GCSE ONLINE TUTORING × RS REMOTE TUTORING

Functional skills level 2 Maths guide

(for all exam boards)

LESSON NOTEBOOK • EDITION 2

LEARN THE METHOD. SHOW THE REASONING. CHECK THE RESULT.

An original teaching and learning notebook for the shared Functional Skills Level 2 mathematics content in England. Use with tutor-led lessons or structured self-study.

28 detailed lessons	28 coached problem journeys
84 short lesson checks	Method notes and misconception clinics
Original diagrams and grids	Tutor prompts and reflection space

Numbers and calculation • Measures, shape and space • Information and data

Name: _____

Tutor / centre: _____

Exam board and date: _____

Edition 2 • Lesson notebook • Revised 13 November 2026

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The supplied physics notebook informed the brand names, website details and general navy-and-teal presentation. The maths teaching sequence and page content were developed independently for this guide.

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“For all exam boards” means coverage of the shared reformed Functional Skills Level 2 mathematics subject content used by Ofqual-regulated awarding organisations in England. It does not mean every board has approved this guide or uses identical papers. This is not a curriculum guide for Scotland’s Core Skills, Wales’s Essential Skills or Northern Ireland’s Essential Skills qualifications.

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Accuracy and examination outcomes

This guide supports preparation; it cannot guarantee a pass. Use your centre’s current specification, official sample assessments and exam instructions alongside it. Sample money, tax, exchange and interest rates are invented teaching data, not current financial guidance.

YOUR STUDY ROUTINE

How to learn with this notebook

A LESSON IS COMPLETE WHEN YOU CAN EXPLAIN IT

Use each lesson as a three-page learning sequence: understand the teaching, discuss the coached problem, then record your own method and complete the short checks.

A repeatable lesson cycle

1. Recall: spend five minutes retrieving an earlier method without notes. Say what the quantities and units mean.

2. Learn: read the explanations and examples. On the coached page, say what you would do before revealing the model route. Explain why each operation is suitable.

3. Record: use the learner notebook page to write a key rule, show a method and complete three short checks. These checks support the lesson; extended drills and timed mocks belong in the separate practice notebook.

4. Check: compare your short checks with the model answers at the back. Explain an error in the misconception clinic and write one change that would improve your own working.

5. Repair: write the cause of each error and redo that question on another day. Repeated success without notes is stronger evidence than recognising an answer.

Build a complete solution

Read the final question first, underline relevant facts, and identify the output and unit. Decide on a route through the information. Calculate in labelled steps, retain precision, and finish with a sentence that answers the question. Check using an estimate, inverse calculation or another representation.

A suggested flexible route

Stage	Focus	Evidence to collect
Start	Diagnostic and foundation repair	Error list and first targets
Number	Lessons 1-12	Accurate methods with and without calculator
Application	Lessons 13-22	Units, diagrams and multi-step decisions
Data	Lessons 23-28	Comparisons supported by figures
Consolidation	Tutor review, then separate practice notebook	Independent explanation, followed by official assessment practice

Take the time you need. Revisit earlier skills within later problems instead of treating each topic as isolated. If foundational calculation remains difficult, work with a tutor on Level 1 and Entry Level gaps before moving faster.

CONTENTS

Find your lesson

Lesson	Topic	Teach / coach / notes
01	Numbers, place value and negatives	11 / 12 / 13
02	Whole-number calculation and checking	14 / 15 / 16
03	Expressions and formulae	17 / 18 / 19
04	Fractions, decimals and percentages	20 / 21 / 22
05	Percentages of amounts	23 / 24 / 25
06	Percentage change and original amounts	26 / 27 / 28
07	Adding, subtracting and ordering fractions	29 / 30 / 31
08	Fractions of quantities	32 / 33 / 34
09	Decimals and sensible accuracy	35 / 36 / 37
10	Calculating with decimals	38 / 39 / 40
11	Ratio and proportion	41 / 42 / 43
12	Order of operations and powers	44 / 45 / 46
13	Money, tax, budgets and compound interest	47 / 48 / 49
14	Units, conversion factors and graphs	50 / 51 / 52
15	Speed, density, pay and time	53 / 54 / 55
16	Perimeter, area and composite shapes	56 / 57 / 58
17	Volume and surface area	59 / 60 / 61
18	Scale drawings and actual dimensions	62 / 63 / 64
19	Coordinates in four quadrants	65 / 66 / 67
20	Solids, nets and views	68 / 69 / 70
21	Drawing solids, plans and elevations	71 / 72 / 73
22	Angles and geometric reasoning	74 / 75 / 76
23	Median, mode and organised data	77 / 78 / 79
24	Estimated mean from grouped data	80 / 81 / 82
25	Comparing data and evaluating charts	83 / 84 / 85
26	Combined events and two-way tables	86 / 87 / 88
27	Probability forms and expected frequencies	89 / 90 / 91
28	Scatter diagrams and correlation	92 / 93 / 94

Lesson-check answers: pages 95-108 • Exam routine and reference: pages 109-110

Progress tracker, additional teaching, paper-review notes and official sources follow. Full practice papers are reserved for the separate practice notebook.

NUMBER AND FINANCE

Curriculum coverage map

References below identify the numbered Level 2 content points in the DfE subject-content document. These short labels are navigation aids, not replacement specification wording. Each point has a teaching page, a coached problem, learner notes and short-check answers. Use the correspondingly numbered lesson.

DfE point	Focus	Lesson pages / answers
1	Signed numbers	11-13 / 95
2	Calculation and checks	14-16 / 95
3	Formula substitution	17-19 / 96
4	Equivalent forms	20-22 / 96
5	Percentage amounts	23-25 / 97
6	Percentage change	26-28 / 97
7	Fraction calculation	29-31 / 98
8	Fraction comparisons	32-34 / 98
9	Decimal accuracy	35-37 / 99
10	Decimal operations	38-40 / 99
11	Ratio and proportion	41-43 / 100
12	Operation order	44-46 / 100
13	Money and interest	47-49 / 101
14	Unit conversions	50-52 / 101

The shared assessment framework

Ofqual requires 25% of marks without a calculator and 75% with a calculator. A separate design target allocates approximately 25% to underpinning skills and 75% to problem solving. These are different classifications: a non-calculator question can still be a contextual problem. [S2]

MEASURES, GEOMETRY AND DATA

Curriculum coverage map

References below identify the numbered Level 2 content points in the DfE subject-content document. These short labels are navigation aids, not replacement specification wording. Each point has a teaching page, a coached problem, learner notes and short-check answers. Use the correspondingly numbered lesson.

DfE point	Focus	Lesson pages / answers
15	Compound measures	53-55 / 102
16	Perimeter and area	56-58 / 102
17	Volume and surface	59-61 / 103
18	Scale drawings	62-64 / 103
19	Coordinates	65-67 / 104
20	Representations	68-70 / 104
21	Plans and elevations	71-73 / 105
22	Angles and positions	74-76 / 105
23	Median and mode	77-79 / 106
24	Grouped mean	80-82 / 106
25	Data comparison	83-85 / 107
26	Combined probability	86-88 / 107
27	Probability forms	89-91 / 108
28	Scatter diagrams	92-94 / 108

Inherited knowledge and complex problems

Lower-level knowledge remains relevant: number facts and rounding, units and time, shape properties, bar charts, pie charts and line graphs. These are revisited across the lessons and foundation pages. Multi-step interpretation and checking are developed in the examples, coached problems and lesson checks. [S1, S2]

Lesson 12 also covers numerical and spatial patterns; lesson 25 includes chart construction and evaluation. The separate practice notebook will provide additional independent questions and mixed assessments. This volume focuses on learning the methods.

QUALIFICATION CHECK • 13 NOVEMBER 2026

Your exam board: shared content

The common DfE curriculum and Ofqual requirements provide the basis for cross-board content coverage. The reviewed sources below support this approach. The level of document verification is stated explicitly; not every current handbook could be retrieved. Do not use this list as a guarantee that a centre can book an exam today.

Organisation / Level 2 QN	Evidence reviewed	What to use next
Pearson Edexcel 603/4268/7	2019 Level 1/2 specification and current qualification page [S3]	Current specification, official samples and centre instructions
City & Guilds 603/4649/8	Maths handbook v1.3 (August 2025) retrieved; page lists October 2026 delivery guidance [S4]	4748-04 handbook and Level 2 delivery guidance
NCFE 603/5060/X	Qualification specification v2.0 and qualification page [S5]	Current assessment materials; some samples need centre access
NOCN 603/5267/X	Level 1/2 specification v1.7, March 2026 [S6]	Current examination and invigilation instructions
Open Awards 603/4806/9	Qualification guide and qualification page [S7]	Current guide and Level 2 sample assessment

What transfers between boards

The maths methods and shared content statements transfer. The paper layout, total marks, duration, permitted calculator functions, onscreen tools, booking rules and reasonable-adjustment procedures must be checked with the relevant board and centre. A board may sample different topics in different assessments.

There is no universal pass mark stated in this guide. Mark boundaries and assessment arrangements must come from your own awarding organisation. Book-specific practice scores are diagnostic information, not an official pass prediction.

CURRENT DOCUMENTS TAKE PRIORITY

Your exam board: further checks

Organisation / QN	Evidence reviewed and limitation
Highfield 603/4992/X	Reformed specification v1.4 retrieved. Recheck later amendments through the centre before using its timings or rules. [S8]
FutureQuals 603/4804/5	Level 2 specification v6 retrieved; current learning-aim listing also checked. Recheck the latest handbook and availability. [S9]
VTCT Skills (formerly Skillsfirst) 603/4808/2	Current qualification page checked; Skillsfirst RFSM2 handbook v3 (2019) retrieved. Shared core checked; confirm later VTCT amendments. [S10]
TQUK / current VQA website 610/2624/4	TQUK Maths Level 1/2 specification v1.2 retrieved. The suite page redirects to VQA and still lists this Maths QN under TQUK. Confirm current awarding name and documentation with the centre. [S11]

Withdrawn and legacy sources

AQA states that its Functional Skills qualifications were withdrawn in August 2025. OCR announced withdrawal of its reformed English and Maths Functional Skills provision in 2020. Historical papers can be useful for selected practice but these boards are not presented as current booking options. [S12, S13]

The Gateway Qualifications PDF located during research is dated February 2018 and relates to legacy qualifications. It was excluded from the current alignment basis. A recently crawled web document is not necessarily a current specification. [S14]

COMPLETE THIS WITH YOUR CENTRE

Awarding organisation and qualification number: _____
Assessment mode and timings: _____
Approved calculator and equipment: _____
Current sample paper completed: _____
Access arrangements confirmed, if needed: _____

Research limitation: this is a shared-content alignment, not an exhaustive live-register audit or certification of every current board handbook. Latest VTCT handbook currency remains to be confirmed with the centre.

TRY WITHOUT NOTES • NO PASS BOUNDARY

Starting-point diagnostic

Use this to identify gaps. Try questions 1-8 without a calculator, then use one for 9-12 if needed. Write your method in the space provided or on separate paper. Tick any topic that needs a lesson revisit.

1. Order -7, 3, -2 and 0.

2. Calculate 46 × 18.

3. Calculate 6.2 - 0.875.

4. Find 3/4 + 2/3.

5. Find 15% of £260.

6. A sale price is £63 after a 10% discount. Find the original.

7. Share £240 in the ratio 2:3.

8. Evaluate $4 + 2 \times 3^2$.

9. Convert 1 hour 42 minutes to decimal hours.

10. A triangle has base 9 m and perpendicular height 4 m. Find its area.

11. Find the mean and median of 4, 7, 7, 10.

12. A fair die is rolled. Find P(a score greater than 4), as a fraction and percentage.

DIAGNOSTIC ANSWERS AND FOUNDATION REPAIR

Check your starting point

1. -7, -2, 0, 3 (lesson 1).

2. 46 × (20 - 2) = 920 - 92 = 828 (lesson 2).

3. 6.200 - 0.875 = 5.325 (lesson 10).

4. 9/12 + 8/12 = 17/12 = 1 5/12 (lesson 7).

5. 10% £26 + 5% £13 = £39 (lesson 5).

6. 63 ÷ 0.90 = £70 (lesson 6).

7. Five parts; one part £48. Shares £96 and £144 (lesson 11).

8. 4 + 2 × 9 = 22 (lesson 12).

9. 1 + 42/60 = 1.7 hours (lesson 15).

10. 9 × 4 ÷ 2 = 18 m² (lesson 16).

11. Mean 28/4 = 7. Median (7 + 7)/2 = 7 (lessons 23-25).

12. 2/6 = 1/3 = 33 1/3%, or about 33.3% (lesson 27).

Foundation repair bank

If this is difficult...	Practise this first
Written arithmetic	Times tables; place-value columns; multiplication/division inverses; regrouping
Fractions and decimals	Equal parts; common multiples; tenths/hundredths/thousandths; equivalent fractions
Units and time	Read scales by counting intervals; 60 minutes = 1 hour; 24-hour clocks; unit labels
Shapes and diagrams	Rectangle, square, triangle, circle, cuboid, cylinder; ruler and protractor use
Charts	Title and units; equal intervals; categorical bars; time-series lines; sector proportions

For an unlabelled scale division, subtract two neighbouring labelled values and divide by the number of gaps, not the number of tick marks. For example, between 20 and 40 with four equal gaps, each gap represents 5.

MY FIRST THREE TARGETS

1. _____ 2. _____ 3. _____

LESSON 01 • UNDERSTAND

Numbers, place value and negatives

LEARNING GOAL • DfE LEVEL 2 POINT 1

Read large numbers, compare signed values and explain changes across zero.

A digit gets its value from its position. In 4,306,082 the 3 represents 300,000; in 4.306 it represents three tenths. The decimal places are tenths, hundredths and thousandths. Zeros hold empty places: 5.07 is five and seven hundredths, not five and seven tenths.

An integer is a whole number, including zero and negative whole numbers. On a number line, values increase to the right. Consequently -2 is greater than -9 even though 9 is the larger distance from zero. A negative balance can represent a debt, and a negative temperature is below zero.

To order decimals, line up the decimal points and add trailing zeros if helpful. Compare the first position that differs: 6.080 is less than 6.105. Trailing zeros do not change value. Leading zeros after the decimal point do matter.

For a change, calculate final value minus starting value. A rise from -7 to 5 is 12 units. A fall from 5 to -7 is a change of -12. The distance between two values is positive; the signed change also records direction.

Worked examples: follow the reasoning

1. Temperature check: a freezer warms from -18°C to -11°C. Change = -11 - (-18) = 7°C, so it rises by 7°C. Check by counting seven steps towards zero.

2. Population comparison: 2.04 million = 2,040,000. It is greater than 2,004,000 by 36,000. Writing both in the same form makes the comparison reliable.

COMMON MISTAKE TO AVOID

Do not order negative numbers by ignoring their signs. State whether a question wants a signed change or a positive difference.

LESSON 01 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

An outdoor display must operate only when the temperature is at least -4°C. The morning reading is -9°C. It rises by 7°C, then falls by 3°C. Decide whether the display may operate after each change. Explain your decisions using the threshold.

Pause and plan with your tutor

1. What does “at least -4°C” allow, including equality?
2. Write the two changes in their actual order. Which direction is a rise?
3. Compare each new reading with -4°C, keeping the negative signs.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. After the rise: -9 + 7 = -2°C. This is warmer than -4°C, so the display may operate.

Step 2. After the fall: -2 - 3 = -5°C. This is colder than -4°C, so it may not operate.

Step 3. Check the whole journey: total change +7 - 3 = +4°C; -9 + 4 = -5°C. This agrees with the final reading.

Misconception clinic

“Five is greater than four, so -5°C meets a minimum of -4°C.”

For negative temperatures the number nearer zero is warmer. Use a number line or compare positions, not digit sizes. Equality at exactly -4°C would be permitted.

SAY IT BACK

A correct comparison includes the sign, the threshold and the direction of the inequality.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 01 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Values increase to the right



Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Write 7.035 million in figures and give the value of its digit 3.

2. Order 0.509, 0.59 and 0.095 from smallest to largest.

3. An account has -£85. A payment of £140 arrives and £32 is then spent. Find the final balance and explain its sign.

My check / correction and why it matters:

Model answers: page 95. Confidence: need support / can use / can explain. Next step: _____

LESSON 02 • UNDERSTAND

Whole-number calculation and checking

LEARNING GOAL • DfE LEVEL 2 POINT 2

Choose efficient written methods, estimate first and interpret remainders.

Addition combines amounts; subtraction finds what is left or the difference. Align place values in column work. Regroup one ten as ten units when necessary. Keep your calculation visible so that a misplaced digit can be identified.

For multiplication, split a factor into useful parts. For 36 × 24, calculate 36 × 20 and 36 × 4, then add. Long multiplication records the same idea; the second row represents tens and must start in the tens column.

Division asks how many groups fit or how large each group is. A remainder needs a decision: a partly filled coach still needs booking, whereas a partly completed pack cannot be sold as a full pack. Do not automatically round every division answer in the same direction.

Estimate before calculating by replacing values with convenient nearby numbers. Check afterwards with the inverse operation or a different method. An estimate checks size, not exact accuracy: two incorrect answers can be close to the same estimate.

Worked examples: follow the reasoning

1. An organisation buys 48 chairs at £27 each. 48 × (20 + 7) = 960 + 336 = £1,296. Estimate 50 × £30 = £1,500, so the exact result is plausible.

