$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Substitute $x=-3$ carefully. Answer: 11. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Rearrange to $y=-2x+5$. Answer: Gradient -2; y-intercept 5. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Linear equations and inequations

Solve equations and inequalities with rational coefficients.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A9. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Solve equations and inequalities with rational coefficients. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $3(2x-5)=4x+7$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Build the method

Linear equations and inequations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1Solve $3(2x-5)=4x+7$.**WORKED REASONING**

1. Expand to $6x-15=4x+7$.
2. Collect variables and constants.

Conclusion: $x=11$.**WORKED EXAMPLE 2**Solve $x/3+5/6=7/2$.**WORKED REASONING**

1. Multiply throughout by 6.
2. Solve $2x+5=21$.

Conclusion: $x=8$.**SELF-EXPLANATION PROMPT**

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 9: Connect and reason

Linear equations and inequations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A9, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $5 - 2x < 13$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding linear equations and inequations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A9 • Linear equations and inequations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $3(2x-5)=4x+7$.

[4]

2. Solve $x/3+5/6=7/2$.

[4]

3. Solve $5-2x<13$.

[4]

4. Solve $-3\leq 2x+1<9$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A9 • Linear equations and inequations

1. WORKED ANSWER

Method: Expand to $6x - 15 = 4x + 7$. Collect variables and constants. Answer: $x = 11$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Multiply throughout by 6. Solve $2x + 5 = 21$. Answer: $x = 8$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Subtract 5 then divide by -2. Reverse the inequality when dividing by a negative. Answer: $x > -4$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Subtract 1 throughout then divide by 2. Answer: $-2 \leq x < 4$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Simultaneous equations

Construct and solve pairs of linear equations algebraically and graphically.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A10. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Construct and solve pairs of linear equations algebraically and graphically. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $2x+y=11$ and $x-y=1$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Build the method

Simultaneous equations

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Solve $2x+y=11$ and $x-y=1$.

WORKED REASONING

1. Add the equations to eliminate y .
2. Substitute $x=4$.

Conclusion: $x=4$, $y=3$.

WORKED EXAMPLE 2

Solve $3x+2y=16$ and $5x-2y=8$.

WORKED REASONING

1. Add equations to get $8x=24$.
2. Substitute back.

Conclusion: $x=3$, $y=7/2$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 10: Connect and reason

Simultaneous equations

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A10, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Tickets cost GBP 6 for adults and GBP 4 for children. Twelve tickets cost GBP 62. Find each number. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding simultaneous equations.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A10 • Simultaneous equations

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $2x+y=11$ and $x-y=1$.

[4]

2. Solve $3x+2y=16$ and $5x-2y=8$.

[4]

3. Tickets cost GBP 6 for adults and GBP 4 for children. Twelve tickets cost GBP 62. Find each number.

[4]

4. Explain the graphical meaning of a pair of simultaneous linear equations with no solution.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A10 • Simultaneous equations

1. WORKED ANSWER

Method: Add the equations to eliminate y . Substitute $x=4$. Answer: $x=4$, $y=3$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Add equations to get $8x=24$. Substitute back. Answer: $x=3$, $y=7/2$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Let $a+c=12$ and $6a+4c=62$. Eliminate one variable. Answer: 7 adults and 5 children. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: They have equal gradients and different intercepts, so they never meet. Answer: The two lines are parallel and distinct. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Changing the subject

Rearrange linear formulae and formulae involving a simple square or square root.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A11. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Rearrange linear formulae and formulae involving a simple square or square root. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Make x the subject of $y=5x-8$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Build the method

Changing the subject

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Make x the subject of $y=5x-8$.

WORKED REASONING

1. Add 8 then divide by 5.

Conclusion: $x=(y+8)/5$.

WORKED EXAMPLE 2

Make r the subject of $A=\pi r^2$.

WORKED REASONING

1. Divide by π then take the positive square root.

Conclusion: $r=\sqrt{A/\pi}$, taking $r \geq 0$ for a radius.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 11: Connect and reason

Changing the subject

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A11, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Make t the subject of $v^2 = u^2 + 2at$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding changing the subject.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A11 • Changing the subject

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Make x the subject of $y=5x-8$.

