GCSE ONLINE TUTORING

IN PARTNERSHIP WITH RS REMOTE TUTORING

HIGHER MATHEMATICS

FORMULAE, METHODS, EXACT VALUES & SELECTION HANDBOOK

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What is supplied vs what must still be understood

The Higher formulae list supplies circle equations, scalar-product formulae, addition/double-angle formulae and standard derivatives/integrals for sin(ax) and cos(ax). [FORM] A formula being supplied does not remove the need to recognise when it applies, handle algebra correctly, use radians in calculus, or combine it with National 5 knowledge such as exact trig values and core identities.

01. Circle

(x-a)^2 + (y-b)^2 = r^2

Use / condition: Centre (a,b), radius r. Use for identifying or constructing a circle.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

02. Circle expanded

x^2+y^2+2gx+2fy+c=0

Use / condition: Centre (-g,-f), radius sqrt(g^2+f^2-c).

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

03. Scalar product

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$

Use / condition: Find angle between vectors or test perpendicularity (dot product 0).

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

04. Scalar product components

$\mathbf{a}\cdot\mathbf{b} = a_1b_1+a_2b_2+a_3b_3$

Use / condition: Coordinate/vector-component route to scalar product.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

05. Trig addition

$\sin(A\pm B)=\sin A\cos B \pm \cos A\sin B$

Use / condition: Use for compound-angle manipulation.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

06. Trig addition

$\cos(A\pm B)=\cos A\cos B \mp \sin A\sin B$

Use / condition: Use for compound-angle manipulation.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

07. Double angle

$\sin 2A=2\sin A\cos A$

Use / condition: Convert products/angles when solving or proving.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

08. Double angle

$\cos 2A=\cos^2 A-\sin^2 A = 2\cos^2 A-1 = 1-2\sin^2 A$

Use / condition: Choose the form that best matches the expression.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

09. Derivative

$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$

Use / condition: Angles in radians for calculus.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

10. Derivative

$d/dx[\cos(ax)] = -a\sin(ax)$

Use / condition: Angles in radians for calculus.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

11. Integral

$\int \sin(ax)dx = -(1/a)\cos(ax)+c$

Use / condition: Remember reciprocal factor and +c for indefinite integrals.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

12. Integral

$\int \cos(ax)dx = (1/a)\sin(ax)+c$

Use / condition: Remember reciprocal factor and +c.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

13. Gradient/angle

$m = \tan\theta$

Use / condition: θ is measured from the positive direction of the x-axis.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

14. Discriminant

$\Delta=b^2-4ac$

Use / condition: Δ>0 two real roots; Δ=0 repeated real root; Δ<0 no real roots.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

15. Straight line

$y-y1=m(x-x1)$

Use / condition: Use for tangent, perpendicular bisector, median/altitude line equations.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

16. Perpendicular gradients

$m1\ m2 = -1$

Use / condition: For non-vertical perpendicular lines.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

17. Chain rule

dy/dx = (dy/du)(du/dx)

Use / condition: Differentiate composite functions.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

18. Power derivative

d/dx(x^n)=n x^(n-1)

Use / condition: Apply to positive, negative and fractional powers.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

19. Power integral

∫x^n dx = x^(n+1)/(n+1)+c, n≠-1

Use / condition: Foundation for algebraic integration.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

20. Definite integral

$\int_a^b f(x)dx = F(b)-F(a)$

Use / condition: Use full brackets around substituted limits.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

21. Recurrence limit

If $u_{n+1}=f(u_n)$ and a limit L exists, then $L=f(L)$

Use / condition: This equation is necessary; existence/convergence still needs interpretation.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

22. Power model

$y=ax^b \Rightarrow \log y = \log a + b \log x$

Use / condition: Straight-line plot of log y against log x: gradient b, intercept log a.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

23. Exponential model

$y=ab^x \Rightarrow \log y = \log a + x \log b$

Use / condition: Straight-line plot of log y against x: gradient log b.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

24. Wave function

$a \cos x + b \sin x = k \cos(x-\alpha)$ or $k \sin(x+\alpha)$

Use / condition: $k=\sqrt{a^2+b^2}$; determine α by coefficient matching with correct quadrant.

Selection cue: Before substituting, identify the mathematical object (line, circle, vector, composite function, trig expression, rate or accumulated area) and state why this relationship fits.

Common trap: Using the correct-looking formula with the wrong angle convention, missing brackets, wrong sign, omitted constant of integration, or inconsistent line-to-line algebra.

Check: Verify dimensions/structure, substitute back where possible, and compare the result with the expected graph/geometry/sign/range.

Exact-value and non-calculator readiness

- Know exact sin/cos/tan values for standard angles inherited from National 5 and be able to work with them in degrees and radians.
- Simplify surds, fractions and negative-number arithmetic without decimal approximation.
- Know $\sin^2x+\cos^2x=1$ and $\tan x=\sin x/\cos x$; these may be needed even when not printed on the Higher formula list.
- Be fluent converting 30°, 45°, 60°, 90°, etc. to radians when a calculus or trig equation is expressed in radians.

Method-selection map

Nature of roots / tangent condition: Discriminant

Angle between vectors: Scalar product

Composite derivative: Chain rule

Area between curves: Intersection points + upper-minus-lower definite integral

Maximum/minimum in interval: Derivative + stationary points + endpoints

Power-law data: Log-log straight-line model

Exponential data: log y against x model

Combined sin/cos expression: Wave function or addition formulae

Circle tangent: Radius perpendicular to tangent + line/circle algebra

References and source notes

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