2. Transport for 173 people uses vehicles with 24 seats. 173 ÷ 24 = 7 remainder 5. Seven vehicles seat 168; eight vehicles are needed. This is a capacity decision, not rounding to the nearest whole number.

COMMON MISTAKE TO AVOID

Write units and interpret the result. A calculator answer such as 7.2083 is not a usable booking instruction.

LESSON 02 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A museum has 286 entry tickets to prepare. A printer produces sheets of 16 tickets. Each sheet costs £1.35 to print. Only whole sheets can be printed. Find the minimum printing cost and the number of tickets left over.

Pause and plan with your tutor

1. Which division finds the number of sheets? What does a remainder mean here?
2. Why would rounding to the nearest whole number risk a shortage?
3. After choosing the sheet count, calculate cost and surplus separately.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. 286 ÷ 16 = 17 remainder 14. Seventeen sheets provide only 272 tickets.

Step 2. Use 18 sheets. Cost = 18 × £1.35 = £24.30.

Step 3. Eighteen sheets provide 288 tickets, giving 288 - 286 = 2 spare. Check that 17 sheets are insufficient and 18 are sufficient.

Misconception clinic

“286 ÷ 16 = 17.875, so 17 sheets should be enough because the decimal is only part of a sheet.”

The missing 0.875 of a sheet still contains required tickets. A whole-sheet purchase must be rounded up. The cost must then use the purchased count, not 17.875.

SAY IT BACK

Interpret the remainder before deciding the whole-number answer.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 02 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Calculate 48,726 + 9,587.

2. Calculate 84 × 36 without a calculator.

3. A club needs 247 badges. They come in packs of 18 costing £6.40. Work out the minimum cost and number of spare badges.

My check / correction and why it matters:

Model answers: page 95. Confidence: need support / can use / can explain. Next step: _____

LESSON 03 • UNDERSTAND

Expressions and formulae

LEARNING GOAL • DfE LEVEL 2 POINT 3

Substitute accurately into word and symbol formulae, including squared values.

An expression describes a calculation, such as $4n + 7$. A formula states a relationship, such as $C = 4n + 7$. Here C might be a charge and n the number of items. A letter stands for a value; adjoining letters or a number and a letter usually mean multiplication.

Translate word formulae before calculating. If total cost = fixed fee + distance × rate, identify each input and its unit. Substitute the values, keep brackets where needed and follow the order of operations. Do not add the fixed fee to the distance before multiplying.

A power applies only to its base unless brackets change the grouping. If $A = 3s^2$ and $s = 4$, then $A = 3 \times 16 = 48$, not 12^2. If a substituted value is negative, brackets prevent sign errors: $(-3)^2 = 9$.

Formulae connect quantities. You may need to find a missing input by reversing steps: if $C = 5n + 12$ and $C = 47$, subtract 12 and then divide by 5. Write each step with an explanation; do not guess repeatedly when an inverse calculation is available.

Worked examples: follow the reasoning

1. A delivery charge is $C = 6 + 1.8d$ pounds, where d is distance in kilometres. For 12 km: $C = 6 + 1.8 \times 12 = 6 + 21.6 = £27.60$.

2. A supplied formula is $V = \pi r^2 h$. With $r = 3$ cm and $h = 8$ cm, $V = \pi \times 3^2 \times 8 = 72\pi = 226.19...$ cm³. To the nearest cm³, $V = 226$ cm³.

COMMON MISTAKE TO AVOID

Units must match the formula. A radius of 30 mm is 3 cm if the height is given in centimetres.

LESSON 03 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A community laundry uses the formula C = 2.4n + 0.15t, where C is cost in pounds, n is the number of wash cycles and t is dryer use in minutes. A customer uses 3 wash cycles and 28 dryer minutes. The budget is £12. Is it sufficient?

Pause and plan with your tutor

1. Identify the meaning and unit of each letter. Which input is counted in minutes?
2. Substitute each value next to its own coefficient. Are brackets required?
3. Find the cost before comparing it with the budget.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. C = 2.4 × 3 + 0.15 × 28.

Step 2. Washing costs £7.20; drying costs £4.20. Total cost = £11.40.

Step 3. £12 - £11.40 = £0.60, so the budget is sufficient with 60p remaining. Check: each part is below £12, but only their total answers the question.

Misconception clinic

“C = (2.4 + 0.15) × (3 + 28).”

The formula has two separate products, which are then added. Combining the coefficients and inputs changes the relationship and introduces charges that were never stated.

SAY IT BACK

A formula is a structure to preserve, not a collection of numbers to combine in any order.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 03 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Evaluate 7x + 4 when x = 6.

2. Use F = 1.8C + 32 when C = -5.

3. Use A = (a + b)h ÷ 2 with a = 5, b = 9 and h = 4. Explain why brackets matter.

My check / correction and why it matters:

Model answers: page 96. Confidence: need support / can use / can explain. Next step: _____

LESSON 04 • UNDERSTAND

Fractions, decimals and percentages

LEARNING GOAL • DfE LEVEL 2 POINT 4

Convert between equivalent forms and choose a useful form for comparison.

A fraction a/b means a divided by b. A decimal records place value in powers of ten. A percentage records an amount per hundred. These are different ways to represent the same share: 3/4 = 0.75 = 75%.

To convert a fraction to a decimal, divide the numerator by the denominator. Multiply that decimal by 100 to write a percentage. To convert a percentage to a fraction, write it over 100 and simplify by dividing both parts by the same common factor.

To write a terminating decimal as a fraction, use its place value: 0.125 = 125/1000 = 1/8. For a percentage to a decimal, divide by 100: 2.5% = 0.025. Percentages above 100 represent more than one whole; 150% = 1.5 = 3/2.

Learn useful benchmarks: 1/2 is 50%; 1/4 is 25%; 1/5 is 20%; 1/8 is 12.5%; 1/10 is 10%. For an unfamiliar comparison convert every amount to the same form. Avoid rounding recurring decimals too early; 1/3 is exactly one third but only approximately 0.333.

Worked examples: follow the reasoning

1. Compare 7/20, 0.36 and 34%. Convert to percentages: 7 ÷ 20 × 100 = 35%; 0.36 × 100 = 36%. Thus 34% < 7/20 < 0.36.

2. Write 0.045 as a percentage and simplest fraction. Percentage: 4.5%. Fraction: 45/1000 = 9/200. Multiplying by 100 changes the notation, not the underlying share.

COMMON MISTAKE TO AVOID

0.6 is 60%, not 6%. Always check against a benchmark such as half.

LESSON 04 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

Three progress indicators describe the share of an induction course completed: Rowan 0.625, Morgan 61%, and Lee 5/8. Identify who has completed the largest share and explain any equal results.

Pause and plan with your tutor

1. Choose one common form for all three values. Which conversion is exact?
2. How can 5/8 be related to the benchmark 1/8?
3. Compare the converted values before ranking the learners.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. 0.625 × 100 = 62.5%. Morgan is already expressed as 61%.

Step 2. 5/8 = 0.625 = 62.5%, using 5 ÷ 8 or five lots of 12.5%.

Step 3. Rowan and Lee tie for the largest share, 62.5%; Morgan has completed 61%. The difference is 1.5 percentage points.

Misconception clinic

“61 is larger than 0.625, so Morgan has completed more.”

The numerical notation uses different scales. Compare like with like: 61% = 0.61, which is less than 0.625. A percentage sign changes the interpretation of the number.

SAY IT BACK

Equal shares can look different until written in a common form.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 04 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Write 3/8 as a decimal and percentage.

2. Write 125% as a decimal and mixed number.

3. A progress chart shows 0.035 of a course completed. A learner calls this 35%. Correct and explain the mistake.

My check / correction and why it matters:

Model answers: page 96. Confidence: need support / can use / can explain. Next step: _____

LESSON 05 • UNDERSTAND

Percentages of amounts

LEARNING GOAL • DfE LEVEL 2 POINT 5

Find a percentage of a quantity and express one quantity as a percentage of another.

A percentage calculation has a base: the amount treated as 100%. Identify this before starting. To find p% of an amount, multiply by p/100. For 18% of £250 use 0.18 × 250. To find what percentage a part is of a total, calculate part ÷ total × 100.

A non-calculator method builds the target percentage from known pieces. Ten per cent is one tenth; one per cent is one hundredth. Five per cent is half of ten per cent. For 17%, combine 10% + 5% + 2%. An efficient alternative is useful when numbers divide easily.

Quantities must have matching units before forming a percentage. For 450 g as a percentage of 2 kg, first write 2 kg = 2,000 g. Then calculate 450/2000 × 100. A percentage is a proportion, so units cancel.

A percentage can exceed 100. If 65 people attend when 50 were expected, attendance is 130% of the forecast. That means 30% more than forecast. It does not mean attendance increased by 130%.

Worked examples: follow the reasoning

1. Find 17% of £360 without a calculator. 10% = £36; 5% = £18; 2% = £7.20. Total = £61.20.

2. A team completes 42 of 56 jobs. Percentage complete = 42 ÷ 56 × 100 = 75%. Check: 42 is three quarters of 56.

COMMON MISTAKE TO AVOID

“A as a percentage of B” uses B as the denominator. Reversing the two amounts changes the answer.

LESSON 05 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A centre sends 160 course invitations. Seventy-two people reply. Of those who reply, 25% request an evening session. Find the response percentage and the number requesting evenings. State the whole used in each calculation.

Pause and plan with your tutor

1. For response rate, which group represents 100%?
2. For the evening preference, does 25% refer to invitations or replies?
3. Check the evening count does not exceed the total replies.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Response percentage = 72 ÷ 160 × 100 = 45%. The base is all 160 invitations.

Step 2. Evening requests = 0.25 × 72 = 18 people. The base is the 72 replies.

Step 3. A check: 18 × 4 = 72, so the evening requests are a quarter of replies. Non-replies are 88, but their preferences are unknown.

Misconception clinic

“25% of 160 is 40, so 40 people requested evenings.”

The second percentage refers only to respondents. Re-read the phrase that defines the group before choosing the base amount.

SAY IT BACK

A multi-step percentage problem can change its base from one stage to the next.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 05 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find 12% of £450 without a calculator.

2. Express 27 as a percentage of 45.

3. A course has 80 learners. 35% study in the morning; the rest study in the evening. Find the evening number and check the total.

My check / correction and why it matters:

Model answers: page 97. Confidence: need support / can use / can explain. Next step: _____

LESSON 06 • UNDERSTAND

Percentage change and original amounts

LEARNING GOAL • DfE LEVEL 2 POINT 6

Use multipliers for increases, decreases, reverse percentages and successive changes.

For an increase of p%, multiply by 1 + p/100. For a decrease of p%, multiply by 1 - p/100. A 12% increase uses 1.12; a 12% decrease uses 0.88. These multipliers include the original amount, so the result is the new total.

To find percentage change from two values, calculate the change divided by the original value, then multiply by 100. Say whether it is an increase or decrease. Using the final value as the denominator answers a different question.

A reverse percentage starts with the changed amount. Write changed amount = original × multiplier. Divide by the multiplier to recover the original. If a sale price is 80% of the original, dividing by 0.8 restores the original 100%. Adding 20% to the sale price does not.

Successive changes act on successive bases. Multiply the multipliers rather than add the percentages. An increase followed by the same percentage decrease usually does not return to the start. Keep full calculator precision until a final money answer is required.

Worked examples: follow the reasoning

1. A bicycle costs £408 after a 15% discount. £408 is 85% of the original. Original = 408 ÷ 0.85 = £480. Check 480 × 0.85 = 408.

2. A £500 machine rises by 10%, then falls by 10%. 500 × 1.10 × 0.90 = £495. The fall is £55 because it acts on £550. Overall change = -£5, a 1% decrease.

COMMON MISTAKE TO AVOID

Reverse percentage means divide by the multiplier, not subtract the advertised percentage from the final price.

LESSON 06 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A maintenance contract costs £864 after a 20% increase. Find the previous price. The manager later negotiates a 10% reduction on the £864 price. Find the negotiated price and compare it with the original.

Pause and plan with your tutor

1. What percentage of the original does £864 represent?
2. Why must the reverse calculation divide by 1.20?
3. Which amount is reduced by 10% in the second negotiation?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. £864 represents 120% of the original. Original price = 864 ÷ 1.20 = £720.

Step 2. Negotiated price = 864 × 0.90 = £777.60.

Step 3. The negotiated price remains £57.60 above £720. Overall increase = 57.60 ÷ 720 × 100 = 8%. Check the combined multiplier: 1.20 × 0.90 = 1.08.

Misconception clinic

“The changes cancel to a 10% increase because 20 - 10 = 10.”

The increase acts on £720 and the reduction acts on £864. Different bases mean the percentage changes cannot simply be subtracted.

SAY IT BACK

Reverse the first multiplier to recover the original; multiply successive multipliers to combine changes.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 06 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Increase £240 by 8%.

2. A weekly cost rises from £48 to £60. Find the percentage increase.

3. A £320 bill has a 10% reduction followed by a 5% increase. Find the final bill and overall percentage decrease.

My check / correction and why it matters:

Model answers: page 97. Confidence: need support / can use / can explain. Next step: _____

LESSON 07 • UNDERSTAND

Adding, subtracting and ordering fractions

LEARNING GOAL • DfE LEVEL 2 POINT 7

Use equivalent fractions to calculate with proper fractions, improper fractions and mixed numbers.

The denominator names the size of each equal part. You can only add or subtract counts of the same-sized part. For different denominators, find a common multiple and rewrite both fractions with it. Multiply the numerator and denominator by the same number so the value stays unchanged.

A proper fraction has a numerator smaller than its denominator. An improper fraction is at least one whole. A mixed number combines a whole number and a proper fraction. Convert a mixed number by multiplying the whole number by the denominator and adding the numerator: 2 3/5 = 13/5.

To add mixed numbers, either add whole and fractional parts separately, or convert both to improper fractions. For subtraction, improper fractions often avoid borrowing mistakes. Simplify the final fraction by dividing numerator and denominator by their highest common factor.

To compare fractions use a common denominator, a reliable decimal conversion or a clear benchmark. A larger denominator does not automatically give a larger fraction. One seventh is smaller than one fifth because the whole has been divided into more pieces.

Worked examples: follow the reasoning

1. 2 1/3 + 1 3/4 = 7/3 + 7/4 = 28/12 + 21/12 = 49/12 = 4 1/12.

2. 3 1/5 - 1 7/10 = 16/5 - 17/10 = 32/10 - 17/10 = 15/10 = 1 1/2. Check by adding 1 1/2 to 1 7/10 to recover 3 1/5.

COMMON MISTAKE TO AVOID

Do not add denominators. 1/3 + 1/4 is 7/12, not 2/7.

LESSON 07 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A small theatre has 4 1/2 hours of rehearsal time. Lighting checks use 1 2/3 hours and sound checks use 1 5/8 hours. Find the rehearsal time remaining as a mixed number of hours. Is a further 1 1/4-hour run-through possible?

Pause and plan with your tutor

1. Which denominator can represent halves, thirds and eighths?
2. Would improper fractions make the subtraction clearer?
3. Compare the remaining time with the required run-through using the same denominator.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Use denominator 24: 4 1/2 = 108/24; 1 2/3 = 40/24; 1 5/8 = 39/24.

Step 2. Remaining = (108 - 40 - 39)/24 = 29/24 = 1 5/24 hours.

Step 3. The run-through needs 1 1/4 = 30/24 hours. There is a shortage of 1/24 hour, which is 2.5 minutes. It does not fit.

Misconception clinic

“There are more than 1 hour left, so a 1 1/4-hour run-through fits.”

More than one hour does not imply at least one and a quarter hours. Compare the fractions exactly before making a capacity decision.

SAY IT BACK

A small fractional difference can determine a real scheduling decision.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 07 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Simplify 18/24 and convert 17/5 to a mixed number.

2. Calculate 4 1/4 - 2 2/3.

3. A roll contains 5 1/2 m of fabric. A maker cuts 1 3/4 m and 2 2/5 m. Find the amount left.

My check / correction and why it matters:

Model answers: page 98. Confidence: need support / can use / can explain. Next step: _____

LESSON 08 • UNDERSTAND

Fractions of quantities

LEARNING GOAL • DfE LEVEL 2 POINT 8 + foundation

Express one amount as a fraction of another, simplify and find a fractional share.

To express one amount as a fraction of another, put the first amount over the reference amount. Simplify both numerator and denominator by the same factor. If 18 out of 30 places are filled, the fraction filled is 18/30 = 3/5. The unfilled fraction is 1 - 3/5 = 2/5.

Match units first. For 250 ml as a fraction of 1.5 litres, use 250/1500 = 1/6. Mixing millilitres and litres in the fraction would produce a meaningless ratio.

To find a fraction of a quantity, divide by the denominator and multiply by the numerator. For 3/7 of 84, one seventh is 12 and three sevenths are 36. The same method works for money, lengths and counts, although a real-world count may require a whole-number decision.

When a known amount is a fractional part, reverse the process. If 3/5 of a budget is £420, one fifth is £140 and all five fifths are £700. Check that the part is smaller than the whole for a fraction between zero and one.

Worked examples: follow the reasoning

1. A £960 budget allocates 5/8 to equipment. One eighth = 960 ÷ 8 = £120. Five eighths = £600. The remainder is £360.

2. A tank contains 45 litres, which is 3/4 of capacity. One quarter = 45 ÷ 3 = 15 litres; full capacity = 4 × 15 = 60 litres.