[4]

2. Make r the subject of $A=\pi r^2$.

[4]

3. Make t the subject of $v^2=u^2+2at$.

[4]

4. Make x the subject of $p=\sqrt{3x+1}$.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A11 • Changing the subject

1. WORKED ANSWER

Method: Add 8 then divide by 5. Answer: $x=(y+8)/5$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Divide by π then take the positive square root. Answer: $r=\sqrt{A/\pi}$, taking $r \geq 0$ for a radius. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Subtract u^2 then divide by $2a$. Answer: $t=(v^2-u^2)/(2a)$, a not equal to 0. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Square both sides, subtract 1 and divide by 3. Answer: $x=(p^2-1)/3$, with $p \geq 0$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Quadratic graphs and forms

Recognise and sketch quadratics from factorised and completed-square forms.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A12. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Recognise and sketch quadratics from factorised and completed-square forms. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “For $y=(x-2)(x+5)$, state the x-intercepts.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Build the method

Quadratic graphs and forms

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

For $y=(x-2)(x+5)$, state the x-intercepts.

WORKED REASONING

1. Set each factor equal to zero.

Conclusion: (2,0) and (-5,0).

WORKED EXAMPLE 2

For $y=2(x+3)^2-8$, state the turning point and axis.

WORKED REASONING

1. Read the completed-square form.

Conclusion: Turning point (-3,-8); axis $x=-3$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 12: Connect and reason

Quadratic graphs and forms

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A12, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

For $y = -3(x-1)^2 + 12$, state the nature of the turning point. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding quadratic graphs and forms.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A12 • Quadratic graphs and forms

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. For $y=(x-2)(x+5)$, state the x-intercepts.

[4]

2. For $y=2(x+3)^2-8$, state the turning point and axis.

[4]

3. For $y=-3(x-1)^2+12$, state the nature of the turning point.

[4]

4. A quadratic has turning point (4,-2) and vertical scale factor 3. Write its equation.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A12 • Quadratic graphs and forms

1. WORKED ANSWER

Method: Set each factor equal to zero. Answer: (2,0) and (-5,0). Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Read the completed-square form. Answer: Turning point (-3,-8); axis $x=-3$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: The negative coefficient means the graph opens downward. Answer: A maximum at (1,12). Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Use $y=k(x-p)^2+q$. Answer: $y=3(x-4)^2-2$. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Solving quadratics by factorising

Solve equations from factorised form or after factorising.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A13. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Solve equations from factorised form or after factorising. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $(x-4)(x+7)=0$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Build the method

Solving quadratics by factorising

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Solve $(x-4)(x+7)=0$.

WORKED REASONING

1. Use the zero-product rule.

Conclusion: $x=4$ or $x=-7$.

WORKED EXAMPLE 2

Solve $x^2+3x-28=0$.

WORKED REASONING

1. Factorise as $(x+7)(x-4)$.

Conclusion: $x=4$ or $x=-7$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 13: Connect and reason

Solving quadratics by factorising

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A13, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $2x^2 - 5x - 3 = 0$. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding solving quadratics by factorising.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A13 • Solving quadratics by factorising

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $(x-4)(x+7)=0$.

[4]

2. Solve $x^2+3x-28=0$.

[4]

3. Solve $2x^2-5x-3=0$.

[4]

4. A rectangle has sides $x+2$ and $x-1$ and area 40. Find its positive side lengths.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A13 • Solving quadratics by factorising

1. WORKED ANSWER

Method: Use the zero-product rule. Answer: $x=4$ or $x=-7$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Factorise as $(x+7)(x-4)$. Answer: $x=4$ or $x=-7$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Factorise as $(2x+1)(x-3)$. Answer: $x=3$ or $x=-1/2$. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Solve $(x+2)(x-1)=40$. Factorise $x^2+x-42=(x+7)(x-6)$, so $x=6$. Answer: 8 and 5. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Quadratic formula

Use the quadratic formula accurately and interpret exact or approximate roots.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A14. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use the quadratic formula accurately and interpret exact or approximate roots. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “Solve $2x^2+3x-4=0$ exactly.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Build the method

Quadratic formula

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

Solve $2x^2 + 3x - 4 = 0$ exactly.