COMMON MISTAKE TO AVOID

A fraction must refer to a named whole. “Three fifths” alone does not tell you which quantity was used.

LESSON 08 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A repair project has spent £540, which is 3/8 of its agreed budget. Find the total budget and the amount still available. A second purchase would cost £650. Decide whether the unspent budget covers it.

Pause and plan with your tutor

1. Is £540 the whole budget or only a fractional part?
2. If three eighths cost £540, what is one eighth?
3. Find the whole and remaining amount before evaluating the purchase.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. One eighth = £540 ÷ 3 = £180.

Step 2. Whole budget = 8 × £180 = £1,440. Remaining amount = £1,440 - £540 = £900.

Step 3. A £650 purchase is covered, leaving £250. Check: £540/£1,440 = 3/8, and £540 + £900 = £1,440.

Misconception clinic

“The budget is 540 × 3/8 = £202.50.”

Multiplying finds three eighths of £540, but £540 already represents three eighths. Recover one part first, then build the whole. The whole must exceed the known proper-fraction part.

SAY IT BACK

When a fractional part is known, divide by its numerator before rebuilding all parts.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 08 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Express 24 as a simplest fraction of 64.

2. Find 7/12 of £540.

3. A class has 32 learners; 3/8 travel by bus and 1/4 walk. Everyone else cycles. Find the cyclist number and fraction.

My check / correction and why it matters:

Model answers: page 98. Confidence: need support / can use / can explain. Next step: _____

LESSON 09 • UNDERSTAND

Decimals and sensible accuracy

LEARNING GOAL • DfE LEVEL 2 POINT 9

Compare, round and report decimals at an accuracy suited to the task.

Decimals extend the place-value system beyond the units column. In 14.286, the 2 is worth 0.2, the 8 is worth 0.08 and the 6 is worth 0.006. Compare decimals by lining up decimal points; zero padding helps you see equal positions.

For rounding, identify the place to keep and inspect the next digit. If that next digit is 5 or more, increase the kept digit by one; otherwise leave it. Remove later digits. A carry can cross the decimal point: 9.996 rounds to 10.00 to two decimal places.

Decimal places count positions after the decimal point. Significant figures count from the first non-zero digit. Thus 0.004862 to two significant figures is 0.0049. Only apply a requested accuracy; do not round every small number to a whole number.

Retain extra digits during multi-step work. Round at the end unless the context requires payment rounding at an earlier stage. Buying sufficient material may require rounding up to an available pack size, which differs from ordinary nearest-value rounding.

Worked examples: follow the reasoning

1. Round 38.475 to two decimal places. Keep hundredths (7); inspect thousandths (5); answer 38.48.

2. A pipe requirement is 12.02 m and pipes are sold in whole 3 m lengths. 12.02 ÷ 3 = 4.0066... . Four lengths give only 12 m; buy five. Nearest-whole rounding would lead to a shortage.

COMMON MISTAKE TO AVOID

Write money to two decimal places when giving a final payment: £8.50, not £8.5 or £8.500.

LESSON 09 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A technician measures three strips: 2.086 m, 2.08 m and 2.105 m. The minimum acceptable length is 2.085 m. Identify the acceptable strips using the original measurements, then report each measurement to 2 decimal places.

Pause and plan with your tutor

1. Write trailing zeros to compare all values at three decimal places.
2. Should acceptance use the original measurements or rounded reports?
3. Which extra digit controls rounding to 2 decimal places?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Write 2.080, 2.086 and 2.105 m. The 2.086 m and 2.105 m strips meet the minimum; 2.080 m does not.

Step 2. To 2 decimal places the reported measurements are 2.09 m, 2.08 m and 2.11 m respectively.

Step 3. Make the acceptance decision before reporting the rounded values. A rounded value may conceal which side of a close threshold an exact reading lies.

Misconception clinic

“Round everything first; 2.085 becomes 2.09, so any reported 2.09 m strip is acceptable.”

Rounding loses information. A reported 2.09 m can come from readings slightly below the exact threshold as well as above it. Use the precision supplied for the decision unless told otherwise.

SAY IT BACK

Distinguish the accuracy used for a decision from the accuracy requested for a report.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 09 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Order 4.09, 4.9, 4.009 and 4.19.

2. Round 0.07384 to two significant figures.

3. A floor needs 19.2 m² of material. Each box covers 1.5 m². Find boxes needed and unused coverage.

My check / correction and why it matters:

Model answers: page 99. Confidence: need support / can use / can explain. Next step: _____

LESSON 10 • UNDERSTAND

Calculating with decimals

LEARNING GOAL • DfE LEVEL 2 POINT 10

Use all four operations with values up to three decimal places.

For addition and subtraction, line up decimal points and append zeros to equalise decimal places. Treat the columns just as whole-number columns, keeping the decimal point aligned in the result. A decimal point is not a movable separator chosen for appearance.

For written multiplication, multiply as whole numbers and then restore the total number of decimal places in the two factors. In 1.25 × 0.8 there are three decimal places altogether: 125 × 8 = 1000, giving 1.000 = 1.

For division by a decimal, multiply both dividend and divisor by the same power of ten until the divisor is whole. For 4.32 ÷ 0.12, use 432 ÷ 12. Scaling both values preserves the quotient; scaling only the divisor does not.

Always estimate direction and size. Multiplying a positive value by a number between zero and one makes it smaller. Dividing by a positive number below one makes it larger. These checks are particularly useful for spotting decimal-placement errors.

Worked examples: follow the reasoning

1. Find 8.406 - 2.79. Write 8.406 - 2.790 = 5.616. Check: 5.616 + 2.790 = 8.406.

2. Six containers each hold 0.375 litres. Total = 6 × 0.375 = 2.25 litres. To fill 0.125-litre cups, calculate 2.25 ÷ 0.125 = 2250 ÷ 125 = 18 cups.

COMMON MISTAKE TO AVOID

When dividing by a decimal, move the decimal point in both numbers by the same number of places.

LESSON 10 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A workshop has 3.6 kg of resin. Each mould uses 0.145 kg and preparation wastes 0.08 kg in total. Find whether there is enough resin for 24 moulds and how much remains or is missing.

Pause and plan with your tutor

1. Which loss is per mould, and which happens once?
2. Calculate the mould requirement, then add the one-off waste.
3. Compare the total requirement with the starting mass using aligned decimal places.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Moulds use 24 × 0.145 = 3.480 kg.

Step 2. Total required = 3.480 + 0.080 = 3.560 kg.

Step 3. Available 3.600 - 3.560 = 0.040 kg, or 40 g. There is enough. Check: ignoring waste would leave 120 g; the 80 g waste reduces this to 40 g.

Misconception clinic

“Each mould uses 0.145 + 0.08 kg, so multiply 0.225 by 24.”

The preparation waste is a single total, not a repeated loss for each mould. Identify how frequently each quantity applies.

SAY IT BACK

Decimal accuracy matters, but the first decision is which amounts repeat.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 10 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Calculate 6.075 + 0.89.

2. Calculate 2.4 × 0.35.

3. A machine uses 0.275 kg of material for each part. Starting with 6 kg, it makes 18 parts. Find the material left and check whether 4 more parts can be made.

My check / correction and why it matters:

Model answers: page 99. Confidence: need support / can use / can explain. Next step: _____

LESSON 11 • UNDERSTAND

Ratio and proportion

LEARNING GOAL • DfE LEVEL 2 POINT 11

Share in a ratio, scale directly and recognise inverse proportion.

A ratio compares quantities in a stated order. Red:blue = 2:3 means two red parts for every three blue parts. It does not mean red is two thirds of the total; there are five parts altogether. Simplify a ratio by dividing every part by the same factor after matching units.

To share a total in a ratio, add the ratio parts, divide the total by that sum and multiply by each part. If one share is given instead, divide it by its own ratio number to find one part. Keep the labels attached throughout.

In direct proportion, multiplying one quantity by a factor multiplies the other by the same factor. For a fixed recipe, ingredients scale with servings. Find the amount for one serving or use a single scale factor. Direct proportion has a constant ratio y/x.

In inverse proportion, one quantity increases as the other decreases so their product stays constant. For a fixed job, equally productive workers × time is constant if everyone works at the same rate with no interference. State this assumption: real projects do not always follow inverse proportion.

Worked examples: follow the reasoning

1. Share £840 in the ratio 3:4:5. Total parts = 12; one part = £70. Shares = £210, £280 and £350. Check their sum is £840.

2. Six equally productive workers finish a job in 15 hours. For ten workers, constant work = 6 × 15 = 90 worker-hours. Time = 90 ÷ 10 = 9 hours. More workers should need less time.

COMMON MISTAKE TO AVOID

First decide whether the total, one share or a fixed amount of work is given. The same numbers can require different methods.

LESSON 11 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

Two identical-format display teams use staff in the ratio assembly:checking = 4:3. There are 18 checkers. Find assembly staff and total staff. For a separate fixed packing job, 7 equally productive workers take 12 hours. Estimate time with 8 such workers.

Pause and plan with your tutor

1. Does 18 represent all ratio parts or just the three checking parts?
2. Find one ratio part, then the requested categories.
3. For the packing job, what quantity stays constant when staffing changes?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Three checking parts = 18 people, so one part = 6. Assembly staff = 4 × 6 = 24; total staff = 42.

Step 2. Packing work = 7 × 12 = 84 worker-hours. With 8 workers, time = 84/8 = 10.5 hours.

Step 3. This is 10 hours 30 minutes. It assumes the same productivity, the same job and no interference or delays. More workers should produce a shorter time under this model.

Misconception clinic

“Eight workers are more than seven, so multiply 12 by 8/7.”

That scales time directly instead of inversely. For a fixed job, use the constant product of workers and hours.

SAY IT BACK

Ratio allocation and inverse proportion use different models; identify the relationship before calculating.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 11 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Simplify 450 ml : 1.2 litres.

2. A recipe for 8 people uses 600 g of rice. Find rice for 14 people.

3. A drink has concentrate:water = 2:7. A caterer needs 27 litres and owns 5 litres of concentrate. Is this enough? Show the required amounts.

My check / correction and why it matters:

Model answers: page 100. Confidence: need support / can use / can explain. Next step: _____

LESSON 12 • UNDERSTAND

Order of operations and powers

LEARNING GOAL • DfE LEVEL 2 POINT 12 + patterns

Evaluate multi-step expressions and use numerical and spatial patterns.

Calculations follow an agreed order: brackets, powers, then multiplication and division from left to right, then addition and subtraction from left to right. Multiplication is not always before division; they have equal priority. Addition and subtraction also share priority.

An index is a power. For example $4^3 = 4 \times 4 \times 4 = 64$. The expression 4×3 is 12 and means something different. Squaring a length changes a length unit into a square unit, so label results carefully.

A numerical pattern may have a constant difference or a multiplying rule. Write the changes between terms and test a proposed rule against more than one term. To use a position rule, substitute the position number. Avoid assuming the first term is the rule itself.

Spatial patterns can be counted by separating fixed and repeating parts. If a row of joined squares uses four sticks for the first square and three extra sticks for each additional square, n squares need $3n + 1$ sticks. Sharing edges explains why the count is not $4n$.

Worked examples: follow the reasoning

1. Evaluate $18 - 2 \times (3 + 1)^2 \div 4$. Brackets give 4; square gives 16; multiply and divide left to right: $2 \times 16 \div 4 = 8$. Final value = 10.

2. A sequence follows $T = 5n - 2$. At $n = 1$, $T = 3$; at $n = 6$, $T = 28$. Each new position adds five, which agrees with the coefficient of n.

COMMON MISTAKE TO AVOID

Do not calculate strictly from left to right before checking for brackets and powers.

LESSON 12 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

An equipment-locker code is calculated from $K = 30 - 2(n + 1)^2 \div 3$. Use $n = 2$ to find K. A separate strip pattern uses $5n + 2$ tiles at stage n. Explain the calculation order and find the tiles at stage 12.

Pause and plan with your tutor

1. What must happen inside the brackets before the square?
2. Which operations have equal priority and must be worked left to right?
3. How does the coefficient of n describe a repeating pattern?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. For K: $n + 1 = 3$, then $3^2 = 9$.

Step 2. Multiply and divide from left to right: $2 \times 9 \div 3 = 6$. Thus $K = 30 - 6 = 24$.

Step 3. For the pattern: $5 \times 12 + 2 = 62$ tiles. Each new stage adds five tiles while the fixed contribution stays two.

Misconception clinic

“30 - 2 = 28, then use the brackets.”

The subtraction must wait until brackets, powers, multiplication and division are complete. Write intermediate expressions so the changing structure remains visible.

SAY IT BACK

Operation order controls the calculation; a pattern rule controls how the structure grows.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 12 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Calculate $7 + 3 \times 4^2$.

2. Calculate $(15 - 3) \div 4 + 2^3$.

3. Joined squares use $3n + 1$ sticks. Find sticks for 25 squares and the greatest complete number of squares possible with 100 sticks.

My check / correction and why it matters:

Model answers: page 100. Confidence: need support / can use / can explain. Next step: _____

LESSON 13 • UNDERSTAND

Money, tax, budgets and compound interest

LEARNING GOAL • DfE LEVEL 2 POINT 13

Build a budget, compare offers and calculate repeated percentage growth.

A budget compares income with expenditure over the same period. Add regular and one-off costs separately, then calculate income minus expenditure. A positive result is a surplus; a negative result is a shortfall. Weekly and monthly amounts need a stated conversion: 52 weeks per year gives an average month of 52/12 weeks, not exactly four.

Tax, discount and interest examples in this book use invented rates for maths practice. Use the rate and taxable amount supplied in a question. A tax on a bill increases the price using a multiplier. If tax applies only above an allowance, subtract that allowance before applying the stated rate.

Simple interest is calculated on the original deposit each period. Compound interest is calculated on the current balance, including earlier interest. After n equal periods at r% per period, balance = initial amount × (1 + r/100)^n. Interest earned = final balance minus initial deposit.

When comparing deals, calculate the final total for the same quantity and time. Include delivery, tax and recurring charges where stated. A lower headline rate may not give a lower final bill. Explain your choice using the difference in pounds.

Worked examples: follow the reasoning

1. £1,500 grows at 4% each year for two years, without withdrawals. Year 1: 1500 × 1.04 = £1,560. Year 2: 1560 × 1.04 = £1,622.40. Interest earned = £122.40, not £120.

2. Equipment is £240 before a 15% discount and a stated 20% tax applied to the discounted price. 240 × 0.85 × 1.20 = £244.80. Keep the order and the tax base explicit.

COMMON MISTAKE TO AVOID

Compound interest repeats the multiplier. Do not multiply the original interest by the number of years unless interest is simple.

LESSON 13 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A training fund receives £2,000. It earns 2.5% compound interest each year for 2 years, with no other transactions. After the interest is added, a £1,480 course bill receives a 7% discount. Find the fund balance remaining after paying the discounted bill.

Pause and plan with your tutor

1. Which amount earns interest, and how many times is the multiplier applied?
2. Does the discount reduce the fund or reduce the course bill?
3. Keep the fund calculation and bill calculation separate until the payment stage.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Fund after interest = $2000 \times 1.025^2 = £2{,}101.25$.

Step 2. Discounted bill = $1480 \times 0.93 = £1{,}376.40$.

Step 3. Remaining fund = 2101.25 - 1376.40 = £724.85. Check the first year adds £50; the second adds £51.25, reflecting growth on the larger balance.

Misconception clinic

“Two years at 2.5% means exactly 5%, so the fund is £2,100.”

That treats interest as simple. Compound interest applies each year to the current balance, including previous interest.

SAY IT BACK

Name each money quantity so interest, discounts and payments act on the correct base.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 13 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. A budget has £1,460 income and costs £620, £185, £245 and £110. Find the surplus.

2. Find £800 after 3 years at 5% annual compound interest, to the nearest penny.

3. A worker earns £240 each week and budgets £910 each average month. Use 52 weeks per year to find average monthly income and surplus.

My check / correction and why it matters:

Model answers: page 101. Confidence: need support / can use / can explain. Next step: _____

LESSON 14 • UNDERSTAND

Units, conversion factors and graphs

LEARNING GOAL • DfE LEVEL 2 POINT 14

Convert within and between systems and read a conversion graph.

Before converting, identify the starting unit, target unit and conversion relationship. Within metric units: 1 km = 1,000 m; 1 m = 100 cm = 1,000 mm; 1 kg = 1,000 g; 1 litre = 1,000 ml. A smaller unit needs a larger numerical count.

A conversion factor links systems. If 1 mile is approximately 1.6 km, multiply miles by 1.6 to obtain kilometres; divide kilometres by 1.6 to obtain miles. Factors used in exam questions may be rounded. Use the supplied relationship rather than replace it with a different remembered approximation.

A conversion graph represents pairs of equivalent measurements. Read the first axis, move to the line, then move across to the other axis. Interpolate between grid marks when needed and report a sensible degree of accuracy. A straight conversion line through the origin represents a fixed multiplying factor.

Area and volume conversions need squared and cubed factors. Since 1 m = 100 cm, a 1 m square is 100 cm by 100 cm, so 1 m^2 = 10,000 cm^2. Similarly 1 m^3 = 1,000,000 cm^3. Capacity link: 1 cm^3 = 1 ml and 1,000 cm^3 = 1 litre.

Worked examples: follow the reasoning

1. Using 1 kg ≈ 2.2 lb, a 77 lb parcel has mass 77 ÷ 2.2 = 35 kg. Check: 35 × 2.2 = 77.

2. On the conversion graph, 5 gallons corresponds to 22.5 litres. The slope is 4.5 litres per gallon. Therefore 18 litres corresponds to 4 gallons. The graph uses the supplied rounded relationship.

COMMON MISTAKE TO AVOID

Label both axes and units. Do not use the conversion factor backwards; test it on the given one-unit relationship.

LESSON 14 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

An exhibition stand can carry a parcel up to 30 kg. A parcel label gives 61.6 lb. Use 1 kg ≈ 2.2 lb. The parcel contains 4 equal pieces, and the buyer removes one before transport. Find the transported mass in kg and decide whether the limit is met.

Pause and plan with your tutor

1. Which direction converts pounds of mass to kilograms? Do not confuse lb with £.
2. Find the original mass before taking a fraction of the contents.
3. What assumption lets each piece be assigned the same mass?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Original mass = 61.6 ÷ 2.2 = 28 kg, using the supplied rounded factor.

Step 2. Removing one of four equal-mass pieces leaves 3/4 of the mass: 28 × 3/4 = 21 kg.

Step 3. The 30 kg limit is met, with 9 kg capacity remaining. This assumes the label refers to the contents and negligible packaging mass, or that all mass is shared equally as stated.

Misconception clinic

“61.6 × 2.2 = 135.52 kg.”

One kilogram corresponds to more than one pound, so converting a fixed mass from lb to kg should reduce its numerical value. Test the method on 2.2 lb, which must become 1 kg.

SAY IT BACK

Use the given relationship as a one-unit test before applying a conversion factor.

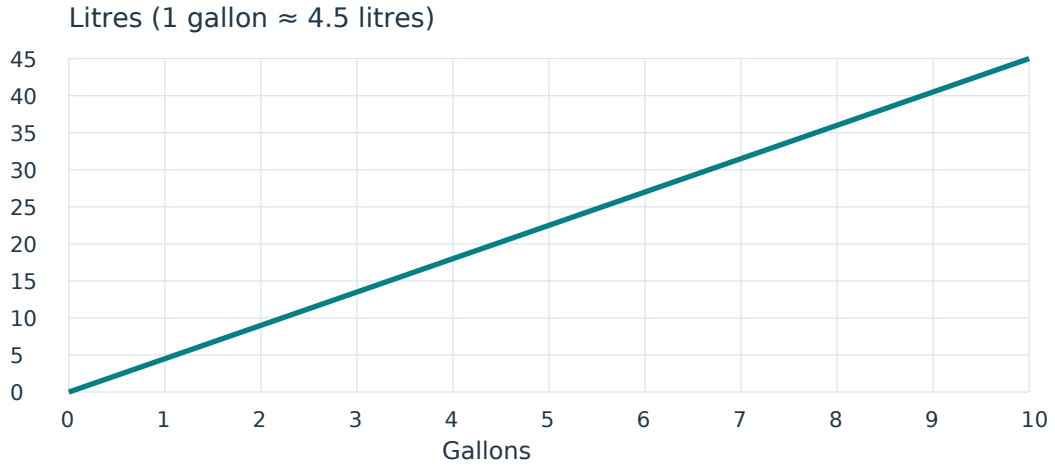
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 14 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Convert 3.75 km to metres and 840 ml to litres.

2. Use the printed graph to convert 7 gallons to litres and 36 litres to gallons.

3. A container has volume 0.018 m³. Find its capacity in litres. Explain why multiplying by 100 is incorrect.

My check / correction and why it matters:

Model answers: page 101. Confidence: need support / can use / can explain. Next step: _____

LESSON 15 • UNDERSTAND

Speed, density, pay and time

LEARNING GOAL • DfE LEVEL 2 POINT 15

Use compound measures and convert minutes into fractions of an hour.

A compound measure combines quantities. Speed = distance ÷ time; density = mass ÷ volume; a rate of pay is money per unit time. The word per usually means division. Keep compatible units throughout: kilometres divided by hours gives km/h, not m/s.

For average speed use total distance divided by total elapsed time. You cannot usually average two speeds directly because the time spent at each speed may differ. To find distance, multiply speed by time. To find time, divide distance by speed.

Minutes are sixtieths of an hour. Thus 1 hour 24 minutes = 1 + 24/60 = 1.4 hours. Conversely 2.35 hours = 2 hours plus 0.35 × 60 = 21 minutes. Clock notation and decimal hours must not be interchanged.

Density compares mass with volume. If mass is in grams and volume in cm^3, density is in g/cm^3. For wages, calculate normal and overtime portions at their own rates, then add. Read whether breaks are paid. A 24-hour timetable may cross midnight, so split the interval at midnight if useful.

Worked examples: follow the reasoning

1. A driver travels 126 km in 1 hour 45 minutes. Time = 1.75 hours. Average speed = 126 ÷ 1.75 = 72 km/h.

2. A shift lasts 08:30 to 17:00 with a 30-minute unpaid break. Paid time = 8 hours. At £12.40 per hour, pay = 8 × 12.40 = £99.20. Check the break is subtracted once.

COMMON MISTAKE TO AVOID

1 hour 30 minutes is 1.5 hours. Using 1.30 in a speed or wages calculation changes the time.

LESSON 15 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A school minibus travels 84 km at an average moving speed of 56 km/h. A stop adds 18 minutes. It departs at 10:35. Find the arrival time and the driver's pay at an invented £13.20 per hour for the full trip including the stop.

Pause and plan with your tutor

1. Find moving time using distance divided by speed. What unit does that give?
2. Convert hours to minutes before adding a stop in minutes.
3. Which total time must be converted back to decimal hours for pay?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Moving time = 84/56 = 1.5 hours = 90 minutes.

Step 2. Total time = 90 + 18 = 108 minutes = 1 hour 48 minutes. Arrival = 12:23.

Step 3. Paid time = 108/60 = 1.8 hours. Pay = 1.8 × £13.20 = £23.76. Check that 0.8 hour equals 48 minutes, not 80 or 8 minutes.

Misconception clinic

"One hour 48 minutes is 1.48 hours, so pay is £19.536."

Clock minutes are sixtieths, not hundredths. Divide the 48 minutes by 60 before combining it with whole hours.

SAY IT BACK

Use one consistent time unit at each calculation stage, then convert deliberately.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 15 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Convert 2 hours 18 minutes to decimal hours.

2. Find the density of a 540 g object with volume 200 cm^3.

3. A journey starts at 23:40 and ends at 01:15 next day, covering 114 km. Find elapsed time and average speed.

My check / correction and why it matters:

Model answers: page 102. Confidence: need support / can use / can explain. Next step: _____

LESSON 16 • UNDERSTAND

Perimeter, area and composite shapes

LEARNING GOAL • DfE LEVEL 2 POINT 16

Choose boundary or coverage calculations and work with circles and composite shapes.

Perimeter measures distance around a boundary and has length units. Area measures coverage and has square units. Draw a labelled sketch before calculating; a question about fencing normally needs perimeter, while paint or flooring usually needs area.

Rectangle area = length × width. Triangle area = base × perpendicular height ÷ 2. The height meets the base at a right angle; a sloping side is not automatically the height. For a circle, diameter d = 2r, circumference = πd = 2πr and area = πr². Keep radius and diameter distinct.

For a composite shape, divide it into simpler shapes with no overlap or subtract a missing piece from a larger whole. Infer missing lengths from totals. For perimeter, follow only exposed outside edges; internal dividing lines do not count.

A semicircle has half a circle's area. Its perimeter includes both the curved part and its straight diameter. In purchasing problems, first calculate the area, then account for waste and pack sizes as instructed. Unless a question says otherwise, keep π at calculator precision until the end.

Worked examples: follow the reasoning

1. An L-shaped floor is an 8 m by 6 m rectangle with a 3 m by 2 m corner removed. Area = 48 - 6 = 42 m². The cut creates two new edges, 3 m and 2 m, replacing equal removed outside lengths; perimeter remains 28 m.

2. A semicircular window has radius 2 m. Area = 0.5 × π × 2² = 2π ≈ 6.28 m². Boundary = π × 2 + 4 ≈ 10.28 m. The straight base is included in the boundary.

COMMON MISTAKE TO AVOID

Before using a formula, name the requested quantity and its unit. Area and perimeter cannot be added together.

LESSON 16 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A rectangular banner is 3.2 m wide and 1.4 m high. A triangular panel of base 1.2 m and perpendicular height 0.8 m is attached without overlap. Printing costs £9 per m² of material used. Find the total printing cost, to the nearest penny.

Pause and plan with your tutor

1. Which two areas must be calculated, and is there overlap to subtract?
2. Which height belongs in the triangle formula?
3. Multiply the total area by the rate only after combining areas.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Rectangle area = 3.2 × 1.4 = 4.48 m².

Step 2. Triangle area = 1.2 × 0.8 / 2 = 0.48 m². Total printed area = 4.96 m².

Step 3. Cost = 4.96 × £9 = £44.64. A useful estimate is about 5 m² × £9 = £45, close to the exact result.

Misconception clinic

“The triangular panel has area 1.2 × 0.8 = 0.96 m².”

That gives the area of its enclosing rectangle. The triangle occupies half the rectangle when the same base and perpendicular height are used.

SAY IT BACK

Decompose a composite area, label the pieces and combine only matching square units.

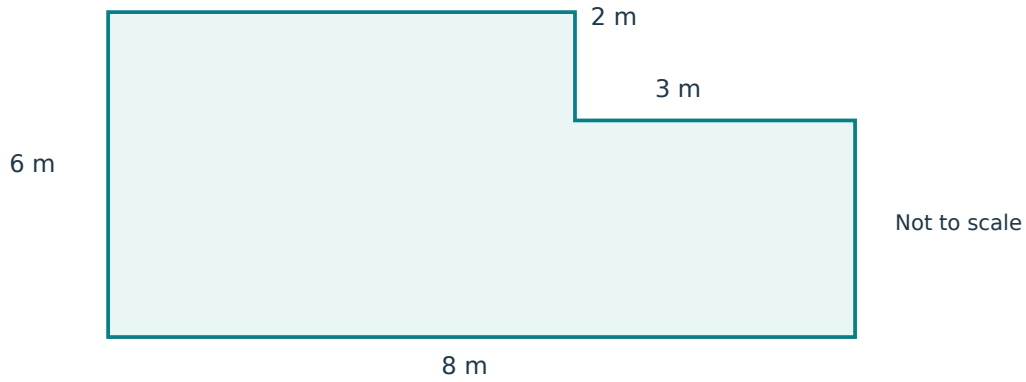
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 16 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find area and perimeter of a 7.2 m by 4.5 m rectangle.

2. Use the printed L-shaped plan to calculate area and perimeter.

3. The 42 m² L-shaped floor needs 8% extra material. Boxes cover 2.5 m² and cost £24 each. Find the minimum material cost.

My check / correction and why it matters:

Model answers: page 102. Confidence: need support / can use / can explain. Next step: _____

LESSON 17 • UNDERSTAND

Volume and surface area

LEARNING GOAL • DfE LEVEL 2 POINT 17

Calculate capacity and coverage of solids, including closed and open cylinders.

Volume measures the space inside a solid, using cubic units. Surface area adds the areas of its outside faces, using square units. A cuboid of dimensions l, w and h has volume lwh. Its three pairs of matching rectangular faces give surface area 2lw + 2lh + 2wh.

A cylinder has a circular cross-section repeated through its height. Volume = $\pi r^2 h$. Unrolling its curved side makes a rectangle with width $2\pi r$ and height h, so curved surface area = $2\pi rh$. A closed cylinder includes two circles: total surface area = $2\pi rh + 2\pi r^2$.

An open container may have one circular base and no lid. Use only the surfaces that actually exist or need covering. State whether internal or external dimensions are given; capacity requires internal dimensions. For other solids, carefully substitute into any formula supplied.

Convert dimensions into one length unit before multiplying. A volume in cm^3 can become litres by dividing by 1,000. Sketching a net helps you count surfaces. Round the final answer as requested, then decide whether a real-world purchase needs to be rounded up.

Worked examples: follow the reasoning

1. A cylinder has radius 3 cm and height 10 cm. Volume = $\pi \times 9 \times 10 = 90\pi \approx 282.74$ cm^3. Closed surface area = $2\pi \times 3 \times 10 + 2\pi \times 9 = 78\pi \approx 245.04$ cm^2.

2. A tank measures 60 cm × 40 cm × 35 cm internally. Capacity = 84,000 cm^3 = 84 litres. At 75% full it holds 63 litres. Converting to litres after multiplying avoids mixed units.

COMMON MISTAKE TO AVOID

A diameter must be halved before substitution for r. Squaring the diameter produces four times the correct circle area.

LESSON 17 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

An open cylindrical display pot has internal radius 6 cm and height 15 cm. Ignore wall thickness. Find its capacity in ml and the area of material needed for the curved side and one base. Give final results to the nearest whole unit.

Pause and plan with your tutor

1. Which requested answer is volume, and which is surface area?
2. How many circular faces are present on an open pot?
3. When does cm^3 convert directly into ml?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Volume = $\pi \times 6^2 \times 15 = 540\pi$ $cm^3 \approx 1{,}696.46$ cm^3, so capacity ≈ 1,696 ml.

Step 2. Curved area = $2\pi \times 6 \times 15 = 180\pi$ cm^2. One base adds $\pi \times 6^2 = 36\pi$ cm^2.

Step 3. Material area = $216\pi \approx 679$ cm^2. Do not add a second base because there is no lid. Capacity has cubic units; material coverage has square units.

Misconception clinic

“The material area is the same number as the capacity because both use the radius and height.”

The formulas describe different quantities. A volume multiplies base area by height; a surface calculation adds the areas of the existing surfaces.

SAY IT BACK

Sketch or list the surfaces before deciding which surface-area terms to include.

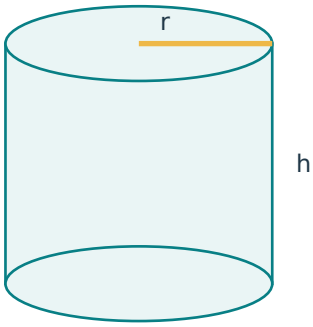
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 17 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Volume = $\pi r^2 h$

Closed area = $2\pi rh + 2\pi r^2$

Open top: remove one πr^2

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find volume and closed surface area of a cuboid 8 cm by 5 cm by 3 cm.

2. Find surface area of a closed cylinder with radius 2 cm and height 5 cm, to 2 decimal places.

3. A tank has internal dimensions 1.2 m by 0.5 m by 0.6 m. It is 80% full. Find litres of water and the time to empty it at 12 litres per minute.

My check / correction and why it matters:

Model answers: page 103. Confidence: need support / can use / can explain. Next step: _____

LESSON 18 • UNDERSTAND

Scale drawings and actual dimensions

LEARNING GOAL • DfE LEVEL 2 POINT 18

Read scale ratios and draw a measured plan to a stated scale.

A scale states how drawing lengths relate to real lengths. At 1:50, one centimetre on the drawing represents 50 centimetres in reality. A ratio scale must compare the same units. For 1 cm representing 2 m, write 1:200 because 2 m = 200 cm.

To find a real length, multiply the drawing length by the scale factor, then convert units if needed. To find a drawing length, convert the real length into the drawing unit and divide by the factor. A table labelled drawing and actual prevents reversing the calculation.

To create a plan, decide a scale that fits the paper, calculate every drawing dimension, use a ruler and mark the scale and units. A scale diagram is measured; an illustrative sketch is only a guide. A scale bar remains useful if a page is resized, but a printed ratio alone can become unreliable.

Length scale factors do not apply directly to area or volume. If drawing lengths double, drawn areas become four times as large. For practical floor area, calculate actual lengths first, then multiply. A drawing's centimetres are not the room's metres.

Worked examples: follow the reasoning

1. A wall is 8.4 cm long on a 1:50 plan. Actual length = 8.4 × 50 = 420 cm = 4.2 m.

2. A room is 6 m by 4.5 m. At 1:100, convert to 600 cm by 450 cm and divide by 100. Draw a rectangle 6 cm by 4.5 cm. Label 1:100, and print at actual size when checking with a ruler.

COMMON MISTAKE TO AVOID

Use the printed measurements rather than measuring an illustration unless it is explicitly a scale drawing. Screens and printers may resize pages.

LESSON 18 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A display board measures 2.4 m wide and 1.8 m high. A designer must draw its front view at 1:20. A 40 cm wide logo is centred horizontally. Calculate the drawing dimensions and how far each side of the logo is from the board edge on the drawing.

Pause and plan with your tutor

1. Convert every real length into centimetres before using a ratio scale.
2. Find drawing widths by dividing by 20.
3. How is the spare width shared when an object is centred?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Board dimensions are 240 cm by 180 cm, so draw a rectangle 12 cm by 9 cm.

Step 2. Logo width 40/20 = 2 cm on the drawing. Spare width = 12 - 2 = 10 cm.

Step 3. For a centred logo, each side margin is 10/2 = 5 cm. Check 5 + 2 + 5 = 12 cm. State 1:20 and print at actual size if measured.