WORKED REASONING

1. Use $a=2$, $b=3$, $c=-4$.
2. Simplify the denominator $2a=4$.

Conclusion: $x = \frac{-3 + \sqrt{41}}{4}$ or $x = \frac{-3 - \sqrt{41}}{4}$.

WORKED EXAMPLE 2

Solve $x^2 - 5x - 1 = 0$ to 3 significant figures.

WORKED REASONING

1. Use the quadratic formula and round only at the end.

Conclusion: x approximately **5.19** or **-0.193**.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 14: Connect and reason

Quadratic formula

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A14, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Solve $3x^2 + 2x + 7 = 0$ over the real numbers. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding quadratic formula.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A14 • Quadratic formula

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. Solve $2x^2+3x-4=0$ exactly.

[4]

2. Solve $x^2-5x-1=0$ to 3 significant figures.

[4]

3. Solve $3x^2+2x+7=0$ over the real numbers.

[4]

4. A projectile model is $h=-5t^2+18t+2$. Find when $h=0$, rejecting an unsuitable time.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A14 • Quadratic formula

1. WORKED ANSWER

Method: Use $a=2$, $b=3$, $c=-4$. Simplify the denominator $2a=4$. Answer: $x=(-3+\sqrt{41})/4$ or $x=(-3-\sqrt{41})/4$. Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Use the quadratic formula and round only at the end. Answer: x approximately 5.19 or -0.193. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: The discriminant is $4-84=-80$, which is negative. Answer: No real roots. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Apply the quadratic formula. Reject the negative time. Answer: t approximately 3.71 seconds. Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Discriminant and integrated algebra

Use $b^2 - 4ac$ to classify roots and connect multiple algebraic representations.

SPECIFICATION ALIGNMENT

Independent teaching summary for Qualifications Scotland National 5 Mathematics: N5-A15. This lesson develops conceptual understanding, communication using symbols, fluency, logical reasoning and strategic competence.

Understand the mathematical object

Use $b^2 - 4ac$ to classify roots and connect multiple algebraic representations. The central idea is that a representation is useful only when its meaning and conditions are explicit. In this national 5 algebraic skills lesson, first identify the quantities, structures or events involved; then connect the words to notation, a diagram, a table or a graph. A reliable solution preserves that meaning at every step and ends with an answer that fits the original context.

KEY VOCABULARY

expression; equation; factor; root; function; domain; range; representation; condition; verification

Three controlling ideas

- Identify the algebraic object, its restrictions and the required form.
- Choose a reversible transformation or a structure-preserving representation.
- Check by expansion, substitution, graph features, sign analysis or an independent method.

PRIOR-KNOWLEDGE CHECK

Can you state the meaning of the key terms, identify the operation or relationship in “State the nature of the roots of $4x^2 - 12x + 9 = 0$.”, and predict the form or approximate size of the answer? Repair any weakness before continuing.

CHECKPOINT

Explain one method condition in a complete sentence. Give a simple valid example and one non-example that breaks the condition.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Build the method

Discriminant and integrated algebra

Reliable routine

- Interpret: name the target, given information, conditions and units.
- Represent: write an equation, diagram, table, graph or exact numerical structure.
- Execute: show valid transformations or calculations in a readable order.
- Verify: test size, sign, units, domain, accuracy and contextual meaning.

WORKED EXAMPLE 1

State the nature of the roots of $4x^2 - 12x + 9 = 0$.

WORKED REASONING

1. The discriminant is $144 - 144 = 0$.

Conclusion: Two equal real roots (one repeated root).

WORKED EXAMPLE 2

Find k so $x^2 + 6x + k = 0$ has equal roots.