Misconception clinic

“A 1:20 drawing makes the 2.4 m board 48 m wide.”

A reduction drawing divides real lengths by the scale factor. Multiplying gives the reverse direction from a drawing to reality.

SAY IT BACK

Centred placement adds a second geometric decision after the scale conversion.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 18 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. On a 1:200 plan, a line is 3.8 cm. Find the actual length in metres.

2. Write “1 cm represents 5 m” as a ratio scale.

3. A rectangular garden is 4.5 cm by 3 cm on a 1:200 plan. Find actual area and fencing cost at £9 per metre, with a 1 m gap for a gate.

My check / correction and why it matters:

Model answers: page 103. Confidence: need support / can use / can explain. Next step: _____

LESSON 19 • UNDERSTAND

Coordinates in four quadrants

LEARNING GOAL • DfE LEVEL 2 POINT 19

Plot, read and calculate positions using positive and negative coordinates.

Coordinates identify a point using an ordered pair (x, y). Start at the origin, move horizontally according to x, then vertically according to y. Positive x is right, negative x left; positive y is up, negative y down. The order matters: (2, -3) and (-3, 2) are different points.

Both axes need a consistent labelled scale. A point on the x-axis has y = 0; a point on the y-axis has x = 0. Use a small cross or dot so the exact grid intersection can be seen. Name points clearly without covering the coordinate location.

For horizontal or vertical distances, subtract the coordinates that change and take the positive distance. Points with the same y lie on a horizontal line. Points with the same x lie on a vertical line. Use these facts to complete rectangles and locate missing vertices.

A midpoint lies halfway between corresponding coordinates. For a horizontal or vertical segment you can count half the distance; averaging x-values and y-values also works. A negative coordinate is a position, not an instruction to make the final distance negative.

Worked examples: follow the reasoning

1. A(-4, 2), B(3, 2) and C(3, -3) are consecutive rectangle vertices. The fourth vertex is D(-4, -3): copy A's x and C's y. Width = 3 - (-4) = 7; height = 2 - (-3) = 5. Area = 35 square units.

2. The midpoint of (-6, 4) and (2, 4) is (-2, 4), because (-6 + 2) ÷ 2 = -2 and the y-value stays 4.

COMMON MISTAKE TO AVOID

Across comes before up or down. Keep the minus sign attached to the correct coordinate.

LESSON 19 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

On a square grid, a noticeboard has corners A(-5,1), B(1,1), C(1,5) and D(-5,5). A fixing point lies at the centre. Give the centre coordinates and the board's area in square grid units.

Pause and plan with your tutor

1. Which coordinate changes across a horizontal edge?
2. How can a midpoint be found when one endpoint coordinate is negative?
3. Find width and height as positive distances before multiplying.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Width = 1 - (-5) = 6 units. Height = 5 - 1 = 4 units.

Step 2. Centre x = (-5 + 1)/2 = -2; centre y = (1 + 5)/2 = 3. Fixing point is (-2,3).

Step 3. Area = 6 × 4 = 24 square units. Check the centre is 3 units from both vertical edges and 2 units from both horizontal edges.

Misconception clinic

"The centre is (2,3) because a midpoint must have positive coordinates."

A position may be negative even though all distances are positive. The board's horizontal midpoint lies left of the y-axis.

SAY IT BACK

Keep positions signed; keep lengths and areas non-negative.

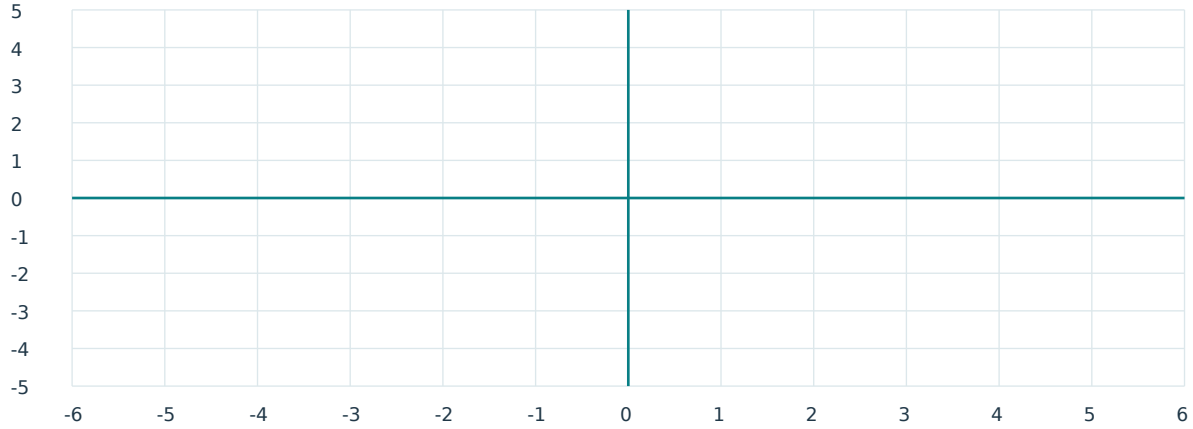
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 19 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Plot A(-3, 2), B(4, 2), C(4, -2) and D(-3, -2) on the grid.

2. Find the area and perimeter of ABCD in grid units.

3. A sensor is at (-2, -4). It moves 5 units right, 7 up and 1 left. Give its final coordinates and explain the change in each coordinate.

My check / correction and why it matters:

Model answers: page 104. Confidence: need support / can use / can explain. Next step: _____

LESSON 20 • UNDERSTAND

Solids, nets and views

LEARNING GOAL • DfE LEVEL 2 POINT 20

Connect 3-D objects with their nets and two-dimensional views.

A three-dimensional object has length, width and height. A face is a flat surface, an edge joins faces and a vertex is a corner. A cube has six square faces, twelve edges and eight vertices. A cylinder has two circular flat faces and one curved surface.

A net is a flat arrangement of faces that can fold into a solid. Each face must be the right shape and size, with matching edge lengths. A cuboid net needs six rectangles in three matching pairs. Count faces, but also check their placement: six correct faces can still overlap when folded.

A cylinder net contains a rectangle and two circles. The rectangle's long dimension is the circle's circumference, not its diameter. Its other dimension is the cylinder's height. This connection explains the curved-surface formula $2\pi rh$.

A plan is a view from above; an elevation is a straight-on view from a stated side. Views show projected outlines, not perspective depth. A circle in a plan can represent a cylinder or cone, so a single view may not uniquely identify an object. Use all supplied views and dimensions.

Worked examples: follow the reasoning

1. A cuboid is 6 cm long, 4 cm wide and 3 cm high. Its six net faces are two 6 × 4 rectangles, two 6 × 3 rectangles and two 4 × 3 rectangles. Their total area is $2(24 + 18 + 12) = 108$ cm².

2. A cylinder has diameter 8 cm and height 10 cm. Its curved net rectangle is 8π cm by 10 cm. Two circles of radius 4 cm complete a closed-cylinder net.

COMMON MISTAKE TO AVOID

A perspective sketch can make parallel edges look slanted. Use stated dimensions and properties, not apparent lengths.

LESSON 20 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A packaging designer is making a closed cuboid carton 9 cm long, 6 cm wide and 4 cm high. List its face pairs and find total card area before overlaps or tabs. Explain why a single 9 cm by 6 cm plan cannot establish the card area.

Pause and plan with your tutor

1. Which three different rectangle sizes make the six faces?
2. How many of each rectangle are required?
3. What dimension is missing from a plan alone?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. The face pairs are two 9 × 6 rectangles, two 9 × 4 rectangles and two 6 × 4 rectangles.

Step 2. Total card area = $2(54 + 36 + 24) = 228$ cm², excluding tabs and waste.

Step 3. A plan shows the 9 by 6 footprint but not the 4 cm height. Cartons of different heights can share the same plan and need different card areas.

Misconception clinic

“A carton has six faces, so its area is 6 × 9 × 6.”

Only a cube has six equal square faces. A cuboid has three potentially different pairs, each of which must be identified from its dimensions.

SAY IT BACK

Use a net or a face list to connect a solid with the areas needed to make it.

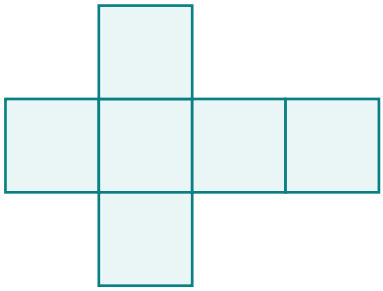
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 20 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



One valid cube net

Six equal square faces

Fold along shared edges

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. State the faces, edges and vertices of a cube.

2. For a cylinder of radius 5 cm and height 12 cm, give the curved net rectangle dimensions.

3. Sketch a valid cube net on squared paper. Explain how you checked that folding will not overlap faces.

My check / correction and why it matters:

Model answers: page 104. Confidence: need support / can use / can explain. Next step: _____

LESSON 21 • UNDERSTAND

Drawing solids, plans and elevations

LEARNING GOAL • DfE LEVEL 2 POINT 21

Construct a 3-D sketch and draw accurate views of a block arrangement.

A drawing should communicate shape and dimensions. In an oblique cuboid sketch, draw a front rectangle, then equal parallel depth lines sloping backwards and join their ends. Mark hidden edges with dashed lines if needed. A sketch can show a shape clearly without being a measured scale drawing.

An isometric drawing uses three sets of parallel directions; vertical edges remain vertical and the other two directions slope evenly. Equal cube edges are represented consistently. Draw the nearest edges clearly and avoid adding visible lines through solid faces.

For a block model, start from a height table. Each table cell describes the stack at one floor position. The plan shows every occupied floor cell. The front elevation shows the greatest height in each left-to-right column; it does not add heights hidden behind one another.

The side elevation similarly shows the greatest height along each front-to-back position. Always mark the viewing direction and which row is front. A plan alone shows a footprint but cannot usually show all stack heights. Check a drawing against the source rather than relying on memory.

Worked examples: follow the reasoning

1. A model has two floor rows. Front row heights, left to right: 1, 3, 2. Back row heights: 2, 1, 0. Total cubes = 1 + 3 + 2 + 2 + 1 = 9. Plan occupies five cells. Front elevation heights are max(1,2), max(3,1), max(2,0) = 2, 3, 2.

2. From the right, read front-to-back positions. The front row's maximum is 3 and back row's maximum is 2, so the right elevation has heights 3 then 2. Hidden stacks do not add extra height.

COMMON MISTAKE TO AVOID

An elevation records the tallest visible outline along a line of sight. It is not the sum of all cubes in that direction.

LESSON 21 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A block display has a front row of stack heights 2,1,3 and a back row 1,0,2, from left to right. Each block is a unit cube. Determine its cube count, footprint, front elevation and right elevation. Mark the front direction.

Pause and plan with your tutor

1. Is a zero-height cell occupied in the plan?
2. Which maximum gives each front-elevation column?
3. For the right elevation, keep front-to-back order rather than left-to-right order.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Cube count = 2 + 1 + 3 + 1 + 0 + 2 = 9. Plan has all three front cells and back-left and back-right cells.

Step 2. Front heights = max(2,1), max(1,0), max(3,2) = 2,1,3.

Step 3. Right-view heights are 3 then 2 from front to back. These are the maximum heights within each row. Sketch unit-square silhouettes and label viewing direction.

Misconception clinic

“The front elevation heights are 3,1,5 because front and back stacks are added.”

Stacks behind one another do not stand on top of one another. An elevation records the maximum visible outline along a line of sight, not the total cubes hidden behind it.

SAY IT BACK

An accurate view requires the footprint, stack heights and viewing direction together.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 21 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Stack heights: viewed from above

2	1	0
1	3	2

FRONT ↑

Back row: 2, 1, 0

Front row: 1, 3, 2

0 means no cube in that cell.

Each other value is the stack height.

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Sketch a cuboid and label length 5 cm, width 3 cm and height 2 cm.

2. Draw its plan, marking the front edge.

3. Draw its right elevation, showing front-to-back order. Explain one piece of information that would be lost if only the plan were supplied.

My check / correction and why it matters:

Model answers: page 105. Confidence: need support / can use / can explain. Next step: _____

LESSON 22 • UNDERSTAND

Angles and geometric reasoning

LEARNING GOAL • DfE LEVEL 2 POINT 22

Use angle facts and shape properties to calculate unknown values.

Angles measure turns in degrees. A right angle is 90°, a straight angle is 180° and a full turn is 360°. Acute angles are below 90°; obtuse angles are between 90° and 180°; reflex angles are between 180° and 360°.

Angles on a straight line sum to 180° and angles around a point sum to 360°. Vertically opposite angles are equal when two straight lines cross. In a triangle, the interior angles sum to 180°. In a quadrilateral, the interior angles sum to 360°.

An isosceles triangle has equal base angles. An equilateral triangle has three 60° angles. A rectangle has four right angles. With parallel lines, corresponding angles are equal, alternate angles are equal and co-interior angles sum to 180°. Only use these parallel-line rules when parallelism is given or established.

Write a reason alongside each calculation. Diagrams may be drawn out of scale. For a shape in a 3-D sketch, use the actual face properties: a rectangular face still has 90° corners even if the sketch looks slanted. Coordinates can help establish horizontal and vertical edges when given.

Worked examples: follow the reasoning

1. An isosceles triangle has apex angle 38°. The two equal base angles sum to 180 - 38 = 142°. Each is 71°.

2. At a point, angles are 90°, 125°, 65° and x. Their sum is 360°, so x = 360 - 90 - 125 - 65 = 80°. State “angles around a point” as the reason.

COMMON MISTAKE TO AVOID

Do not estimate an unknown angle from its appearance when it can be calculated. Mark the angle you are finding.

LESSON 22 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A triangular support is isosceles with an apex angle of 44°. At one base vertex, an exterior angle lies on the straight extension of the base. Find each interior base angle and that exterior angle, giving reasons.

Pause and plan with your tutor

1. Which angles are equal because the triangle is isosceles?
2. What total is available for the three interior angles?
3. How does the exterior angle relate to the adjacent interior base angle?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Interior base angles together = 180 - 44 = 136°, using the triangle angle sum.

Step 2. Equal base angles are 136/2 = 68° each.

Step 3. The adjacent exterior angle = 180 - 68 = 112°, using angles on a straight line. Check the three triangle angles: 44 + 68 + 68 = 180°.

Misconception clinic

“The exterior angle is 360 - 68 = 292°.”

The indicated exterior angle and its adjacent interior angle lie on a straight line, not alone around a complete point. The drawing’s relationship determines the rule.

SAY IT BACK

State the geometric reason next to each angle calculation.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 22 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Two adjacent angles lie on a straight line. One is 124°. Find the other.

2. An isosceles triangle has equal base angles of 54°. Find its apex angle.

3. Two parallel lines are crossed by a transversal. One acute angle is 63°. Give the corresponding angle and the adjacent obtuse angle, explaining both rules.

My check / correction and why it matters:

Model answers: page 105. Confidence: need support / can use / can explain. Next step: _____

LESSON 23 • UNDERSTAND

Median, mode and organised data

LEARNING GOAL • DfE LEVEL 2 POINT 23

Find positional averages and distinguish them from the mean.

Data are recorded observations or measurements. Discrete data are separate counts; continuous data can take values within an interval. Organise raw values before calculating, and keep any units. Sorting does not change the observations, only their order.

The median is the middle value in an ordered list. For an odd count n, use position (n + 1)/2. For an even count, average the values in positions n/2 and n/2 + 1. Count positions carefully; the middle value is not the midpoint of the minimum and maximum.

The mode is the most frequent value. There may be no single mode or more than one mode. If two different values share the greatest frequency, both are modes. The modal class is the most frequent group in grouped data; it is not an exact individual value.

The mean uses every value: add the observations and divide by their count. A frequency table shortens repeated lists, but frequency is the number of times each value occurs. To find the median from a frequency table, track cumulative counts until the middle position or positions are located.

Worked examples: follow the reasoning

1. Waiting times are 8, 3, 7, 3, 10 and 5 minutes. Ordered: 3, 3, 5, 7, 8, 10. Median = (5 + 7)/2 = 6 minutes. Mode = 3 minutes. Mean = 36 ÷ 6 = 6 minutes.

2. Scores 1, 2, 3, 4 have frequencies 2, 5, 1, 2. There are ten results. Positions 5 and 6 are both score 2, so the median is 2. The mode is also 2.

COMMON MISTAKE TO AVOID

The median need not occur in the original list. For an even count, calculate the average of the two middle values.

LESSON 23 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A library records daily requests: 0,4,2,4,5,0,6,3. Find the median and all modes. Explain why discarding the zero days changes the data rather than simplifying the calculation.

Pause and plan with your tutor

1. Put every recorded day into order, including zero days.
2. With eight observations, which two positions determine the median?
3. Does more than one value share the greatest frequency?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Ordered data: 0,0,2,3,4,4,5,6. Positions four and five contain 3 and 4.

Step 2. Median = (3 + 4)/2 = 3.5 requests. Modes are 0 and 4 because each appears twice.

Step 3. A zero is a recorded day with no requests, not a missing observation. Removing it changes both frequencies and positions, so it would describe a different set of days.

Misconception clinic

“Zero means nothing, so the sorted list should begin with 2.”

Zero is a valid observation. Only genuinely missing or invalid data should be handled separately, with the reason explained.

SAY IT BACK

Organise the entire data set before selecting positions or frequencies.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 23 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find the median and mode of 4, 9, 4, 7, 6.

2. Find the modes of 2, 3, 2, 4, 3, 5.

3. An outlying value 100 is added to 2,3,4,5,6. Find the new median and mean. Explain which is more affected.

My check / correction and why it matters:

Model answers: page 106. Confidence: need support / can use / can explain. Next step: _____

LESSON 24 • UNDERSTAND

Estimated mean from grouped data

LEARNING GOAL • DfE LEVEL 2 POINT 24

Use class midpoints and frequencies to estimate a mean and explain its limits.

A grouped frequency table replaces exact values with intervals. For discrete counts, groups such as 0-4, 5-9 and 10-14 include whole-number values. Groups must not overlap or leave unintended gaps. A frequency records how many observations belong to each group.

The midpoint represents a typical value for a class: add its endpoints and divide by two. For the class 5-9 the midpoint is 7. Multiply each midpoint x by its frequency f to estimate the combined total for that class.

Add all midpoint-frequency products and divide by the total frequency: estimated mean = total of fx ÷ total of f. Do not divide by the number of groups. A class with many observations should contribute more than a class with only one.

The result is an estimate because the exact values within a group are unknown. It is exact only in special cases where the midpoint approximation happens to match the actual total. Do not claim that every individual has the midpoint value. Use context and units to interpret the answer.

Worked examples: follow the reasoning

1. Books read: 0-4 has frequency 6; 5-9 has 10; 10-14 has 4. Midpoints are 2,7,12. Products are 12,70,48. Estimated total = 130 books; total people = 20. Estimated mean = 130/20 = 6.5 books.

2. The modal class is 5-9 because its frequency is largest. We cannot say the exact mode is 7: the class midpoint is a representative value, not necessarily the most frequent observation.

COMMON MISTAKE TO AVOID

Use the endpoints of the actual class definition. Discrete class 0-4 has midpoint 2, while continuous interval $0 \leq x < 5$ has midpoint 2.5.

LESSON 24 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A makerspace groups completed items per session: 0-3 items has frequency 4; 4-7 has frequency 10; 8-11 has frequency 6. Estimate the mean items completed per session and explain why it is not the exact mean.

Pause and plan with your tutor

1. For discrete classes 0-3, 4-7 and 8-11, what are the midpoints?
2. How does frequency affect a class's contribution to the total?
3. Which denominator counts sessions rather than classes?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Midpoints are 1.5,5.5,9.5. Products with frequencies are 6,55,57.

Step 2. Estimated total = 118 items across 20 sessions. Estimated mean = 118/20 = 5.9 items per session.

Step 3. The exact within-group values are unknown. Half-integer midpoints are still valid representatives of a group of integer counts. Do not round midpoints before using them.

Misconception clinic

"The mean is (1.5 + 5.5 + 9.5)/3 = 5.5."

That gives each class equal importance despite differing frequencies. Weight each midpoint by the number of sessions it represents, then divide by all 20 sessions.

SAY IT BACK

A grouped mean uses both representative values and the frequency weights.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 24 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Class	Frequency f	Midpoint x
0-4	6	2
5-9	10	7
10-14	4	12

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find midpoints of the discrete classes 1-5, 6-10 and 11-15.

2. For those classes with frequencies 4,8,3, calculate the products fx.

3. Explain why an estimated mean of 4.15 items does not imply that any person recorded exactly 4.15 items.

My check / correction and why it matters:

Model answers: page 106. Confidence: need support / can use / can explain. Next step: _____

LESSON 25 • UNDERSTAND

Comparing data and evaluating charts

LEARNING GOAL • DfE LEVEL 2 POINT 25 + inherited charts

Compare typical values and spread, then communicate data honestly.

The range is the largest value minus the smallest. It measures spread, not a typical value. A small range suggests results are more consistent; it does not automatically mean better performance. Whether high or low results are desirable depends on what was measured.

A useful comparison normally combines an average with a measure of spread. Name the statistic, quote values and explain the meaning in context. The median is often helpful when a few extreme values distort the mean. The mode is useful for the most common category or size.

Choose a chart that matches the data. Bar charts compare categories; bars have consistent widths and usually gaps. Line graphs show changes in an ordered variable such as time. Pie charts show parts of a whole: sector angle = category frequency ÷ total × 360°. Frequencies must sum to the stated total.

Evaluate title, labels, units, scale, sample size and source. A truncated vertical axis can exaggerate a small difference in bar heights. A chart can be numerically correct but misleading. Equal time gaps should normally have equal horizontal spacing. Distinguish observations from predictions.

Worked examples: follow the reasoning

1. Team A times are 8,9,10,11,12 minutes. Team B times are 6,9,10,11,14 minutes. Both means and medians are 10 minutes. Range A = 4 and range B = 8, so A is more consistent, with the same typical completion time.

2. In a survey of 40 people, 10 choose bus travel. Pie angle = 10/40 × 360 = 90°. A quarter of the people corresponds to a quarter of the circle.

COMMON MISTAKE TO AVOID

“Better” needs context. A greater average is desirable for output, but often undesirable for delays or faults.

LESSON 25 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

Two repair teams report job times in minutes. Team A: 14,16,18,20,22. Team B: 10,15,18,21,26. Recommend the team with more consistent performance, and state whether one has a lower mean time.

Pause and plan with your tutor

1. Choose a statistic for typical time and a statistic for spread.
2. Calculate both statistics for both teams before writing the recommendation.
3. Explain the meaning of a smaller range in this context.

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Team A mean = 90/5 = 18 minutes; range = 22 - 14 = 8 minutes.

Step 2. Team B mean = 90/5 = 18 minutes; range = 26 - 10 = 16 minutes.

Step 3. Both teams have the same mean time, so neither is faster on that measure. Team A is more consistent because its observed spread is smaller: range 8 rather than 16 minutes.

Misconception clinic

“A is faster because its range is smaller.”

Range measures variability, not average speed. Use the mean or another suitable typical value for a speed comparison, and use range for consistency.

SAY IT BACK

Make two supported comments: one about typical value and one about spread.

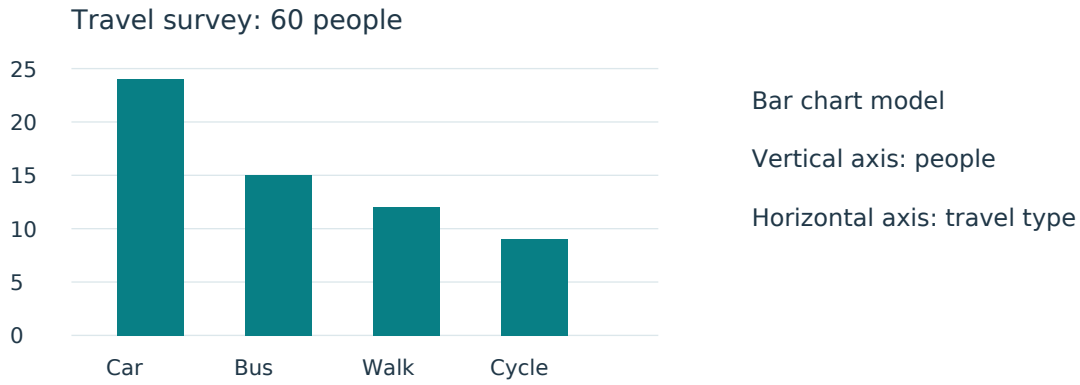
Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 25 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Bar chart model

Vertical axis: people

Horizontal axis: travel type

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Find the mean and range of 5,7,8,8,12.

2. A survey of 60 records car 24, bus 15, walk 12, cycle 9. Calculate pie angles.

3. A bar chart compares sales of 98 and 102 units but starts the vertical axis at 97. Explain the possible misleading impression and suggest an improvement.

My check / correction and why it matters:

Model answers: page 107. Confidence: need support / can use / can explain. Next step: _____

LESSON 26 • UNDERSTAND

Combined events and two-way tables

LEARNING GOAL • DfE LEVEL 2 POINT 26

Organise combined outcomes, select the correct denominator and calculate probabilities.

A probability compares favourable outcomes with all possible outcomes in the relevant set, when these are equally likely. For two events, organise outcomes using a list, sample-space grid, two-way table or tree. This avoids missing outcomes or counting one twice.

A two-way table sorts the same people or items by two characteristics. Row totals and column totals must agree with the grand total. The probability of “A and B” uses their intersection cell. For “A or B”, add the relevant counts but subtract the overlap so it is counted only once.

The words “given that” or “from those who” restrict the group. If choosing from bus users, the denominator is the number of bus users rather than everyone surveyed. Read the selection rule before deciding your denominator.

For independent repeated events, multiply probabilities along a path. With replacement, the original probabilities stay the same. Without replacement, update both relevant counts and the total after a selection. Add the probabilities of distinct, non-overlapping paths if either path satisfies the event.

Worked examples: follow the reasoning

1. Of 40 learners, 12 bus users attend mornings and 8 bus users attend evenings; 6 non-bus users attend mornings and 14 evenings. P(bus and morning) = 12/40 = 3/10. P(morning given bus) = 12/20 = 3/5.

2. A bag has 3 red and 2 blue counters. Without replacement, P(two red) = 3/5 × 2/4 = 3/10. With replacement it is 3/5 × 3/5 = 9/25. Removing a counter changes the second draw.

COMMON MISTAKE TO AVOID

Do not multiply probabilities as though events are independent when the first selection changes the second.

LESSON 26 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A club surveys 50 members. Workshop attendees: 16 use public transport and 9 do not. Non-attendees: 8 use public transport and 17 do not. A member is chosen at random from workshop attendees only. Find the chance they use public transport. Compare with selection from the whole club.

Pause and plan with your tutor

1. Which phrase restricts the selection group?
2. How many workshop attendees form the conditional denominator?
3. Which event in the whole club corresponds to the single intersection count 16?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Workshop attendees total 16 + 9 = 25. Among attendees, P(public transport) = 16/25 = 64%.

Step 2. From all 50 members, P(workshop and public transport) = 16/50 = 32%.

Step 3. If asking public transport regardless of attendance, the count is 16 + 8 = 24, so 24/50 = 48%. These are three different selection questions, not conflicting answers.

Misconception clinic

“All probability questions about this survey must use denominator 50.”

The denominator is the size of the eligible selection group. A restricted group changes it even though the original survey still contains 50 people.

SAY IT BACK

Write the selection group before the fraction so a conditional denominator is visible.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 26 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

	Morning	Evening	Total
Bus	12	8	20
Non-bus	6	14	20
Total	18	22	40

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Using the printed two-way table, find P(evening).

2. Find P(evening given non-bus).

3. A bag has 4 green and 3 yellow counters. Two are drawn without replacement. Find P(both yellow) and P(one of each colour).

My check / correction and why it matters:

Model answers: page 107. Confidence: need support / can use / can explain. Next step: _____

LESSON 27 • UNDERSTAND

Probability forms and expected frequencies

LEARNING GOAL • DfE LEVEL 2 POINT 27

Move between probability forms, use complements and interpret likelihood.

Probability lies from zero to one inclusive. Zero represents impossible and one represents certain. For equally likely outcomes, probability = favourable count ÷ total count. A probability can be written as a fraction, decimal or percentage, using the same conversions as ordinary proportions.

The complement is the event “not A”. Since either A or not A must happen, P(not A) = 1 - P(A). This is useful when the opposite event is easier to calculate. Mutually exclusive events cannot happen together in one trial; their probabilities can be added.

A theoretical probability comes from a model, such as equally likely faces on a fair die. An experimental relative frequency is observed successes divided by trials. A larger representative sample often gives a more stable estimate, but no finite sample guarantees the exact theoretical proportion.

Expected frequency = number of trials × probability. It is a long-run prediction, not a promise. Independent trials do not remember earlier results. If a fair coin lands heads five times, the next toss still has probability 1/2 of heads; it is not forced to balance immediately.

Worked examples: follow the reasoning

1. A defect probability is 3/200. Decimal = 0.015; percentage = 1.5%. For 4,000 independently produced items under the same model, expected defects = 4000 × 0.015 = 60.

2. If P(rain) = 0.28 for the stated situation, P(no rain) = 0.72 = 72%. This is a complementary event; it is not obtained by dividing 1 by 0.28.

COMMON MISTAKE TO AVOID

A probability of 0.2 is 20%, not 0.2%. Expected counts describe an average prediction across repetitions.

LESSON 27 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A quality model gives a probability of 0.035 that a notebook has a printing fault. Predict faults in a batch of 800 and write the fault probability as a percentage and simplest fraction. Explain why the batch need not contain exactly the predicted count.

Pause and plan with your tutor

1. How is a decimal converted to a percentage?
2. Which fraction represents 0.035 before simplification?
3. Does an expected frequency guarantee a single batch result?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Probability = 0.035 = 3.5% = 35/1000 = 7/200.

Step 2. Expected faults = 800 × 0.035 = 28.

Step 3. This is a model-based long-run average. An actual batch can differ because the count is random and the model may only approximate the production process. Expected non-faulty copies are 800 - 28 = 772.

Misconception clinic

“The probability is 0.035%, so practically no faults are expected.”

The decimal 0.035 is 3.5%; retaining the same digits with a percentage sign makes the value one hundred times smaller. Check that 3.5% of 800 agrees with 28.

SAY IT BACK

Convert the probability correctly before interpreting the predicted frequency.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 27 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision

Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Write 7/20 as a decimal and percentage.

2. If P(late) = 0.12, find P(not late).

3. A coin produces 62 heads in 100 tosses. Find experimental P(head). Explain why this does not prove the next toss is more likely to be tails if the coin is known to be fair.

My check / correction and why it matters:

Model answers: page 108. Confidence: need support / can use / can explain. Next step: _____

LESSON 28 • UNDERSTAND

Scatter diagrams and correlation

LEARNING GOAL • DfE LEVEL 2 POINT 28

Plot paired data, recognise correlation and use a sensible line of best fit.

A scatter diagram displays paired numerical measurements. Each point belongs to the same person, item or occasion: an x-value and its matching y-value. Keep the pairs together. Sorting one list alone would destroy the relationship.

Choose sensible axis scales that use the available space, label variables and units, then plot each pair accurately. Do not join the points in time order unless the task is specifically asking for a time-series graph. A scatter diagram investigates association between variables.

Positive correlation means larger x-values tend to accompany larger y-values. Negative correlation means larger x-values tend to accompany smaller y-values. With no clear relationship there is no apparent correlation. Strength describes how closely the points follow a trend, not whether the trend is rising or falling.

A line of best fit follows the centre of an approximately linear trend with points on both sides; it need not pass through every point or the origin. Read an estimate within the data range (interpolation). Predicting far outside the range (extrapolation) is less reliable. Correlation alone cannot show that one variable causes the other.

Worked examples: follow the reasoning

1. Training hours and scores are (1,42), (2,48), (3,53), (4,62), (5,65), (6,73). Plot hours horizontally and score vertically. The pattern has positive correlation. A reasonable trend line gives a score near 58 for 3.5 hours.

2. A temperature-versus-heating-cost chart may show negative correlation: higher temperatures accompany lower costs. This does not mean a negative cost. Another factor, such as building insulation, may influence both the pattern and its variation.

COMMON MISTAKE TO AVOID

A line of best fit is an estimate. Do not claim a precise future score or causation from a small observational data set.

LESSON 28 • CHOOSE, CALCULATE, EXPLAIN

A coached problem journey

ORIGINAL EXAM-STYLE TEACHING TASK

A facilities team records room temperature and heater energy used per day: (8,34), (10,30), (12,27), (14,22), (16,19), (18,15), with temperature in °C and energy in kWh. Explain the association and estimate use at 13°C from a reasonable trend line.

Pause and plan with your tutor

1. Which paired value belongs on each axis? Keep each day’s measurements together.
2. Would energy tend to rise or fall as temperature increases?
3. Is 13°C inside the observed range, and what would a sensible estimate imply?

Before reading the model route, describe the first useful step and its purpose.

Model route: make each link visible

Step 1. Plot temperature horizontally and energy vertically. The points follow a strong downward trend: negative correlation.

Step 2. A straight line through the centre of the points gives roughly 24-25 kWh at 13°C. Reasonable lines and estimates vary slightly.

Step 3. This is interpolation because 13°C lies between 8°C and 18°C. The association alone does not prove causation; occupancy and insulation may also affect energy use.

Misconception clinic

“Negative correlation means the energy readings must be negative.”

Negative describes the direction of association, not the signs of the measured values. Here every energy measurement is positive.

SAY IT BACK

State direction, use the plotted range and distinguish an estimate from a measured result.

Tutor prompt: What would change if a value, condition or required unit changed? Ask the learner to explain before calculating.

LESSON 28 • RECORD AND CHECK

My lesson notebook

Date: _____ Recall from the last lesson: _____

My key rule or decision



Three short checks: use the method you have just learned. Longer independent practice is in the companion practice notebook.

1. Plot the six training-hour/score pairs in the worked example on the printed grid.

2. Draw a sensible straight line of best fit and estimate score at 3.5 hours.

3. A learner says “this proves each extra hour causes exactly six extra marks”. Evaluate the claim using the graph and study design.

My check / correction and why it matters:

Model answers: page 108. Confidence: need support / can use / can explain. Next step: _____

LESSONS 01 AND 02

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

01 • Numbers, place value and negatives

1. 7,035,000; the 3 represents 30,000.