WORKED REASONING

1. Set the discriminant $36 - 4k$ equal to zero.

Conclusion: $k = 9$.

SELF-EXPLANATION PROMPT

Which line contains the main mathematical decision? Which line checks interpretation, reasonableness or accuracy? Explain how the method would change if one condition changed.

COMPREHENSIVE LESSON NOTEBOOK

Chapter 15: Connect and reason

Discriminant and integrated algebra

Why the method works

The method works because each stage preserves the defining relationship in the problem. A calculation is not merely a sequence of button presses: it is a chain in which every symbol, operation and representation has a stated meaning. An independent check uses a different feature - such as an inverse operation, an alternative representation, a bound, a unit or a limiting case - so it can reveal an error rather than repeat it.

COMMON MISCONCEPTION

A common error is to use a familiar rule before checking its conditions. For N5-A15, this can produce a plausible-looking answer with the wrong structure, scale, sign, unit or interpretation. State the condition first, then apply the rule.

GUIDED REASONING TASK

Show that $x^2 + 4x + 8$ is always positive. Before calculating, write: (a) what is known, (b) what is required, (c) the controlling relationship, and (d) one check. Then complete the solution.

COMMUNICATION STANDARD

Use one mathematical statement per line. Define symbols, label units and diagrams, retain exact values until approximation is requested, and close with a sentence that answers the actual question.

EXTENSION OR REPAIR

Change one value or condition in the guided task. Predict which steps remain valid and which must change. This is the difference between memorising a routine and understanding discriminant and integrated algebra.

COMPREHENSIVE LESSON NOTEBOOK

Independent practice

N5-A15 • Discriminant and integrated algebra

Complete without notes. Show the setup, decisive working and final conclusion. Marks are independent teaching allocations, not official Qualifications Scotland / former SQA marks.

1. State the nature of the roots of $4x^2 - 12x + 9 = 0$.

[4]

2. Find k so $x^2 + 6x + k = 0$ has equal roots.

[4]

3. Show that $x^2 + 4x + 8$ is always positive.

[4]

4. A line $y = x + 1$ meets $y = x^2 - 3x + 5$. Find the intersection point.

[4]

REFLECTION

Record the question number of your least secure answer. Name the error type and write the first corrected line that would repair it.

COMPREHENSIVE LESSON NOTEBOOK

Worked answers and repair notes

N5-A15 • Discriminant and integrated algebra

1. WORKED ANSWER

Method: The discriminant is $144 - 144 = 0$. Answer: Two equal real roots (one repeated root). Check: confirm the result against the original conditions, units and expected scale.

2. WORKED ANSWER

Method: Set the discriminant $36 - 4k$ equal to zero. Answer: $k = 9$. Check: confirm the result against the original conditions, units and expected scale.

3. WORKED ANSWER

Method: Complete the square. Answer: $x^2 + 4x + 8 = (x + 2)^2 + 4$, which is at least 4. Check: confirm the result against the original conditions, units and expected scale.

4. WORKED ANSWER

Method: Equate the expressions to get $x^2 - 4x + 4 = 0$. Since $(x - 2)^2 = 0$, $x = 2$ and $y = 3$; the line is tangent to the curve. Answer: (2, 3). Check: confirm the result against the original conditions, units and expected scale.

COMPREHENSIVE LESSON NOTEBOOK

Algebra mastery review

Explain, demonstrate, connect and revisit.

I can...

- N5-A1 Expanding brackets: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A2 Common factors and difference of squares: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A3 Factorising unitary trinomials: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A4 Factorising non-unitary trinomials: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A5 Completing the square: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A6 Simplifying algebraic fractions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A7 Operations with algebraic fractions: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A8 Straight-line equations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A9 Linear equations and inequations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A10 Simultaneous equations: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A11 Changing the subject: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A12 Quadratic graphs and forms: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A13 Solving quadratics by factorising: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A14 Quadratic formula: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.
- N5-A15 Discriminant and integrated algebra: explain the idea, reproduce a valid method, solve an unfamiliar problem and check the result.

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