2. 0.095, 0.509, 0.590. Compare tenths, then hundredths.

3. -85 + 140 - 32 = £23. A positive balance means £23 remains.

02 • Whole-number calculation and checking

1. 58,313. Keep units, tens and hundreds aligned.

2. 84 × 30 + 84 × 6 = 2,520 + 504 = 3,024.

3. 247 ÷ 18 = 13 remainder 13, so buy 14 packs. Cost 14 × £6.40 = £89.60; 252 - 247 = 5 spare.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 03 AND 04

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

03 • Expressions and formulae

1. 7 × 6 + 4 = 46.

2. 1.8 × (-5) + 32 = -9 + 32 = 23.

3. (5 + 9) × 4 ÷ 2 = 28 square units. Add the parallel lengths before multiplying by height.

04 • Fractions, decimals and percentages

1. 3 ÷ 8 = 0.375 = 37.5%.

2. 1.25 and 1 1/4.

3. 0.035 × 100 = 3.5%. The stated percentage is ten times too large.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 05 AND 06

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

05 • Percentages of amounts

1. 10% = £45 and 2% = £9, giving £54.

2. 27 ÷ 45 × 100 = 60%.

3. Morning: 0.35 × 80 = 28. Evening: 80 - 28 = 52. Check 28 + 52 = 80.

06 • Percentage change and original amounts

1. 240 × 1.08 = £259.20.

2. (60 - 48) ÷ 48 × 100 = 25%.

3. 320 × 0.90 × 1.05 = £302.40. Reduction £17.60; 17.6 ÷ 320 × 100 = 5.5%.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 07 AND 08

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

07 • Adding, subtracting and ordering fractions

1. 18/24 = 3/4. 17/5 = 3 2/5.

2. 17/4 - 8/3 = 51/12 - 32/12 = 19/12 = 1 7/12.

3. 110/20 - 35/20 - 48/20 = 27/20 = 1 7/20 m.

08 • Fractions of quantities

1. 24/64 = 3/8.

2. 540 ÷ 12 × 7 = £315.

3. Bus 12, walk 8, cycle 12. Cyclist fraction 12/32 = 3/8; the three counts total 32.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 09 AND 10

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

09 • Decimals and sensible accuracy

1. 4.009, 4.090, 4.190, 4.900.

2. 0.074.

3. 19.2 ÷ 1.5 = 12.8, so 13 boxes. Coverage 19.5 m^2; unused 0.3 m^2.

10 • Calculating with decimals

1. 6.075 + 0.890 = 6.965.

2. 24 × 35 = 840; three decimal places gives 0.840 = 0.84.

3. 18 × 0.275 = 4.95 kg used; 1.05 kg left. Four parts need 1.10 kg, so no; shortage 0.05 kg.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 11 AND 12

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

11 • Ratio and proportion

1. 450:1200 = 3:8.

2. 600 ÷ 8 × 14 = 1,050 g.

3. 9 parts; each is 3 litres. Need 6 litres concentrate and 21 litres water. Not enough concentrate: 1 litre short.

12 • Order of operations and powers

1. 7 + 3 × 16 = 55.

2. 12 ÷ 4 + 8 = 11.

3. 3 × 25 + 1 = 76 sticks. For 100 sticks, (100 - 1) ÷ 3 = 33 squares.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 13 AND 14

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

13 • Money, tax, budgets and compound interest

1. Costs £1,160; surplus £300.

2. $800 \times 1.05^3 = £926.10$; interest £126.10.

3. $240 \times 52 \div 12 = £1{,}040$ per average month. Surplus £130.

14 • Units, conversion factors and graphs

1. 3,750 m and 0.84 litres.

2. 31.5 litres and 8 gallons.

3. $0.018 \times 1{,}000{,}000 = 18{,}000\ cm^3 = 18$ litres. Volume uses a cubic conversion; $1\ m^3 = 1{,}000$ litres.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 15 AND 16

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

15 • Speed, density, pay and time

1. 2 + 18/60 = 2.3 hours.

2. 540 ÷ 200 = 2.7 g/cm³.

3. 20 min to midnight + 75 min = 95 min = 19/12 hours. Speed 114 ÷ (19/12) = 72 km/h.

16 • Perimeter, area and composite shapes

1. Area 32.4 m²; perimeter 2(7.2 + 4.5) = 23.4 m.

2. Area 8 × 6 - 3 × 2 = 42 m². Boundary 8 + 4 + 3 + 2 + 5 + 6 = 28 m.

3. 42 × 1.08 = 45.36 m². 45.36 ÷ 2.5 = 18.144, so 19 boxes. Cost £456.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 17 AND 18

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

17 • Volume and surface area

1. Volume 120 cm³. Surface 2(40 + 24 + 15) = 158 cm².

2. $2\pi \times 2 \times 5 + 2\pi \times 2^2 = 28\pi \approx 87.96$ cm².

3. Volume 0.36 m³ = 360 litres. Water 288 litres; 288 ÷ 12 = 24 minutes.

18 • Scale drawings and actual dimensions

1. 3.8 × 200 = 760 cm = 7.6 m.

2. 1:500.

3. Actual 9 m by 6 m; area 54 m². Perimeter 30 m; fence 29 m. Cost £261.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 19 AND 20

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

19 • Coordinates in four quadrants

1. A top left, B top right, C bottom right, D bottom left. All points should be on grid intersections.

2. Area 28 square units; perimeter 22 units.

3. x = -2 + 5 - 1 = 2; y = -4 + 7 = 3. Final (2, 3).

20 • Solids, nets and views

1. 6 faces, 12 edges, 8 vertices.

2. 10π cm (about 31.42 cm) by 12 cm.

3. One valid net has four squares in a row, with one above and one below the second square. They fold into four side faces, a top and a base.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 21 AND 22

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

21 • Drawing solids, plans and elevations

1. Front 5 by 2 rectangle, depth 3 indicated along parallel receding edges; all dimensions labelled.

2. Three front cells, two back cells at left and centre; back-right cell absent.

3. Heights 3 then 2 from front to back. The plan loses stack heights; several different models can share that footprint.

22 • Angles and geometric reasoning

1. 180 - 124 = 56°; angles on a straight line.

2. 180 - 54 - 54 = 72°.

3. Corresponding angle 63° because lines are parallel; adjacent obtuse angle 180 - 63 = 117° because angles on a straight line sum to 180°.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 23 AND 24

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

23 • Median, mode and organised data

1. Ordered 4,4,6,7,9: median 6, mode 4.

2. 2 and 3, each occurring twice.

3. Ordered 2,3,4,5,6,100: median 4.5; mean 120/6 = 20. Original median and mean both 4; mean rises much more.

24 • Estimated mean from grouped data

1. 3, 8, 13.

2. 12, 64, 39.

3. Counts are whole numbers; a mean can be fractional. Grouping replaces exact observations with midpoints, so the mean is also an estimate.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 25 AND 26

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

25 • Comparing data and evaluating charts

1. Mean 40/5 = 8; range 12 - 5 = 7.

2. Each person = 6°. Car 144°, bus 90°, walk 72°, cycle 54°; sum 360°.

3. The bar heights may suggest a much larger proportional difference. Start a bar-chart frequency axis at zero, or very clearly explain any break and show the numbers.

26 • Combined events and two-way tables

1. 22/40 = 11/20.

2. 14/20 = 7/10.

3. Both yellow = 3/7 × 2/6 = 1/7. One each = 4/7 × 3/6 + 3/7 × 4/6 = 24/42 = 4/7.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

LESSONS 27 AND 28

Lesson-check answers

These answers support lesson review, not a timed test. Compare the reasoning as well as the result. For drawings, check dimensions, labels, scale and the viewing direction.

27 • Probability forms and expected frequencies

1. 0.35 and 35%.

2. 1 - 0.12 = 0.88 or 88%.

3. 62/100 = 0.62. Independent fair tosses still have P(head) = P(tail) = 1/2; earlier imbalance does not force compensation.

28 • Scatter diagrams and correlation

1. Plot (1,42),(2,48),(3,53),(4,62),(5,65),(6,73), with labelled axes.

2. A line through the centre gives about 57-59 marks; reasonable lines vary. Do not force it through the origin.

3. The data show an association, not proof of cause. Different learners and other factors may affect scores; the points do not all follow an exact six-mark increase.

Discuss your method

Which step was a decision rather than arithmetic? Explain it in a sentence. If your answer differs, decide whether the issue was reading, method, calculation, units, rounding or interpretation. Correct the cause before redoing the calculation.

READY FOR THE NEXT LESSON?

I can name the method, explain why it fits, keep units consistent and check the final result. If not, revisit the coached example with a tutor.

TURN KNOWLEDGE INTO CLEAR EVIDENCE

Before, during and after the exam

Prepare with the correct conditions

Ask your centre to confirm the awarding organisation, qualification number, paper or onscreen mode, section timings, approved calculator functions and equipment. Practise on the actual interface or official familiarisation test when available. Arrange any approved access support through your centre in advance.

Use a calculator deliberately

Estimate the answer first. Type one complete stage at a time, check the display and record the result with its meaning. Use brackets for a numerator or denominator that contains several terms. Test how your permitted calculator handles powers, negative signs and repeated operations. If a function is unavailable, use repeated multiplication or write intermediate steps.

Read command words as actions

Instruction	What your response should do
Calculate / work out	Show the numerical route, give the answer and units.
Estimate	Use sensible rounded values or an approximate model and make that clear.
Compare	Quote figures for both groups and explain a difference or similarity.
Explain / justify	Support your decision with a calculation, property or relevant evidence.
Draw / construct	Use appropriate tools, labels, scales and any stated accuracy.
Check	Use a different route, inverse operation, estimate or contextual test.

When you get stuck

Write the target quantity and list useful facts with units. Draw a diagram, form a table or find a one-unit value. Break the problem into smaller connected stages. If you still cannot finish, leave clear useful working and return after other questions; follow the permitted navigation rules of your assessment.

Final review

Check missing parts, sign errors, calculator entries, copied values, units and required rounding. Check that pack and capacity decisions provide enough, totals match, percentages use the intended base and conclusions answer the actual question. Do not change a supported correct answer merely because it looks unfamiliar.

REFERENCE BANK • NOT AN EXAM INSERT

Methods to recall

This page is for revision. Do not assume it can be taken into an assessment. Some formulae are expected knowledge; other shapes may come with a supplied formula. Follow your own board's instructions.

Topic	Relationship or method
Percentages	p% of A = A × p/100; part as % = part/whole × 100
Change	% change = change/original × 100; original = final/multiplier
Compound growth	Final = initial × (1 + r/100)^n, for fixed rate r per period
Fraction of amount	Divide by denominator, then multiply by numerator
Ratio share	One part = total/sum of parts; multiply by each ratio part
Inverse proportion	For fixed work at equal rates: workers × time = constant
Rectangle / triangle	Rectangle A = lw; triangle A = bh/2, h perpendicular
Circle	d = 2r; circumference = 2πr = πd; A = πr²
Cuboid	V = lwh; surface area = 2(lw + lh + wh)
Cylinder	V = πr²h; closed surface area = 2πrh + 2πr²
Compound measures	Speed = distance/time; density = mass/volume
Mean and range	Mean = total/count; range = maximum - minimum
Grouped mean	Estimate = sum(midpoint × frequency)/sum(frequency)
Probability	Favourable/total (equally likely); P(not A) = 1 - P(A)
Pie sectors	Angle = category frequency/total frequency × 360°
Angles	Line 180°; point 360°; triangle 180°; quadrilateral 360°

Useful conversions: 60 min = 1 h; 1 km = 1,000 m; 1 m = 100 cm; 1 kg = 1,000 g; 1 litre = 1,000 ml; 1 ml = 1 cm³; 1 m³ = 1,000 litres. For metric-imperial or currency conversions, use the factor supplied.

EVIDENCE OF LEARNING

Track progress and plan revisits

Lesson	First attempt	Retry date	Secure without notes?
01 Numbers, place value and negatives			
02 Whole-number calculation and check			
03 Expressions and formulae			
04 Fractions, decimals and percentage			
05 Percentages of amounts			
06 Percentage change and original amo			
07 Adding, subtracting and ordering f			
08 Fractions of quantities			
09 Decimals and sensible accuracy			
10 Calculating with decimals			
11 Ratio and proportion			
12 Order of operations and powers			
13 Money, tax, budgets and compound i			
14 Units, conversion factors and grap			
15 Speed, density, pay and time			
16 Perimeter, area and composite shap			
17 Volume and surface area			
18 Scale drawings and actual dimensio			
19 Coordinates in four quadrants			
20 Solids, nets and views			
21 Drawing solids, plans and elevatio			
22 Angles and geometric reasoning			
23 Median, mode and organised data			
24 Estimated mean from grouped data			
25 Comparing data and evaluating char			
26 Combined events and two-way tables			
27 Probability forms and expected fre			
28 Scatter diagrams and correlation			

Secure means you can choose a method, calculate correctly, explain why and check the result. A copied solution does not demonstrate independent performance. Record what you can do before starting extended practice.

EXPLAIN YOUR THINKING

Maths language and error log

Word	Meaning in this guide
Approximation	A value close enough for a stated purpose, often found by rounding.
Base amount	The reference quantity treated as the whole or 100%.
Compound measure	A measure combining quantities, such as distance per time.
Correlation	A pattern of association between two numerical variables.
Denominator / numerator	Bottom / top parts of a fraction; equal part size / number of those parts.
Frequency	The number of observations in a category or class.
Interpolation	Estimating within the range of recorded values.
Perpendicular	Meeting at a right angle.
Proportion	A relationship between quantities or a share of a whole.
Scale factor	A multiplier linking corresponding lengths or quantities.
Surplus / shortfall	Amount remaining / amount missing when comparing funds and costs.

Record the cause, not just the correction

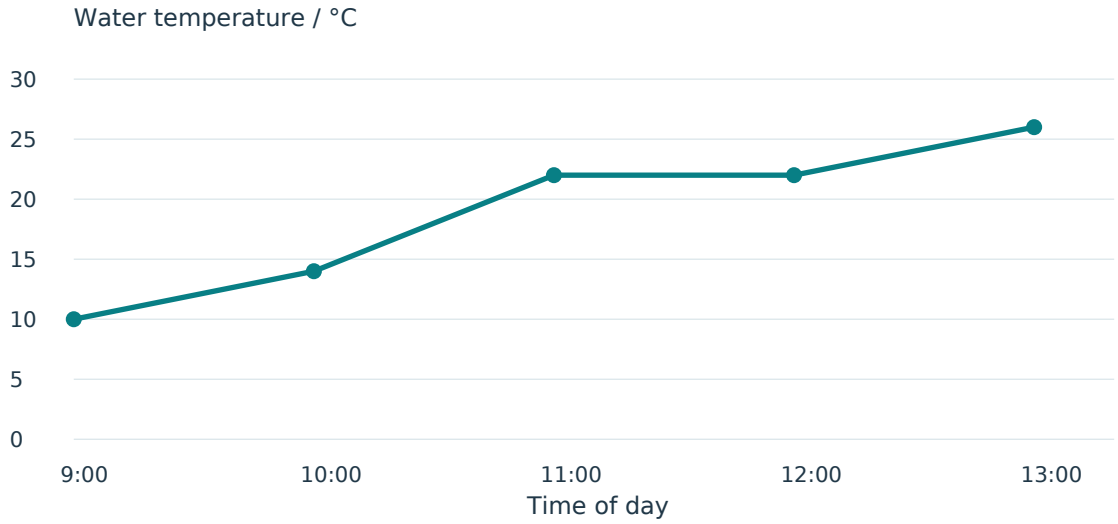
Question	What went wrong?	New rule or check	Retry date

EXTENSION A • LINK TO LESSON 25

Line graphs: change over time

A line graph suits an ordered variable such as time. Put time on the horizontal axis with equal spacing for equal durations. Plot each pair, then join consecutive points with straight segments if that representation suits the measurements. A connecting segment assumes a simple change between readings; it does not reveal every fluctuation that actually occurred.

A steep upward segment indicates a faster increase than a shallow upward segment when the axes use the same scales. A horizontal segment means no recorded change. Compare changes over equal durations, or divide change by elapsed time to compare rates. Read the units: a vertical rise of 10 could mean £10, 10°C or 10 people.



Worked interpretation

From 10:00 to 11:00 the water rises from 14°C to 22°C, an increase of 8°C in one hour. From 11:00 to 12:00 it stays at 22°C. A straight-segment estimate at 10:30 is 18°C, halfway between 14 and 22. This is an interpolation, not a measured reading.

1. Find the total temperature increase from 09:00 to 13:00 and the average rise per hour.

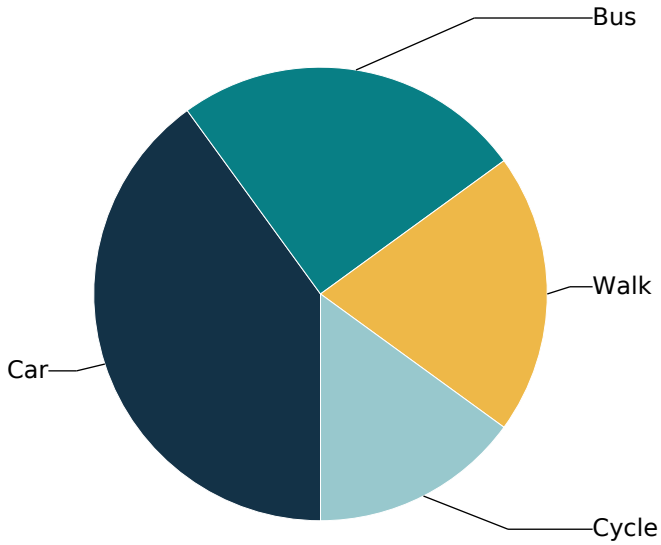
2. Estimate the temperature at 12:30. Explain why a reading at 15:00 cannot be reliably obtained from this graph alone.

EXTENSION B • LINK TO LESSON 25

Pie charts: construct a complete picture

Use a pie chart when categories form one complete whole. First verify that every person or item is counted once. Find each category fraction and multiply it by 360° for the sector angle. Draw a circle with compasses, mark a starting radius, then use a protractor for each angle. Add a title and labels or a key.

Use the travel survey from lesson 25: car 24, bus 15, walk 12, cycle 9; total 60. One person represents 360/60 = 6°. Angles are therefore 144°, 90°, 72° and 54°. Check the angles total 360° and the largest category has the largest sector.



Travel choices: 60 learners

Car: 40% → 144°

Bus: 25% → 90°

Walk: 20% → 72°

Cycle: 15% → 54°

1. A separate survey of 80 people records tea 32, coffee 28 and water 20. Calculate the three sector angles and construct a labelled pie chart on separate paper.

2. Two pie charts have equal printed size but represent surveys of 40 and 400 people. Explain why equal-sized sectors do not necessarily represent equal numbers of people.

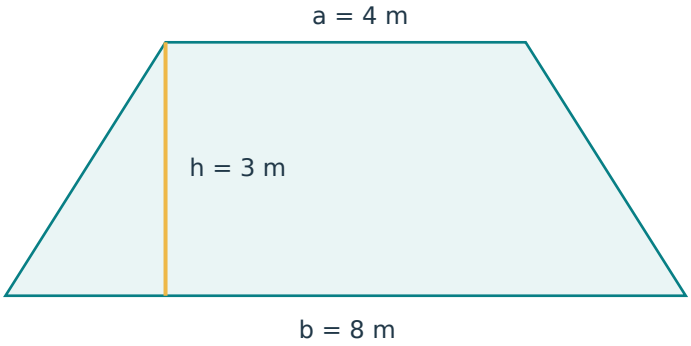
EXTENSION C • LINK TO LESSONS 16-17

Using supplied shape formulae

A formula may be supplied for an unfamiliar shape. You still need to match symbols to the diagram, convert units, select the requested quantity and apply operation order. Record each substituted value before calculating. A formula is not a reason to skip interpreting the shape.

TRAPEZIUM AREA

For parallel sides a and b and perpendicular separation h: A = (a + b)h/2. The non-parallel sloping side is not the perpendicular height.



Not to scale

Worked example: A = (4 + 8) × 3 / 2 = 18 m². To coat this surface twice at a coverage rate of 6 m² per litre requires 18 × 2 / 6 = 6 litres. Perimeter cannot be obtained from these three measurements alone without further side-length information or methods.

CONE VOLUME • SUPPLIED FOR THIS EXERCISE

V = πr²h/3. Here r is the circular base radius and h is the perpendicular height, not the sloping side.

For radius 3 cm and height 10 cm: V = π × 3² × 10 / 3 = 30π ≈ 94.25 cm³. This is one third of the volume of a cylinder with the same base and height. The units remain cubic because an area has been multiplied by a length.

1. A trapezium has parallel sides 6 m and 10 m, perpendicular height 4 m. Find its area, then the cost of covering it at £7.50 per m².

2. A cone has diameter 8 cm and perpendicular height 9 cm. Use the supplied formula to find volume to 1 decimal place.

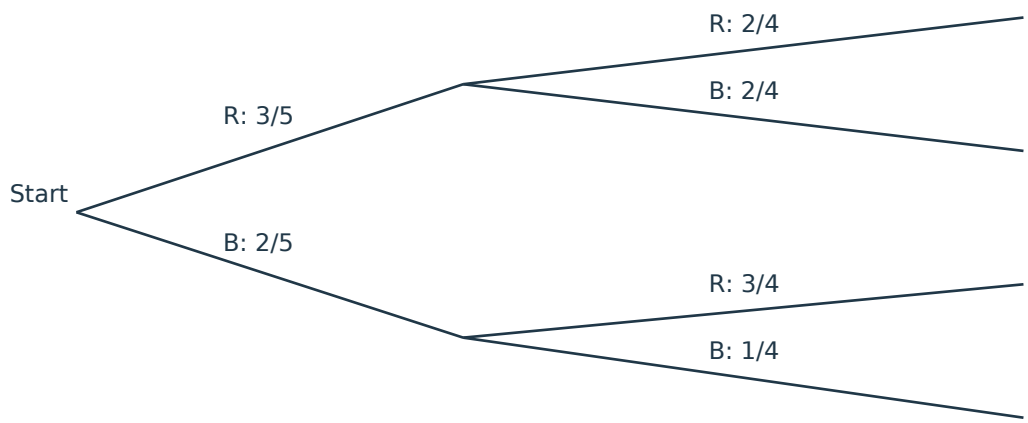
EXTENSION D • LINK TO LESSON 26

Sample spaces and probability trees

A sample-space table shows every pair of possible outcomes. Put one event along the columns and the other down the rows. If a fair coin and fair three-section spinner numbered 1, 2, 3 are independent, the six cells H1,H2,H3,T1,T2,T3 are equally likely. Head with an odd spinner number occupies two cells, giving 2/6 = 1/3.

Coin / spinner	1	2	3
Head	H1	H2	H3
Tail	T1	T2	T3

A tree diagram records conditional probabilities along branches. At each branching point the probabilities of every possible next outcome sum to one. Multiply along a complete path; add distinct successful paths when several qualify. The second-stage probabilities can differ between first-stage branches.



This tree is for 3 red and 2 blue counters drawn twice without replacement. P(one of each) = P(R then B) + P(B then R) = 3/5 × 2/4 + 2/5 × 3/4 = 12/20 = 3/5. These paths cannot both occur in the same two-draw outcome, so adding is valid.

1. List the sample space for two fair coin tosses. Find P(exactly one head).

2. For the counter tree, find P(same colour). Check it against the complement of P(one of each).

EXTENSION E • INTEGRATED NUMBER PRACTICE

Currency, unit prices and pay comparisons

Exchange rates are conversion relationships. If the exercise states £1 = €1.18, multiply pounds by 1.18 to get euros and divide euros by 1.18 to get pounds. Real providers may use different buy and sell rates and fees. For exam practice use precisely the rates and fee rules stated; the examples here do not quote current exchange rates.

Worked example: exchange £350 at €1.18 per pound after a £5 fee is deducted from the pounds. Exchangeable money = £345, so received euros = 345 × 1.18 = €407.10. If a fee were deducted from euros afterwards, the calculation would be different. Identify the order and currency of every fee.

Compare like with like

A unit price divides cost by quantity. A 750 g bag for £2.40 costs £3.20 per kg because £2.40/0.75 = £3.20. A 1.2 kg bag for £3.72 costs £3.10 per kg and is cheaper per kilogram, although its till price is higher. If the buyer needs an exact minimum amount, compare whole-pack totals as well.

Pay can combine rates and fixed charges

Quote A charges £25 plus £16 per hour. Quote B charges £19 per hour with no fixed fee. For a six-hour job, A costs £121 and B costs £114. For a ten-hour job, A costs £185 and B costs £190. There is no single cheaper quote without a stated job duration.

1. Use £1 = €1.18. Exchange £500 with a £6 fee deducted first. Find euros received. Then convert €236 back to pounds at the same stated rate, with no fee.

2. You need at least 3 kg of rice. Offer A is 750 g for £2.40; offer B is 1.2 kg for £3.72. Compare unit prices and the minimum whole-pack spend for each offer. Explain your choice if lowest spend is the only criterion.

METHODS FOR EXTENSIONS A-E

Extension answers and checking

A • Line graphs

1. Increase 26 - 10 = 16°C over 4 hours. Average rise 4°C per hour. This average does not imply a steady rise each hour.

2. At 12:30 the straight segment gives 24°C. A 15:00 value is outside the observed range and the temperature may stop rising or change at a different rate.

B • Pie charts

1. One person represents 360/80 = 4.5°. Tea 144°, coffee 126°, water 90°. Check sum 360°. Construct with protractor, title and labels.

2. Equal angles mean equal proportions. A quarter-sector represents 10 people in a 40-person survey and 100 in a 400-person survey.

C • Supplied formulae

1. Area = (6 + 10) × 4/2 = 32 m². Cost = 32 × £7.50 = £240.

2. Radius = 8/2 = 4 cm. Volume = π × 4² × 9/3 = 48π = 150.8 cm³ to 1 dp.

D • Probability diagrams

1. HH,HT,TH,TT are equally likely. Exactly one head: HT or TH, so 2/4 = 1/2.

2. Same colour: 3/5 × 2/4 + 2/5 × 1/4 = 8/20 = 2/5. Check 1 - 3/5 = 2/5.

E • Currency and value

1. (£500 - £6) × 1.18 = €582.92. €236/1.18 = £200.

2. A: £3.20/kg; four packs give 3 kg for £9.60. B: £3.10/kg; three packs give 3.6 kg for £11.16. B has the lower unit price, but A meets the requirement for £1.56 less at the till.

MORE THAN A CORRECT NUMBER

For a comparison, state the criterion. For a model, state an assumption. For a conversion, state the unit. For a diagram, use labels and a scale.

ASSESSMENT LITERACY • TEACHING, NOT PREDICTION

What the official papers ask you to do

Official materials reviewed for this edition include City & Guilds Level 2 Sample 4 calculator paper; Open Awards Level 2 sample assessment with mark scheme; Pearson Level 2 scaffolded long-question support and assessor exemplars; and NCFE's report on the December 2019 assessments. These are samples, teaching resources and a historical examiner report, not a set of current live papers. [P1-P5]

The sources support a teaching approach that develops short calculations as well as longer connected decisions. They do not establish how often a topic will appear in your next paper. Exact mark awards and accepted alternatives depend on the relevant official mark scheme.

Question demand	What to learn here	Where it is developed
Select and interpret information	Name the required output; locate relevant values, units and conditions.	Every coached page; lessons 3, 5, 14, 26
Connect several processes	Write intermediate quantities and carry them into the next calculation.	Lessons 2, 6, 13, 15-18
Construct or interpret a diagram	Use a scale, labels, dimensions and the correct view or axes.	Lessons 18-22, 25, 28; extensions
Justify a conclusion	Use a figure and a comparison criterion, rather than only yes or no.	Lessons 1, 9, 13, 25, 28
Make method visible	Record substitutions, missing dimensions, units and checking steps.	All model routes and notebook pages

Access limitations and rights

City & Guilds lists released past papers publicly, but the Past Paper 5 calculator PDF could not be read reliably during this review. It was not used as evidence for question content. NCFE states that some newer assessment materials need centre access. No attempt was made to use restricted or live papers. [P6, P7]

All coached tasks in this notebook are independently written, with their own contexts, data and solution routes. They practise comparable mathematical skills rather than reproducing official questions, tables, diagrams or mark schemes. Follow the official links for authentic full-paper experience.

TUTOR GUIDE AND LEARNER HANDOVER

From a lesson to independent practice

1. Make thinking audible

Ask the learner to explain what the question wants and why a calculation will help. If they name an operation without a reason, ask them to describe the quantities it connects. Use the planning prompts before revealing the model route, then compare a different valid method where possible.

2. Reduce support gradually

First model a step with a reason. Next let the learner supply a missing step while you ask only a prompt. Finally cover the model and have them explain the whole route. Do not treat a correct copied result as independent understanding. Use the three short checks as a conversation about understanding, not a pass prediction.

3. Diagnose a specific misconception

The clinic gives an incorrect claim followed by a repair. Ask the learner to identify the exact point where the reasoning fails, then provide a counterexample or a unit check. Extend this by changing one condition: a discount becomes an increase; selection becomes restricted; a container loses its lid; a time interval includes a break.

4. Move to the separate practice notebook

Keep lesson numbers aligned between the two books. The companion should begin with topic practice, progress to connected unfamiliar problems, then include mixed non-calculator and calculator sets. Remove scaffolding gradually. Add timed mock papers only after the learner can choose methods without prompts. The practice notebook has not been created as part of this edition.

MY LEARNING HANDOVER

Lesson: ____ Method I can explain: _____
A condition I must notice: _____
An error I can now diagnose: _____
Practice I need next: _____
Tutor comment and revisit date: _____

Before using an official assessment, confirm its current status and your own board’s rules. Historical examiner feedback identifies useful teaching priorities, but is not evidence of a current universal pass mark or current exam timing.

OFFICIAL SOURCES • 1 OF 3

Sources and alignment record

Sources accessed 13 November 2026. Links are clickable. Review the version and current status before use; publication dates and qualifications can change. Source references document the curriculum basis, not endorsement of this publication.

S1 • DfE: Maths Functional Skills subject content

Updated 8 November 2024. Shared Level 2 content points 1-28 and lower-level foundations.

Open official source

S2 • Ofqual: Functional skills mathematics conditions and requirements

Updated 1 May 2024. Regulatory content compliance, assessment weightings and qualification rules.

Open official source

S3 • Pearson Edexcel: Mathematics Level 1 and Level 2 specification

2019 reformed specification retrieved; qualification page confirms Level 2 QN 603/4268/7.

Open official source

S4 • City & Guilds: Functional Skills 4748 documents

Handbook v1.3, August 2025, retrieved; qualification page also lists October 2026 Level 2 delivery guidance.

Open official source

S5 • NCFE: Maths Level 1 and Level 2 qualification specification

Specification v2.0 retrieved from the official Level 2 qualification page.

Open official source

OFFICIAL SOURCES • 2 OF 3

Sources and alignment record

S6 • NOCN: Maths Level 1 and Level 2 specification

Version 1.7, March 2026, retrieved; URL filename still contains an earlier version number.

Open official source

S7 • Open Awards: Level 2 Mathematics qualification guide

Guide retrieved from the current qualification page for 603/4806/9.

Open official source

S8 • Highfield: Reformed Maths qualification specification

Version 1.4 retrieved. Confirm any later amendments before relying on assessment arrangements.

Open official source

S9 • FutureQuals: Level 2 Mathematics specification

Version 6 retrieved for 603/4804/5. The current learning-aim record also identifies this qualification; latest delivery rules need centre confirmation.

Open official source

S10 • VTCT Skills: Level 2 Mathematics qualification page

Current award page retrieved for 603/4808/2. Skillsfirst RFSM2 handbook v3, 23 August 2019, also retrieved; confirm later amendments with VTCT Skills.

Open official source

Open older Skillsfirst RFSM2 handbook

OFFICIAL SOURCES • 3 OF 3

Sources and alignment record

S11 • TQUK: Maths Level 1 and Level 2 specification

Version 1.2 retrieved. Suite webpage redirects to vqa.org/functional-skills-suite; confirm current awarding name and documents with the centre.

Open official source

S12 • AQA: Functional Skills withdrawal status

Official page states qualifications withdrawn in August 2025.

Open official source

S13 • OCR: Reformed Functional Skills withdrawal

Official announcement dated 19 February 2020.

Open official source

S14 • Gateway Qualifications: legacy Maths specification

Version 3.1, February 2018, QN 601/7860/7. Located but excluded from current alignment.

Open official source

Editorial scope and further verification

The guide maps all 28 shared Level 2 content points to original teaching and practice. This establishes a cross-board content approach; it is not a complete audit of all current awarding-organisation specifications. Document retrieval limitations are recorded on pages 7-8. Confirm centre-specific rules and current versions before printing an edition for learners.

The attached commercial maths book was not used as a text or question source. The supplied branded physics notebook provided the brand identity and broad visual direction. No commercial page or official assessment item is reproduced.

Before commercial release: confirm the publishing entity and rights notice, arrange an independent subject-specialist proofread, and confirm the latest VTCT handbook and any other later amendments. Keep a dated record of amendments so learners can identify the edition they hold.

PAPER AND TEACHING RESOURCE LINKS • 13 NOVEMBER 2026

Official assessment materials reviewed

P1 • City & Guilds: Level 2 Sample 4, calculator paper

Public sample paper, not a current live exam. Reviewed for breadth of question demand.

Open official material

P2 • Open Awards: Level 2 sample assessment and mark scheme

Published sample package retrieved from the current qualification page; internal sample dates identify September 2019.

Open official material

P3 • Pearson: Level 2 scaffolded calculator long questions, Paper 1

Teaching resource dated 2023; reviewed for how connected calculations can be broken into teachable steps.

Open official material

P4 • Pearson: You Are The Assessor, Level 2 Maths Question 1

Assessor exemplars reviewed for the value of showing method; no marks or learner responses reproduced.

Open official material

P5 • NCFE: Chief examiner report, 9-13 December 2019

Historical assessment feedback published January 2020. Used to identify teaching priorities; not current assessment rules.

Open official material

P6 • City & Guilds: Functional Skills 4748 resources

Lists released past papers. Past Paper 5 PDF was not reliably readable during this review.

Open official material

P7 • NCFE: Level 2 Mathematics qualification and assessment materials

Some newer sample materials require authorised centre access.

Open official material